Supplementary runaway greenhouse problem

January 27, 2014

Problem 0.1 *Mathematical properties of the integral for gray-gas limiting OLR.*

The limiting OLR at high temperatures for a saturated single-component atmosphere is $OLR_\infty = \sigma T_0^4 f(A, \tau_\infty)$, where

$$f = \int_{\Delta\tau=0}^{\tau_\infty} \frac{\exp(-\Delta\tau)}{(1 - A \ln \Delta\tau)^4} d\Delta\tau,$$

where T_0 is the saturation ("dew point") temperature at pressure p_0 and p_0 is chosen such that $\kappa p_0/g = 1$. With this definition of T_0, $A = RT_0/L$, which is generally a small number. Note that because p_0 depends on the absorption coefficient κ, A depends on the radiative as well as the thermodynamic properties of the gas making up the atmosphere. The object of this exercise is to understand why f, and hence OLR_∞, is essentially independent of τ_∞.

(a) To get a feel for what range of $\Delta\tau$ determines the value of the integral, plot the integrand vs $\Delta\tau$ for $A = .01$, $A = .1$ and $A = 1$. (Note that the latter value is physically unrealistic but is included to make a mathematical point). Carry your plot out to the point where the integrand becomes singular. Based on these plots, for what range of τ_∞ to you expect the integral to be independent of τ_∞? How does this depend on the value of A?

(b) Evaluate the integral numerically and plot the results as a function of τ_∞ for various values of the coefficient A. Discuss your results.

(c) The τ_∞ dependence of the integral at very large τ_∞ is a spurious artifact of the $T(p)$ singularity that comes from having neglected the critical point. Suppose that at very large p, $T(p)$ follows some formula that remains finite for all p, but increases without bound as p increases. What condition does $T(p)$ have to satisfy in order for the integral to be independent of τ_∞ out to arbitrarily large values?