

Chapter 6

On the Relative Humidity of the Atmosphere

Raymond T. Pierrehumbert, H  l  ne Brogniez,
and R  my Roca

6.1. Introduction

Water is a nearly miraculous molecule that enters into the operation of the climate system in a remarkable variety of ways. A wide-ranging overview can be found in Pierrehumbert (2002). In the present chapter, we will be concerned only with the radiative effects of atmospheric water vapor on climate, and with the kinematics of how the water vapor distribution is maintained. These effects are important because feedbacks due to changes in atmospheric water vapor amplify the climate system's response to virtually all climate forcings, including anthropogenic and natural changes in CO₂, changes in solar luminosity, and changes in orbital parameters. In contrast to cloud feedbacks, which differ greatly amongst general circulation models, clear-sky water vapor feedback is quite consistent from one model to another. Essentially all general circulation models (GCMs) yield water vapor feedback consistent with that which would result from holding *relative humidity* approximately fixed as climate changes (Held and Soden 2000; Colman and McAvaney 1997). This is an emergent property of the simulated climate system; fixed relative humidity is not in any way built into the model physics, and the models offer ample means by which relative humidity could change.

Why should models with such diverse representations of moist processes yield such similar results when it comes to water vapor feedback? The answer to this question has a considerable bearing on the extent to which one can trust models to faithfully reproduce the water vapor feedback occurring in nature, and in particular in climates that haven't yet been directly observed. There is much indirect evidence that the water vapor feedback in models is correct, and indeed no compelling reason has emerged to doubt it. Nonetheless, it has proved difficult to articulate cleanly and convincingly from basic principles exactly why one should have confidence in this aspect of the models. If the atmosphere were saturated at all levels, understanding water vapor feedback would offer few challenges. The difficulty arises from the prevalence of highly unsaturated air in the atmosphere. To make progress, one needs better conceptual models of the factors

governing the distribution of subsaturated air in the atmosphere. Once mechanisms are understood, prospects for determining whether key processes operate in the same way in models as in the real world become brighter.

Another reason for seeking a better mechanistic understanding of the relative humidity distribution is that such an understanding is a prerequisite for credible incorporation of water vapor feedback in idealized climate models. Despite advances in computer power, energy balance models and their somewhat embellished cousins the “intermediate complexity models” continue to have an important role in exploration of new ideas. When simulations of tens or hundreds of thousands of years are called for, they are indispensable. In thinking about representation of the hydrological cycle in idealized models, one must distinguish between the requirements of modeling water for such purposes as computing precipitation and latent heat transport, and the requirements of modeling water for the purpose of determining its radiative impacts. The former is a challenging enough task, but involves primarily low-altitude processes where most of the atmosphere’s water is found. The latter is yet more formidable, as it requires modeling the relatively small proportion of water found in the upper reaches of the troposphere. Even in idealized models that attempt some representation of the hydrological cycle (Petoukhov et al. 2000; Weaver et al. 2001), the dynamic influences on water vapor feedback are generally neglected.

The aim of this chapter is to present some simple but quantifiable ideas concerning the way in which the atmosphere’s population of unsaturated air emerges from the interplay of transport and condensation. The key insight is to think of the water content of a parcel of air as resulting from a stochastic process operating along air parcel trajectories. The various incarnations of this idea that will be discussed in this chapter account for the prevalence of dry air, offer some insight as to how the relative humidity distribution may change in response to a changing climate, and yield some diagnostic techniques that can be used as a basis for comparing moisture-determining processes in models and the real world. Through detailed analysis of an idealized version of the stochastic problem, we expose some of the mathematical challenges involved in formulating moisture parameterizations for idealized climate models, and suggest a possible approach to stochastic modeling of moisture. The class of theories we consider here deals only with those aspects of moisture that can be inferred on the basis of large-scale wind and temperature fields such as can be explicitly resolved by general circulation models and global analysis/assimilation datasets. This is only a piece of the puzzle, but we think it is a large piece. The whole complex of important issues concerning the way deep convection injects moisture into the atmosphere, and the sensitivity of such processes to microphysics and parameterization (e.g., *Tompkins and Emanuel [2000]*; see also chapters 7 and 11 in this volume) is left untouched for the moment, but must at some point be brought back into the picture.

We begin by reviewing some basic material concerning the radiative importance of water vapor in sections 6.2 and 6.3. The essential ideas governing the generation of

subsaturated air are introduced in section 6.4. A simple class of models embodying these ideas is analyzed in section 6.5, where we also analyze a more conventional diffusive model for the sake of comparison and contrast. In section 6.6 the concepts are applied to diagnosis of moisture dynamics in the present climate, and in GCM simulations of warming in response to elevation of CO₂.

6.2. Basic Properties of Water Vapor

There are two basic aspects of the physics of water vapor that give it a distinguished role in determining the sensitivity of the Earth's climate. The first aspect is common to all substances that undergo a phase change: the maximum partial pressure of water vapor that can be present in a volume of atmosphere in thermodynamic equilibrium is a strongly increasing function of temperature. This maximum is known as the *saturation vapor pressure*, e_s , and is governed by the Clausius-Clapeyron relation. For water vapor at modern terrestrial temperatures, the Clausius-Clapeyron relation is well approximated by the exponential form

$$e_s(T) = e_s(T_o) \exp^{-\frac{L}{R_v}(\frac{1}{T} - \frac{1}{T_o})}, \quad [6.1]$$

where L is the latent heat of the appropriate phase transition (vapor to liquid at warm temperatures, vapor to solid at sufficiently cold temperatures), R_v is the gas constant for water vapor, and T_o is a reference temperature. At the freezing point, e_s is 614 Pa or 6.14 mb; $L/R_v = 5419$ K for condensation into liquid and 6148 K for condensation into ice. The formula predicts a very strong dependence of saturation vapor pressure on temperature. At 300 K, e_s rises to 3664 Pa, whereas at 250 K, e_s drops to a mere 77 Pa. Once the partial pressure of water vapor reaches the saturation vapor pressure, any further addition of water vapor will lead to condensation sufficient to bring the vapor pressure back down to saturation. The condensed water may remain in suspension as cloud droplets, or it may aggregate and be removed from the atmosphere in the form of precipitation. Because specific humidity is a materially conserved quantity until condensation occurs, it is convenient to define the *saturation specific humidity*, q_s , which is the ratio of the mass of water vapor that could be held in saturation to the total mass of saturated air. In terms of saturation vapor pressure, $q_s = 0.622e_s(T)/(p_a + 0.622e_s(T))$, where p_a is the partial pressure of dry air. Condensation occurs whenever the specific humidity q exceeds q_s .¹

The Clausius-Clapeyron relation provides a powerful constraint on the behavior of water vapor, but it is not at all straightforward to tease out the implications of this constraint for climate, for the reason that it only gives an *upper bound* on the water vapor content for any given temperature, and tells us nothing about how closely that bound might be approached.

The second key aspect of water vapor is that it is a potent greenhouse gas. Like most greenhouse gases, the effect of water vapor on outgoing longwave radiation (OLR) is approximately logarithmic in specific humidity, once the concentration is sufficiently large to saturate the principal absorption bands. The logarithmic effect of water vapor is somewhat more difficult to cleanly quantify than is the case for well-mixed greenhouse gases like CO_2 , but if one adopts a base-case vertical distribution and changes water vapor by multiplying this specific humidity profile by an altitude-independent factor, one finds that each doubling of water vapor reduces OLR by about 6 W/m^2 (Pierrehumbert 1999). This is about 50% greater than the sensitivity of OLR to CO_2 . A comprehensive analysis of the latitude and altitude dependence of the sensitivity of OLR to water vapor can be found in Held and Soden (2000). The idea that small quantities of water vapor can have a lot of leverage in climate change has a fairly long history, and is now widely recognized. Water vapor feedback was included in the very first quantitative calculations of CO_2 -induced warming by Arrhenius, and the importance of water vapor aloft was implicit in such calculations, as it was in the first comprehensive radiative-convective simulation of the problem by Manabe and Wetherald (1967). However, consciousness of the importance of free tropospheric humidity, and of the difficulty of understanding it, did not really awaken until the early 1990s. Significant early work on the subject appeared in Soden and Bretherton (1993) and Shine and Sinha (1991), partly in response to some salutary, if ultimately easily dismissed, skepticism about global warming expressed in Lindzen (1990). Related points have been discussed in Spencer and Braswell (1997), Pierrehumbert (1999), and Held and Soden (2000), among others.

One important consequence of the logarithmic dependence is that the relatively small amount of water vapor aloft nonetheless has a great influence on the radiation budget. Another consequence is that the degree of dryness of dry air also has a great effect on the radiation budget: reducing free tropospheric humidity from 5% to 2.5% in a dry zone has nearly the same radiative impact as reducing humidity from 40% to 20%, even though the former reduction involves a much smaller quantity of water. A related consequence of the logarithmic dependence is that the radiative effect of water vapor cannot be accurately determined on the basis of the mean specific humidity alone. Fluctuations in specific humidity, e.g., fluctuations at scales smaller than resolved in a climate model, affect the OLR to some extent. If we denote the specific humidity by q and time or space averages by angle brackets, then $\langle \log q \rangle \neq \log \langle q \rangle$. Specifically, writing $q = \langle q \rangle + q'$,

$$\left\langle \log \left(\langle q \rangle \left(1 + \frac{q'}{\langle q \rangle} \right) \right) \right\rangle \approx \log \langle q \rangle - \frac{1}{2} \left\langle \left(\frac{q'}{\langle q \rangle} \right)^2 \right\rangle < \log \langle q \rangle \quad [6.2]$$

Since OLR is proportional to $-\log q$, fluctuations in water vapor *increase* the OLR and have a cooling effect. Having some very dry air and some very moist air allows more infrared cooling than would the same amount of water spread uniformly over the

atmosphere. To provide some idea of the magnitude of this effect, consider a region of the atmosphere within which the temperature is horizontally homogeneous, and where the relative humidity is 50% everywhere. Next, increase the humidity in one half of the region to 87.5% while keeping the total water in the system constant. This requires a reduction of the rest of the region to 12.5% relative humidity. One not-quite doubles the humidity in half of the region, while reducing the humidity in the rest by a factor of 1/4, yielding a net increase of OLR relative to the uniform state of 3.6 W/m^2 , based on the sensitivity factor given above. If we make the dry air still drier, reducing it to 6.25% while increasing the moist air to 93.75%, the OLR increase becomes 11 W/m^2 .

Water vapor affects climate sensitivity through its effect on the slope of OLR vs. surface temperature. Following Held and Soden (2000), the change ΔT in temperature caused by a change ΔF in radiative forcing can be written $\Delta T = \Lambda \cdot \Delta F$, where the sensitivity coefficient is given by

$$\Lambda = \frac{1}{(1 - \beta) \frac{\partial}{\partial T} \text{OLR}|_{\text{specific humidity fixed}}}. \quad [6.3]$$

If free tropospheric water vapor increases with temperature, the additional greenhouse effect reduces OLR compared to what it would be with water vapor fixed, whence the water vapor feedback factor β is positive, and climate sensitivity is increased.

General circulation model simulations of the change in the present climate caused by increasing CO_2 universally show *polar amplification*, in the sense that the temperature increase at high latitudes is greater than that at low latitudes. Observations of climate change over the past century seem to conform to this expectation. It is important to note that water vapor feedback does not contribute to polar amplification. In fact, for the present pole-to-equator temperature range, water vapor feedback makes the sensitivity Λ *smaller* at cold temperatures than at warm temperatures (see Pierrehumbert [2002]), and hence would lead to tropical rather than polar amplification. Certainly, ice albedo feedback plays a role in polar amplification, but dynamical heat transport and clouds may also contribute. These feedbacks must be sufficiently strong to overcome the tendency of water vapor feedback to put the greatest warming in the Tropics.

6.3. The Importance of Water Vapor to the Radiation Budget

In this section we present a few reminders of the central role that water vapor radiative effects play in determining the Earth's climate. We begin by examining the way in which CO_2 , water vapor, and the presence of subsaturated air individually affect the radiation balance of the Earth, as measured by OLR. To do this, we used the NCAR CCM3 radiation model (Kiehl et al. 1998) to compute what the Earth's OLR would be under various assumptions. For each case, clear-sky OLR was computed on a latitude-longitude grid, with the 3-D temperature pattern held fixed at the January monthly mean climatology derived from NCEP reanalysis data (Kalnay et al. 1996) for

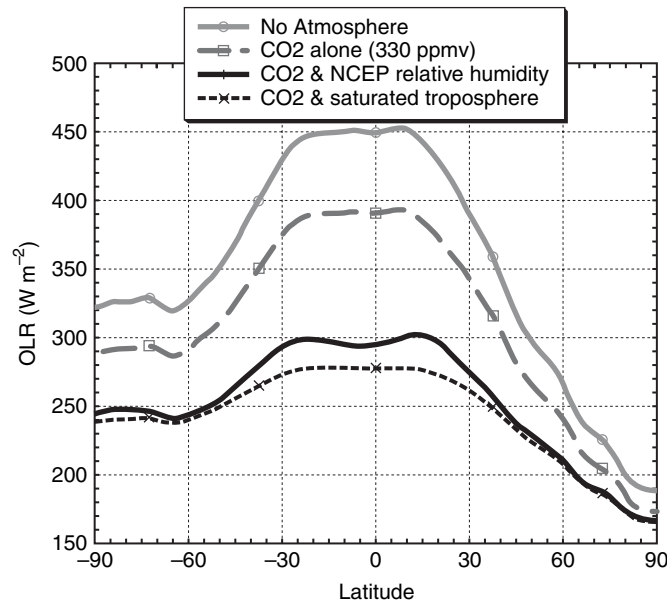


FIGURE 6.1. The effect of atmospheric composition on OLR, with atmospheric and surface temperature held fixed at January climatological values, 1960–1980.

1960–1980. The zonal mean results, shown in Fig. 6.1, are derived from averages of latitude-dependent OLR, rather than being computed on the basis of zonal-mean temperature fields. This is important, because radiation is nonlinear in temperature and humidity. We have not, however, taken the effect of daily fluctuations into account. This introduces some error into the analysis, particularly in the midlatitudes.

As compared to the calculation with no atmospheric greenhouse effect whatsoever, CO_2 by itself brings the tropical OLR down by 60 W/m^2 . The OLR reduction decreases with latitude in the extratropics, falling to 25 W/m^2 at 60°S in the summer hemisphere, and even smaller values in the winter extratropics. The latitudinal variation in the CO_2 greenhouse effect derives from the vertical structure of the atmosphere: in the Tropics there is more contrast between surface and tropopause temperature than there is in the extratropics, and the summer extratropics has more contrast than the winter extratropics. When OLR is recalculated with the observed humidity content of the atmosphere (based on NCEP) in addition to the CO_2 from the previous case, OLR drops by an additional 100 W/m^2 in the Tropics, and a lesser amount in the extratropics. In fact, at each latitude the greenhouse effect of water vapor is approximately twice that of CO_2 . As noted previously, the atmosphere is highly unsaturated. If we recompute the OLR assuming a saturated troposphere, the OLR is further reduced. The effect is considerable in the Tropics, amounting to over 20 W/m^2 of additional greenhouse effect. In the summer midlatitudes, the effect of saturating the atmosphere is much weaker, amounting to only 7 W/m^2 at 45°S . This is because the monthly mean relative humidity above the boundary layer is already on the order of 50% in the summer extratropics,

so one is only doubling the assumed moisture content there. The winter extratropics shows a somewhat weaker effect, on the order of 5 W/m^2 . These results underestimate the effect of dry air on midlatitude OLR because the more extreme dry-moist contrast seen in daily data is associated with mobile synoptic systems that average out in the monthly means.

Thus, a much drier atmosphere has the potential to be considerably colder than the present climate, whereas a saturated atmosphere has the potential to be considerably warmer. To translate this statement into more quantitative terms, we have carried out a series of GCM experiments in which the water vapor content of the atmosphere has been artificially altered in the radiation calculation alone. This approach allows one to study the radiative impacts of water vapor in isolation, without needing to confront the formidable task of disentangling them from the myriad other complex influences of the hydrological cycle on climate. It is essentially the same technique employed by Hall and Manabe (1999) in their study of water vapor feedback. Our experiments were carried out with the Fast Ocean-Atmosphere Model (FOAM) GCM coupled to a mixed-layer ocean and thermodynamic sea-ice model, employing realistic present-day geography.² Each simulation started from a control-run modern climate with unaltered water vapor. The evolution of the climate following the imposition of each altered water vapor assumption is shown in Fig. 6.2. When the radiative effect of water vapor is eliminated, the Earth falls into a globally glaciated snowball state after 8 years. To probe the opposite extreme, we forced the radiation to be calculated under the assumption that the entire troposphere is saturated. Note that this does not mean that the specific humidity was fixed at the value the control climate would have had in saturation. Rather, each time the radiation module is called, the saturation specific humidity is computed using the model's temperature field at that instant of time; this specific-humidity field is then used in the radiation calculation. This technique yields additional warming because the radiative forcing caused by saturating the atmosphere at any given temperature is *itself* subject to amplification by water vapor feedback, as it should be. In the saturated experiment the tropical temperature rises to an impressive 320 K after 8 years (left panel). It appears to have leveled off at this time, but we do not know if the climate would continue to warm if the integration were carried out for a longer time, since the simulation halted owing to numerical instabilities arising from extreme temperature contrast between the Antarctic glacier and the surrounding warm ocean waters. The tropical warming in the saturated GCM simulation is similar to what one of us predicted earlier on the basis of an idealized two-box model of the tropics (Pierrehumbert 1999). The saturated case shows the potential for water vapor feedbacks to make the climate response to CO_2 much more extreme than found in the extant climate models; there is plenty of dry air around, and if it somehow became saturated, the consequences for climate would be catastrophic. The saturated case provides only an upper bound on how bad things could get, but it is nonetheless an upper bound that should be kept in mind when thinking about climate prediction uncertainties.

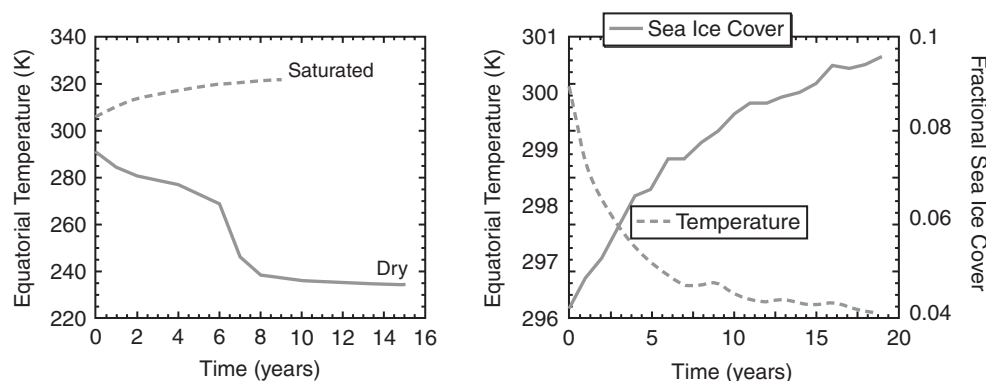


FIGURE 6.2. Results of GCM simulations in which the radiative impact of water vapor has been artificially altered. Left panel: time evolution of equatorial temperature for completely saturated and completely dry atmospheres. Right panel: time evolution of equatorial temperature and global ice cover for a case in which the water vapor used in computing radiation is reduced by a factor of two compared to the standard parameterization.

We also examined the impact of a less-extreme assumption about water vapor, results for which are shown in the right panel of Fig. 6.2. In this case, the water vapor was computed using the GCM's standard explicit and parameterized physics package, but the resulting specific humidity was multiplied by one-half before being handed to the radiation scheme. In this run, the equatorial temperature drops 4 K, to 296 K. Moreover, the fractional sea-ice cover doubles, to nearly 10%. The summer snow line on land (not shown) also moves considerably equatorward. All in all, the resulting climate rather resembles the Earth's climate during the Last Glacial Maximum.

Removing the atmosphere's water provokes a Snowball Earth. Saturating the atmosphere provokes an extreme hothouse climate warmer than any encountered in the past several hundred million years. Reducing water to half that produced by standard parameterizations is sufficient to provoke an ice age. Water vapor is a dynamic and fluctuating quantity with very little intrinsic persistence beyond the monthly time scale. It is striking that, despite this fact, our climate doesn't undergo massive fluctuations in response to spontaneously generated fluctuations in water vapor content. The lack of such fluctuations is a testament to the precision with which water vapor statistics are slaved to the large-scale state of the climate.

Water vapor is the atmosphere's single most important greenhouse gas, and it is correct to say that an accurate prediction of climate change hinges on the ability to accurately predict how the water vapor content of the atmosphere will change. It would be a gross error, however, to conclude that the effect of CO_2 is minor in comparison to that of water vapor. The greenhouse effect of CO_2 accounts for fully a third of the total, and this is a very considerable number. Eliminating the 50 W/m^2 of tropical CO_2 greenhouse effect would drop the tropical temperature by about 25 K, once amplified by water vapor feedback. When further amplified by ice-albedo feedback, this would certainly cause the Earth to fall into a snowball state. A warmer, water-rich world

subjected to much greater solar radiation than the Earth could indeed be in a state where the addition or removal of a few hundred ppm of CO_2 would make little difference to the climate, but this state of affairs does not by any stretch of the imagination apply to the Earth. Another important difference between CO_2 and water vapor is that the former has a much longer response time because on Earth it is removed only by slow biogeochemical processes as opposed to the rapid condensation and precipitation processes affecting water vapor. Water vapor responds quickly to temperature changes induced by CO_2 , whereas CO_2 would take thousands of years to respond to a change in the hydrological cycle.

6.4. How Saturated Is the Atmosphere?

Above the boundary layer, much of the atmosphere is highly unsaturated, with relative humidities below 10% occurring frequently in both the Tropics and the extratropics. The satellite snapshot of midtropospheric humidity on 16 January 1992, shown in Fig. 6.3, serves as a reminder of the prevalence of dry air. The prevalence of dry air can be quantified in terms of the probability distribution of relative humidity. This diagnostic has been extensively studied in the Tropics (Spencer and Braswell 1997; Zhang et al. 2003; Brogniez 2004). The midlatitude distribution has not received as much attention, but there is ample evidence that highly undersaturated air is common there as well (e.g., Soden and Bretherton 1993). Early work suggested a lognormal distribution of relative humidity (Soden and Bretherton 1993) but further study has revealed a greater variety of shapes of the relative humidity histogram, including even bimodality (Zhang et al. 2003; Brogniez 2004). For now, the detailed shape of the probability distribution will not concern us; it is enough to know that most of the free troposphere is significantly undersaturated. This widespread occurrence of unsaturated air is a manifestation of the fact that from a thermodynamic standpoint, the atmosphere is a nonequilibrium system. In a state of thermodynamic equilibrium with the oceanic moisture reservoir, the entire atmosphere would be saturated. The prevalence of subsaturated air, and the sorting of air into moist and dry regions, yield a state of relatively low thermodynamic entropy. Evaporation of liquid water or ice into the subsaturated regions, or diffusion of moisture from regions of high vapor pressure into subsaturated regions of lower vapor pressure, would increase the entropy of the atmosphere (Pauluis and Held 2002).

It is essential to understand the processes that determine the degree of subsaturation of the atmosphere, and to come to an understanding of how the net result of these processes might change as climate changes. In thinking about atmospheric water vapor content, one must draw a distinction between the net column water content and the water content in the mid to upper troposphere. The latter accounts for a small fraction of the total water content of the atmosphere because the lower warmer parts of the atmosphere have much higher saturation vapor pressure and are moreover sufficiently

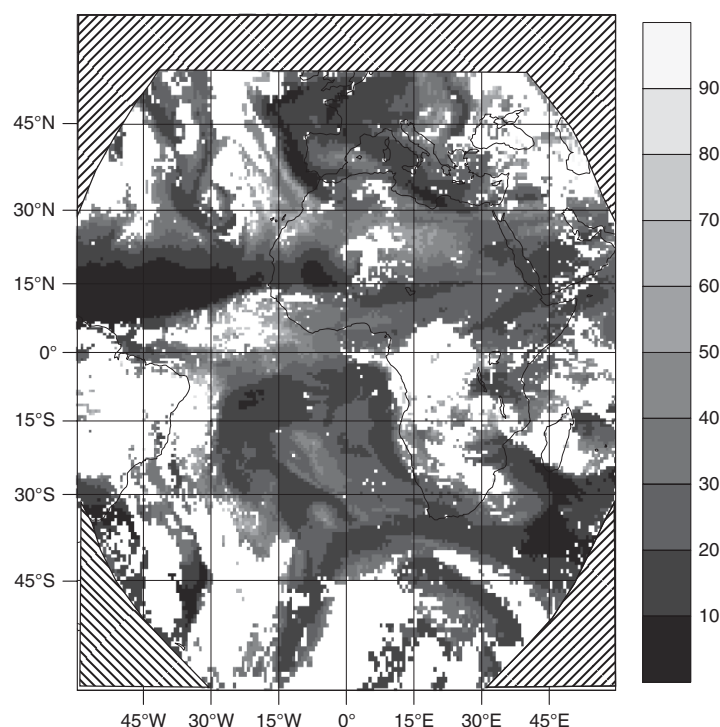


FIGURE 6.3. Free tropospheric relative humidity on 16 January 1992, retrieved from the Meteosat water vapor channel, based on retrieval in Brogniez (2004). Cloudy regions where the infrared retrieval could not be done were filled in with 100% relative humidity for the purposes of this figure.

close to the surface water reservoir that they tend to stay relatively saturated. Yet water vapor, or any greenhouse gas, placed near the surface has little effect on OLR because the low-level air temperature is not much lower than the surface temperature. To get a significant greenhouse effect, one must increase the infrared opacity of a portion of the atmosphere that is significantly colder than the surface. Because the radiative effect of water vapor is logarithmic in its concentration, small quantities of water vapor can accomplish this task aloft. This leads to the concept of free tropospheric humidity (FTH) or upper tropospheric humidity (UTH), which may be loosely defined as the water content of the portion of the atmosphere where water vapor has a considerable effect on the radiation budget. Diversion of a tiny proportion of the atmosphere's net water vapor content would be sufficient to saturate the mid to upper troposphere and radically warm the climate. For example, consider a 50 mb thick layer of saturated air near the surface of the tropical ocean, having a temperature of 295 K. Less than 3% of the water content of this layer would suffice to completely saturate a layer of equal mass having a temperature of 250 K, such as would be encountered in the tropical mid troposphere, or at lower altitudes in the extratropics. Clearly, the magnitude of the boundary layer water vapor reservoir is not the limiting factor in determining the free tropospheric humidity.

We pause now to dismiss two fallacies, which unfortunately have not yet completely disappeared from discussions of the subject at hand. The first is the *evaporation fallacy*, typified by statements like: “In a warmer world there will be more evaporation, which will increase the supply of moisture to the atmosphere and increase its humidity.” This is wrong on at least two counts. It is not inevitable that warmth increases evaporation, since evaporation is determined by wind speed and boundary-layer relative humidity, as well as the boundary-layer saturation vapor pressure (which does unquestionably go up with temperature). It is entirely possible for a warmer climate to have less evaporation. An even more serious objection to the evaporation fallacy is that it hopelessly confuses fluxes and reservoirs. Evaporation gives the *rate* at which moisture fluxes through the atmosphere, which is distinct from, and even has different units from, the amount of water left behind in the atmosphere (or more specifically the free troposphere). In this regard, the evaporation fallacy is as absurd as saying that, in comparing the water content of two vessels, the one with the higher rate of filling will contain more water, regardless of whether one of them is a sieve while the other is a watertight bucket. The second fallacy is the *saturation fallacy*, which states flatly that a warmer atmosphere will tend to become moister because saturation vapor pressure increases with temperature. That is like saying that one always expects to find more water in a bigger bucket than a smaller bucket, no matter how leaky either might be. Clausius-Clapeyron does indeed govern the relation between temperature and water content, but in a far subtler way that will begin to become clear shortly.

So what really determines the water vapor content of the free troposphere? It is easiest to think about this problem in a Lagrangian sense, tracking the water content of an air parcel as it wanders about the atmosphere. The fluctuating water content of the parcel results from a balance between the rate at which water is added to the parcel against the rate at which water is removed. Water vapor is removed either by condensation or by diffusion into a neighboring drier air parcel. Let us suppose for the moment that diffusivity is so low that the latter mechanism is unimportant. In that case, water vapor is removed when the air parcel wanders into a region where the local saturation specific humidity is lower than the current specific humidity of the parcel, at which time the specific humidity is reset to the lower local saturation value and the balance is rained out. The net result is that the specific humidity of an initially saturated parcel after time τ is equal to the minimum q_s encountered along the trajectory during that time. By definition, this is a nonincreasing function of τ , though there will be long periods of time over which the minimum remains constant between those times at which new minima are encountered. Define $q_{\min}(t_0, \tau) \equiv \min(q_s(t); t_0 < t < t_0 + \tau)$. Then the rate at which q_{\min} decreases, which is different for each trajectory, determines the drying rate on that trajectory. The drying is not a continual process, but occurs in fits and starts as new minima are encountered. If no new water is added to the parcel, new condensation events eventually become very infrequent, or cease altogether if the global minimum is encountered. The drying rate determined by this process must be balanced

against whatever processes episodically add new water to the air parcel, determining the statistically fluctuating water content of the parcel.

The importance of nonlocal control of tropospheric humidity was clearly revealed in the case study described in Kelly et al. (1991). Drawing on this insight, Yang and Pierrehumbert (1994) examined the general problem of midlatitude dry air production from a Lagrangian standpoint, and pointed out the importance of the q_{\min} statistic. The most prominent dry air pool in the atmosphere is found in the subtropics, and over the next several years a number of investigators developed an interest in large-scale control of subtropical humidity, employing Lagrangian techniques (Emanuel and Pierrehumbert 1996; Salathe and Hartmann 1997; Soden 1998; Pierrehumbert 1998; Pierrehumbert and Roca 1998; Sherwood and Dessler 2000; Galewsky et al. 2005). It was an idea whose time had come, and these studies provided a reassuring counterpoint to the then-emerging suspicion that everything was controlled by parameterized microphysics, and everything was inscrutable. In recent years, the term *time of last saturation model* has gained currency as a name for the general approach. We prefer the broader term *advection-condensation model*. If one knows the specific humidity of the air parcel at its destination, then one can determine the time of last saturation by following the back trajectory until that specific humidity becomes saturated. However, if the aim is to *predict* the specific humidity at the destination, determining the time of last saturation requires some assumption about the processes which, at some time in the past, added moisture to the air parcel.

Figure 6.4 shows temperature and pressure following a typical midlatitude trajectory. The temperature and pressure are nearly perfectly correlated, so minima in q_s are very nearly coincident with minima in T or p . The trajectory executes a wave-like motion under the influence of midlatitude synoptic eddies, swinging between the subtropical surface where the temperature reaches 290 K and the high-latitude tropopause where the temperature drops to 230 K. If there were no moisture sources in the free troposphere, then we would know that when the trajectory reaches the 800 mb level on day 5, its specific humidity would be no greater than $1.4 \cdot 10^{-4}$, the value corresponding to the minimum at day 0 (computed with 232 K and 500 mb). If the parcel becomes saturated when it encounters the ground at day 7, then it remains at saturation through day 13 since the temperature and saturation specific humidity decrease monotonically through that part of the trajectory. However, when the parcel lands on the 800 mb surface on day 18, its specific humidity would be $6.6 \cdot 10^{-4}$, corresponding to the minimum at day 16. Based on the local saturation specific humidity on day 18, the relative humidity would be 25%. This trajectory illustrates an important principle: in order to know the specific humidity at a given point on the trajectory, one only needs to know the history going back to the time of last saturation. Information about earlier times is erased whenever saturation occurs. The trajectory also illustrates the fact that midlatitude trajectories typically dip into the subtropical boundary layer every 15 days or so, where they have an opportunity to pick

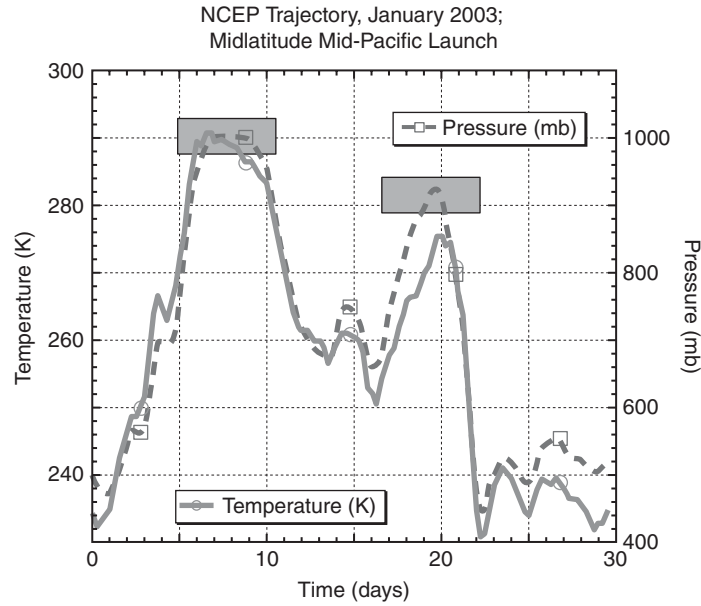


FIGURE 6.4. Temperature and pressure along a 3-D air parcel trajectory launched at 500 mb in the midlatitude Pacific. The trajectory was computed on the basis of 4 times daily NCEP horizontal and vertical velocity fields. The gray boxes indicate encounters with the boundary layer, where moistening is presumed to occur.

up new moisture. This is probably the main moisture source for the midlatitude free troposphere.

Most midlatitude trajectories have a character qualitatively like that shown in Fig. 6.4, but tropical trajectories are quite different. Isentropes are flat there, and there is less baroclinic eddy activity. Trajectories are dominated by organized rising motion in convective regions, ejection due to steady outflow or tropical transient eddies, and slow descent in the large-scale organized subsidence regions (see Pierrehumbert 1998; Pierrehumbert and Roca 1998). The tropical model of Minschwaner and Dessler (2004) provides perhaps the simplest realization of this class of models, in that the trajectories consist simply of ejection of saturated air from convective regions, followed by monotone subsidence at a rate determined by infrared radiative cooling. This idealized trajectory model neglects the lateral mixing incorporated in Pierrehumbert and Roca (1998), but nonetheless yields a number of insights concerning the nature of water vapor feedback. Generally speaking, one can expect the statistics characterizing trajectories to differ considerably between the Tropics and midlatitudes.

If we consider an ensemble of trajectories launched from a given location, the behavior of the minimum q_s statistic can be characterized by the probability density $Q_{\min}(q_{\min}|q_o, \tau, t)$, which gives the probability that the minimum q_s encountered between time t and $t + \tau$ is near q_{\min} , given that $q_s = q_o$ at the place where the particle is located at time t . Obviously $Q_{\min} = 0$ for $q_{\min} > q_o$. If the process is statistically

stationary, Q_{\min} will be independent of t . If one is trying to understand the water vapor at a specified point, it is most convenient to deal with back trajectories, corresponding to negative τ . We are more interested in where the air arriving at the target point *came from* than in where it is going. If the trajectory process is statistically reversible, all statistics of back trajectories have the same behavior as the corresponding statistics of forward trajectories, and in particular $Q_{\min}(q_{\min}|q_o, \tau, t) = Q_{\min}(q_{\min}|q_o, -\tau, t)$. Somewhat counter-intuitively, Brownian motion is statistically reversible in this sense. It is widely assumed, and probably true, that the atmospheric trajectory problem is statistically reversible, though we will not explicitly make use of the assumption in our calculations below.

As an example of the use of back trajectory statistics, let's suppose that we wish to know the probability distribution of specific humidity q in some patch of the 500 mb surface at a given time t , and are willing to assume that the entire atmosphere was saturated at time $t - \tau$, and moreover that there were no moisture sources in the intervening time. To solve this problem, we use Q_{\min} for the ensemble of trajectories that arrive at the patch at the specified time. For simplicity, we will assume that q_o is constant within the patch, though this is an assumption that can be easily relaxed. Because we assumed that all parcels are saturated at time $t - \tau$ (though each has a different q_s , appropriate to its location at that time), the specific humidity each parcel winds up with by time t is simply the minimum q_s encountered along the trajectory. Hence, the probability density function (PDF) of q in the patch is $Q_{\min}(q|q_o, -\tau, t)$, which is concentrated on progressively drier values as τ is made larger. Note that this distribution is entirely distinct from the distribution of *initial* saturations at time $t - \tau$. These could all be in the tropical boundary layer and have very high values, and the final humidity at time t would still become small. A *passive* tracer, with no sources or sinks, would retain its initial value, so that its PDF at later times is determined solely by its initial PDF, with no knowledge of the nature of the intervening path required. In contrast, the statistics of moisture are sensitive to the entire history of the path taken. The sensitivity to probabilities that depend on entire paths is one of the chief mathematical novelties of the water vapor problem, and the source of most of the theoretical challenges.

The probability density Q_{\min} characterizes the drying process, and one needs a corresponding probabilistic description of the moisture source in order to complete a theory of the atmospheric humidity. A moisture source such as evaporation of precipitation falling through dry air could add just a bit of moisture to an air parcel without saturating it. However, let's idealize the moisture source as a series of saturation events, which occur randomly in time, with the chance that a saturation event will occur after waiting for a time τ described by a probability distribution $P_{sat}(\tau)$. Then, the PDF of specific humidity is given by the convolution

$$Q(q|q_o, t) = \int_0^\infty P_{sat}(\tau) Q_{\min}(q|q_o, -\tau, t) d\tau, \quad [6.4]$$

where q_o is the saturation specific humidity at the point under consideration. If the trajectory process is statistically stationary, Q will be independent of t .

As an example of the application of equation (6.4) in the simplest possible context, we consider a uniform subsidence model similar to that in Minschwaner and Dessler (2004). In this case, since the trajectories always go downward to regions of larger q_s , the minimum q_s in the back trajectory always occurs at the time of saturation. To be definite, let's assume that the vertical coordinate is pressure p , that trajectories subside at a constant rate ω , and that $q_s = q_s(p)$, i.e., that the saturation field is horizontally homogeneous, as is approximately the case in the Tropics. Then Q_{\min} is a δ -function concentrated on the value of q_s at the pressure the parcel was at when it was resaturated. If p_o is the pressure at the target point (i.e., $q_o = q(p_o)$) then the pressure at the saturation point is $p_o - \omega\tau$ and hence $Q_{\min}(q|q_o, -\tau, t) = \delta(q - q_s(p_o - \omega\tau))$. Substituting this into equation (6.4) and using the relation $\delta(g(\tau))d\tau = (g'(\tau))^{-1}\delta(g)dg$, we find

$$Q(q|q_o, t) = \frac{1}{\omega \left. \frac{dq_s}{dp} \right|_{p_o - \omega\tau^*}} P_{sat}(\tau^*), \quad [6.5]$$

where $\tau^*(q)$ is the solution to $q_s(p_o - \omega\tau^*) = q$. In this case, the specific humidity PDF is determined by the saturation statistics and the vertical structure of q_s .³ In the general case, the moisture dynamics is characterized by the two PDFs P_{sat} and Q_{\min} . In order to fully understand water vapor feedbacks, we need to understand how these two PDFs change as climate changes. This is a tall order. Some further examples of the trajectory statistics in action will be given in section 6.5 for idealized trajectory models and section 6.6 for realistic trajectories.

There are three difficulties with the trajectory approach, two of them technical and one of a more fundamental nature. The first difficulty is that there may be mixing of moisture amongst air parcels arising from small-scale turbulent motions. Because large-scale resolved strain causes exponential amplification of gradients (Yang and Pierrehumbert 1994; Pierrehumbert 1998), even a weak effective diffusivity would eventually cause significant mixing. The mixing is likely to be dominated by vertical rather than horizontal mixing processes, for the reasons discussed in Haynes and Anglade (1997). Incorporation of mixing greatly complicates the calculation because the moisture evolution on one trajectory becomes dependent on the moisture evolution of all other trajectories that pass in its vicinity during the past. Explicit calculation then calls for either Eulerian methods (at the expense of the need to confront the difficult problem of unwanted numerical diffusion) or simultaneously integrating the problem on a sufficiently large swarm of trajectories. Mixing between moist and dry air parcels is important because it both reduces the frequency of very dry air and because the dilution increases the subsaturation of the moist air, and delays condensation. It is of utmost importance to determine how much small-scale mixing enters into the tropospheric

moisture problem in the real atmosphere, and comparison of observations with results of the non-diffusive trajectory calculation can provide a means of doing so. There has been considerable attention to the diagnosis of small-scale mixing in the stratosphere (see Haynes and Anglade [1997]; Legras et al. [2003] and references therein), but the analogous question for tropospheric water vapor is unresolved. The second difficulty with the trajectory method is similar to the mixing problem: precipitation generated by large-scale condensation along a trajectory may evaporate as it falls through dry air, adding moisture to trajectories it encounters. This, too, couples trajectories. It can be regarded as just another form of vertical mixing.

The third difficulty comes at the cloud scale, where we confront a much harder problem, particularly in tropical convective regions. Models and analyses produce a large-scale rising motion in such regions, diagnostically associated with the moisture convergence required to feed the convective precipitation. This upward motion, which lifts all trajectories, must be regarded as wholly fictitious. In reality, most of the air in the convective region is subsiding, and the upward mass flux is concentrated in cumulus towers covering only a small fraction of the area of the convective region. The resolved large-scale vertical velocity correctly represents the net upward mass flux, but is not typical of the velocity of individual fluid parcels in the convective region. The net result is that the detrainment obtained from large-scale trajectory simulations is always saturated, whereas the cloud-scale motions offer ample opportunities for the detrained air to be substantially undersaturated. The mixing of moisture between moist and dry air at cloud and sub-cloud scales engages microphysical issues of the sort discussed by Tompkins and Emanuel (2000), and ultimately determines the degree of subsaturation. The success of models with saturated detrainment at reproducing water vapor observed in nonconvective regions (e.g., Minschwaner and Dessler 2004; Pierrehumbert and Roca 1998) suggests that the subsaturation must not be too extreme in the typical case, but the whole matter requires further study.

6.5. A Few Illustrative Models

It is instructive to consider some simple one-dimensional models of the connection between mixing, transport, and drying. These models are offered in order to highlight some of the basic issues in cases that can be solved completely. We do not pretend that any of these models yield good representations of the atmosphere's actual water vapor distribution. Both models that we consider in this section are formulated in terms of an abstract spatial coordinate y . The only thing we need to know about y is the saturation specific humidity q_s as a function of y . We may think of y as the north-south distance following an isentropic surface, which yields a decreasing q_s because the surface becomes generally higher (hence colder) as the pole is approached. The coordinate could represent north-south distance at a fixed midtropospheric pressure

level, as in typical one-layer energy balance models. With a suitable increase in mixing rates, y could equally well represent altitude, with latitude and longitude held fixed; in this case the mixing parameterization is to be thought of as a surrogate for convection rather than the larger scale, more ponderous, large-scale advection. If temperature decreases linearly with y , then the Clausius-Clapeyron relation implies that q_s decreases approximately exponentially with y . A decrease of pressure with y somewhat offsets the exponential decay, but not so much as to prevent us from using $q_s(y) = \exp(-y)$ as a useful conceptual model of the atmosphere's saturation specific humidity.

6.5.1. The Diffusion-Condensation Model

We wish to study the interplay of transport and condensation. Diffusion is the simplest model of transport, and is moreover employed as a surrogate for eddy transport of water vapor in many idealized climate models (e.g., Weaver et al. 2001; Petoukhov et al. 2000). Hence, we first examine a model in which moisture is represented by the mean specific humidity $q(y, t)$, which is stipulated to satisfy a diffusion equation

$$\partial_t q - D \partial_{yy} q = -S(q, q_s). \quad [6.6]$$

In this equation, the sink $S(q)$ instantaneously resets q to q_s whenever diffusion causes it to exceed $q_s(y, t)$. The instantaneous sink may be taken as a limiting form of the function

$$S(q, q_s) = \begin{cases} \frac{1}{\tau}(q - q_s(y, t)) & \text{for } q > q_s \\ 0 & \text{for } q \leq q_s \end{cases} \quad [6.7]$$

as $\tau \rightarrow 0$. The mathematical novelty that a sink of this form adds to the problem is that it makes the problem nonlinear. Condensation can be thought of as a particularly simple form of unary chemical reaction, and in fact many of the issues we encounter in the condensation problem are generic to a broad class of nonlinear chemical reactions.

In general, q_s could be a function of both y and t , but henceforth we shall assume $q_s = q_s(y)$. In the instantaneous removal limit ($\tau \rightarrow 0$), $q(y) = q_s(y)$ is a steady solution to equation (6.6) in any region where $d^2 q_s / dy^2 > 0$. If this condition is satisfied at time t , then $S = 0$ at that time but the diffusion term makes $\partial_t q > 0$, so that the moisture sink will be activated at the next instant of time, and reset the moisture to q_s , keeping the moisture in the region fixed at $q_s(y)$.⁴ The moisture loss is balanced by moisture flux from regions of larger q .

Next, we consider a simple initial-value problem. Suppose that y extends from 0 to ∞ , and impose the no-flux boundary condition $\partial_y q = 0$ at $y = 0$ so that there are no sources of new moisture. In this case, the condensation caused by diffusion of moisture from regions of large q_s into regions of small q_s will deplete the moisture content. We shall assume that $d^2 q_s / dy^2 > 0$ throughout the domain, and that q_s attains its maximum value at $y = 0$. Given the curvature assumption, this implies that q_s is

monotonically decreasing. If the atmosphere is initially saturated, then there is some $Y(t)$ such that $q(y) = q_s(y)$ for $y > Y(t)$; this is the region in which condensation is taking place. For $y < Y(t)$ the air is undersaturated, and satisfies the diffusion equation with no sources or sinks. As moisture is drawn out of the system by condensation, $Y(t)$ increases. Under the assumptions on q_s , this quantity approaches a minimum value C from above as $y \rightarrow \infty$. As $t \rightarrow \infty$, we have $Y(t) \rightarrow \infty$ and $q \rightarrow C$. The question of the long-time asymptotic behavior of Y is thus well posed, and in fact has a fairly simple and general answer. Since the advection-condensation equation is invariant under the transformation $q \mapsto q - C$, $q_s \mapsto q_s - C$, we can assume that $C = 0$, without any loss of generality.

Examination of some numerical solutions suggests that q is approximately parabolic in the non-condensing region after a sufficiently long time has passed. Let us then look for a solution with $q = a - by^2$ for $y < Y(t)$. What we shall present here is not a complete and rigorous asymptotic analysis, for we shall not derive the conditions under which the assumed form of q in the non-condensing region is valid. The matching conditions at Y and the requirement that q satisfy the diffusion equation in the subsaturated region imply

$$\begin{aligned} a - bY^2 &= q_s(Y) \\ -2bY &= q'_s(Y) \\ \frac{da}{dt} &= -2bD. \end{aligned} \tag{6.8}$$

Given the assumed form of q , we can only exactly satisfy the diffusion equation at one point, and in writing the third of these relations we have chosen to do so at $y = 0$. Taking the time derivative of the first equation and substituting from the other two results in the following differential equation for Y :

$$-2bD + \frac{1}{2}q'_s(Y)\frac{dY}{dt} + \frac{1}{2}q''_s(Y)Y\frac{dY}{dt} = q'_s(Y)\frac{dY}{dt} \tag{6.9}$$

or equivalently

$$Y \cdot \left(1 - Y \frac{q''_s(Y)}{q'_s(Y)} \right) \frac{dY}{dt} = 2D. \tag{6.10}$$

The quantity q''_s/q'_s has dimensions of inverse length. For $q_s = \exp(-y^2/\sigma^2)$ it is $-2Y/\sigma^2$ at large Y , for $q_s = \exp(-y/L)$ it has the constant value $-1/L$, and for $q_s = A/y^\alpha$ it is $-(\alpha + 1)/Y$. Suppose that $q''_s/q'_s = AY^n$ at large Y . Then, if $n \leq -1$ the second term multiplying the derivative on the left-hand side of equation (6.10) is at most order unity, and the equation can be integrated to yield $Y(t) \sim (Dt)^{1/2}$. On the other hand, if $n > -1$ the second term dominates and we find $Y \sim (Dt)^{1/(n+3)}$. It is interesting that rapid decay of q_s can cause the thickness of the subsaturated region to grow more slowly than the diffusive length scale $(Dt)^{1/2}$.

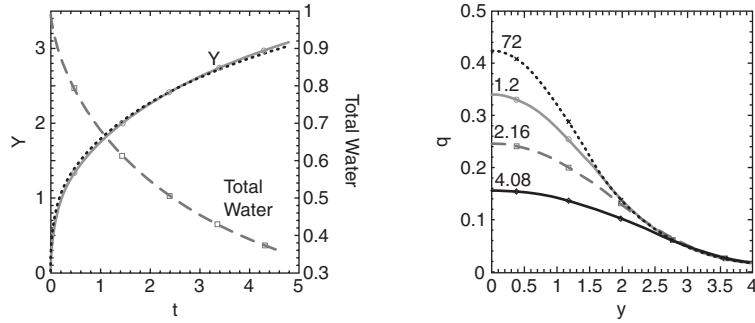


FIGURE 6.5. Numerical results for the freely decaying diffusion-condensation model with a no-flux barrier at $y = 0$. Left panel: time evolution of the point $Y(t)$ bounding the subsaturated region, and of total moisture in the system. The short-dashed line gives the fit to the asymptotic result $Y \sim t^{1/3}$. Right panel: the profile of specific humidity at the times indicated on the curves.

Once $Y(t)$ is known, the humidity decay curve is obtained using

$$q(0) = a = q_s(Y) - \frac{1}{2} Y q'_s(Y) = \left(1 - \frac{1}{2} Y q'_s(Y) / q_s(Y) \right) q_s(Y). \quad [6.11]$$

At large Y , q'_s/q_s will generally have the same scaling as q''_s/q'_s ; by assumption, it is always negative. If q_s decays exponentially, then q'_s/q_s is a negative constant and the second term in the factor multiplying q_s dominates; in this case $q(0) \sim Y q_s(Y)$, and hence decays like $t^{1/3} \exp(-ct^{1/3})$ for some constant c . If q_s decays algebraically, then both terms in the factor are order unity, so $q(0) \sim q_s(Y)$ and hence decays like $t^{-\alpha/2}$, assuming q_s decays like $y^{-\alpha}$ at large y . In either case, it can easily be shown by integrating $q(y)$ that the total water vapor remaining in the atmosphere decays like $Y(t)q(0, t)$ provided q_s has a finite integral.

Thus, moisture is drawn out of the system by diffusion from subsaturated regions neighboring the boundary (the ground, in many cases) into regions of saturated air with lower specific humidity at larger y , where it can condense. The rate of decay is determined by the rate at which q_s decays with increasing y . Even for exponentially decaying q_s , the moisture decay is slower than exponential, owing to the time required for moisture to diffuse into the ever-retreating condensing region. Figure 6.5 shows numerical solutions to the problem with $q_s = \exp(-y)$, together with the theoretical result for $Y(t)$. The agreement between simulation and theory for this quantity is excellent.

As the next stage in our exploration of the diffusion-condensation equation (6.6), we exhibit the steady-state response to a moisture source. We introduce the moisture source by holding $q = rh \cdot q_s(0)$ at $y = 0$ (with $rh < 1$). If we assume $d^2 q_s / dy^2 > 0$ as before, the equilibrium moisture distribution has the simple form

$$q(y) = \begin{cases} q_s(y) & \text{for } y > y_s \\ rh \cdot q_s(0) + \frac{(q_s(y_s) - rh \cdot q_s(0))y}{y_s} & \text{for } y < y_s, \end{cases} \quad [6.12]$$

where y_s is chosen to make the flux continuous, i.e.,

$$\frac{dq_s}{dy}(y_s) = \frac{(q_s(y_s) - rh \cdot q_s(0))}{y_s}. \quad [6.13]$$

All condensation is at $y > y_s$, and $y_s \rightarrow 0$ as $rh \rightarrow 1$, in which case the whole atmosphere becomes saturated. Further, although the undersaturation for $y < y_s$ relies on mixing, $q(y)$ is independent of the magnitude of D . However, the rate of moisture flux into the condensation region, and hence the precipitation rate, is proportional to D . This solution underscores the point we made earlier with regard to the evaporation fallacy: the factors governing the atmospheric relative humidity distribution are quite distinct from those governing the rate at which water fluxes through the system. Note also that the moisture sink produces air that has a lower specific humidity than the source, even in regions where no condensation is taking place. In effect, this is due to dilution of moist air with drier air from larger y . Still, the diffusive model cannot produce an undersaturated layer unless the source air at $y = 0$ is undersaturated.

Another interesting configuration is the *cold trap*, in which $q_s(y) = q_c < 1$ in a small region near $y = 0$, while $q_s(y) = 1$ elsewhere. A local minimum of this sort could be taken as a crude representation of the minimum occurring at the tropopause, with y measuring altitude. However, the cold trap is also relevant to the case in which y represents horizontal distance on a fairly level surface, such as the tropical tropopause; in this case, the minimum corresponds to the cold conditions occurring over the high tropopause region above the west Pacific warm pool, and we seek to understand the global drying effect of this region. In the freely decaying case subject to initial condition $q(y) = 1$ in an unbounded domain, the cold trap creates a zone around itself in which q approaches q_c . The width of this dry zone increases in proportion to $(Dt)^{1/2}$. Alternately, we can seek an equilibrium solution by imposing $q = 1$ at $y = -1$ and a no-flux condition $\partial_y q = 0$ at $y = 1$. The steady solution in this case is simply $q = 1 - (1 - q_c)(y + 1)$ for $-1 \leq y \leq 0$ and $q = q_c$ for $y \geq 0$. In accord with intuition, moisture is reset to the cold trap specific humidity when air passes through it. We can also impose a moisture source at the right-hand boundary by replacing the no-flux condition by $q = q_1$ at $y = 1$. In this case the solution is

$$q = \begin{cases} 1 - (1 - q_c)(y + 1) & \text{for } -1 \leq y \leq 0 \\ q_c + (q_1 - q_c)y & \text{for } 0 \leq y \leq 1 \end{cases} \quad [6.14]$$

provided that $q_c \leq \frac{1}{2}(1 + q_1)$. If this condition is met, the solution takes the form of a bent stick, with the cold trap depressing the value at the center of the domain. However, if $q_c > \frac{1}{2}(1 + q_1)$, diffusive dilution of the moist air reduces the humidity at the midpoint of the domain to such an extent that no condensation occurs; in this case, the solution is simply a straight line linking the limiting values at the two boundaries, and is a solution of the conventional diffusion equation. In section 6.5.3 we will treat the

same configuration using a stochastic model of water vapor, and find some intriguing differences in the behavior.

There can be no sustained condensation in a region where $d^2q_s/dy^2 < 0$. In such a region, if $q = q_s$ initially, then diffusion will immediately reduce q to below its saturation value everywhere, halting condensation. For example, suppose $q_s = \exp(-y^2)$ in an infinite domain, and that $q = q_s$ initially. After the initial instant, condensation halts for $|y| < 1$, though it continues for $|y| > 1$ where the curvature is positive.

The summary behavior of the one-dimensional diffusion-condensation problem is quite simple. It creates saturated regions embedded in regions where q_s'' is positive and q_s is small. These drain moisture out of the surrounding air, creating subsaturated regions in the vicinity. Apart from the possible inadequacies of diffusion as a representation of the mixing, the main shortcoming of the diffusion-condensation model is that it represents the moisture field in terms of a single concentration q at each y . In reality, a small box drawn about y will contain an intermingling of moist and dry air. The diffusion model is incapable of predicting a probability distribution function for moisture; worse, the neglect of fluctuations fundamentally misrepresents the drying process itself, owing to the nonlinear nature of condensation. The next class of models we shall study rectifies this shortcoming of the diffusion-condensation model.

6.5.2. Stochastic Drying: Initial-Value Problem

As a counterpoint to the diffusion-condensation problem we pose a simplified random walk version of the stochastic drying process introduced in section 6.4. Suppose that an ensemble of particles execute independent random walks in an unbounded one-dimensional domain with coordinate y . Particle j is located at point $y_j(t)$, and is tagged with a specific humidity q_j , which can change with time if condensation occurs. The saturation specific humidity field $q_s(y)$ is assumed to be monotonically decreasing with y . The particles are initially saturated (i.e., $q_j(0) = q_s(y_j(0))$), but whenever they find themselves at a place where $q_j > q_s(y_j)$, then q_j is instantaneously reset to the local q_s . Under these conditions, what is the probability distribution of specific humidity for the particles located at point y at time t ?

This problem is readily solved in terms of the maximum-excursion statistics for Brownian motion (i.e., a random walk with velocity δ -correlated in time). We make use of the fact that the statistics for forward trajectories are identical to those for backward trajectories, and pose the question as follows: for a particle that is located at y_b at time $t = 0$, what is the probability that it was located at y_a at time $t = -\tau$? What is the probability that the maximum y visited in the time interval $[-\tau, 0]$ was y_{\max} ? Since q_s is monotonically decreasing, the latter probability gives us the probability of q_{\min} . If we define the random walk as satisfying $dy/dt = v(t)$ with $\langle v(t)v(t') \rangle = 2D\delta(t-t')$, then the probability that the parcel landing at y_b came from y_a is given by the familiar Gaussian form, just as if we were running the trajectory forward rather than backward

(Karatzas and Shreve 1991; Lesigne 2005):

$$p(y_b|y_a, \tau) = p(y_b - y_a, \tau) = \frac{1}{\sqrt{4\pi D\tau}} e^{-(y_b - y_a)^2/(4D\tau)}. \quad [6.15]$$

This probability density satisfies a diffusion equation with diffusivity D .

It is a remarkable fact that the probability of the maximum excursion along the path, which is a probability distribution on the space of *paths*, can be determined in terms of the probability of the endpoint given in equation (6.15). This is a consequence of the *reflection principle*, discussed in Karatzas and Shreve (1991) and Lesigne (2005), and holds for a broad class of random walk processes. For the unbounded Brownian motion, the maximum excursion probability is found to be simply

$$P_{\max}(y_{\max}|y_b, \tau) = \begin{cases} 2p(y_{\max} - y_b, \tau) & \text{for } y_{\max} \geq y_b \\ 0 & \text{otherwise,} \end{cases} \quad [6.16]$$

where p is the probability defined in equation (6.15). We know that the maximum position visited in the past can be no less than the position at which the particle finds itself at $t = 0$, which is why the probability vanishes for $y_{\max} < y_b$. For $y_{\max} > y_b$, the reflection principle implies that the probability that y_{\max} is the maximum position visited is twice the probability that the particle *originated* at y_{\max} . This seems too good to be true, but the mathematics is incontrovertible. Somewhat surprisingly, the most probable y_{\max} is the terminal position y_b . In other words, the most probable situation is that the value of y where the particle is found at $t = 0$ is the greatest it has visited over the past time τ . Going backwards in time, most trajectories wander to smaller values of y and never come back to the starting position. This is a consequence of the domain being unbounded below, which allows plenty of room in the domain $y < y_b$ for the trajectories to get lost in. Note, however, that the probability of the most probable excursion decays like $(D\tau)^{-1/2}$ as $\tau \rightarrow \infty$, so that trajectories with $y_{\max} = y_b$ become rather improbable. For back trajectories of length τ , values of y_{\max} as large as $y_b + (D\tau)^{1/2}$ are nearly as probable as the most probable one.

Back trajectories with $y_{\max} = y_b$ are saturated when they reach the target position y_b , since the particle has never visited a place with lower q_s than it has at its destination. Because the probability of such trajectories decays very slowly with τ , the mean moisture content of an ensemble of trajectories landing at y_b also decays very slowly. To make these ideas more explicit, let's consider the case $q_s = \exp(-y)$. In this case, a trajectory with maximum excursion y_{\max} will produce a specific humidity $q = \exp(-y_{\max})$ when it lands at y_b . Solving for y_{\max} yields $y_{\max} = -\ln(q)$. In addition, $y_b = -\ln(q_s(y_b))$, whence $y_{\max} - y_b = -\ln(q/q_s(y_b))$. This can be substituted for the argument of p in equation (6.16). From this, we learn that the PDF of $-\ln(q/q_s)$ at a given point y_b is in fact the PDF of $y_{\max} - y_b$ for back trajectories landing at that point. The PDF of humidity may be described as a “truncated lognormal” in the sense that $-\ln(q)$ is normally distributed for $q \leq q_s$, but the probability is identically zero for $q > q_s$.

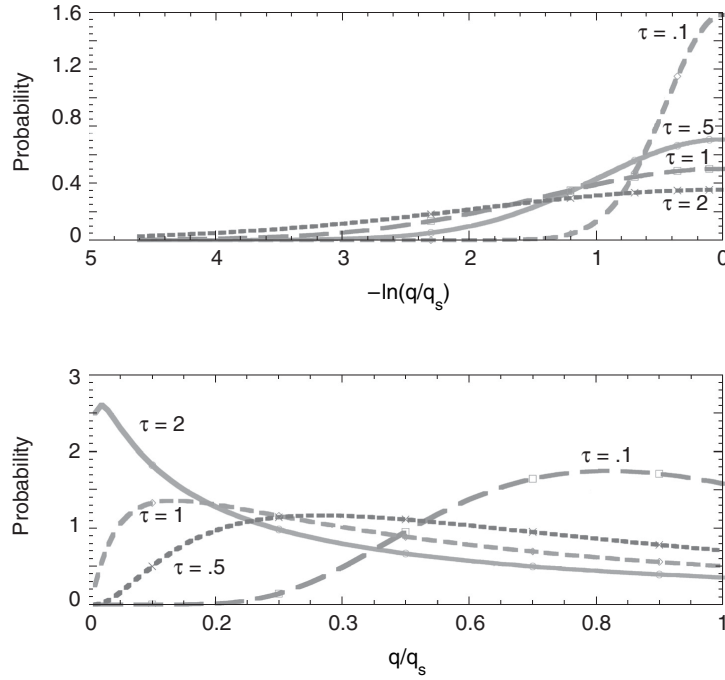


FIGURE 6.6. Probability density functions for $-\ln q/q_s(y_b)$ (top panel) and for q/q_s (bottom panel) for particles executing an unbounded random walk in a saturation field $q_s(y) = \exp(-y)$. Results are shown at $t = 0$, under the assumption that the entire domain was saturated at the time $t = -\tau$ indicated on each curve. Results are shown for $\tau = 0.1, 0.5, 1$, and 2 . For the exponential saturation profile used in this calculation, the PDF of $-\ln(q/q_s(y_b))$ is identical to the PDF of the maximum excursion of back trajectories, i.e., $y_{\max} - y_b$.

If we want the PDF of q itself, we must transform the density using $-d \ln(q/q_s) = -q^{-1}dq$. Then, letting $Q(q, y_b, \tau)$ be the PDF of q at point y_b , at a time τ after an everywhere-saturated state, we have

$$Q(q, y_b, \tau) = \begin{cases} 2q^{-1}p(-\ln(q/q_s(y_b)), \tau) & \text{for } q \leq q_s(y_b) \\ 0 & \text{otherwise.} \end{cases} \quad [6.17]$$

The behavior of P_{\max} and Q are shown in Fig. 6.6. Note that while the probability of $y_{\max} - y_b$ (or equivalently of $-\ln(q/q_s)$) has its peak at zero, corresponding to saturated air, the peak probability of q shifts toward dry air as time progresses. This is a consequence of the q^{-1} factor involved in transforming the probability density from a density in $\ln(q)$ to a density in q . The population shifts toward unsaturated air as time progresses, though there is a long moist tail owing to the persistent high probability of trajectories that do not visit cold places. The stochastic model generates dry air in the unbounded domain, whereas the diffusion-condensation model does not. The difference in the two models lies in the fact that the latter represents moisture by a single mean value at any y , and therefore cannot reflect the fact that amongst the ensemble of

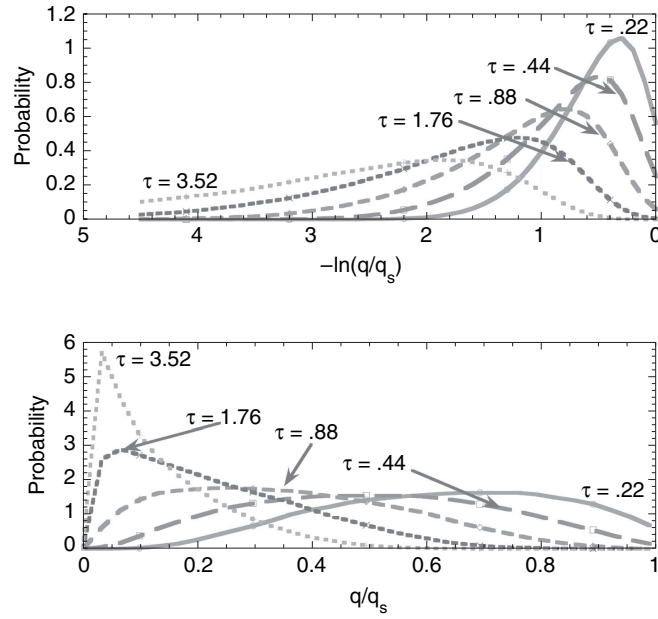


FIGURE 6.7. As for Fig. 6.6, but for the case with a reflecting barrier at $y = 0$. In this case, the results depend on y_b . These PDFs were computed at $y_b = 0.5$. Results are shown for $\tau = 0.22, 0.44, 0.88, 1.76$, and 3.52 . As before, the top panel can be regarded as the PDF of $y_{\max} - y_b$.

particles making up such an average, some have visited very cold places in the past, and therefore have become very dry.

The mean moisture for the ensemble of particles landing at y_b is obtained by carrying out the integral

$$\langle q \rangle = \int_0^{q_s} q Q(q, y_b, \tau) dq = \int_0^{q_s} 2p(-\ln(q/q_s(y_b), \tau)) dq. \quad [6.18]$$

At large τ , p becomes nearly constant outside of an infinitesimal interval near $q = 0$, which contributes nothing to the integral in the limit of $\tau \rightarrow \infty$. Since $p \sim (4\pi D\tau)^{-1/2}$ in this limit, we conclude that $\langle q \rangle \sim 2q_s(y_b)/(4\pi D\tau)^{1/2}$ at large times. The humidity in the unbounded random walk model exhibits a slow algebraic decay owing to the high probability of back trajectories wandering off to regions with large q_s and never returning. The fact that the moisture decays at all nonetheless stands in stark contrast to the diffusion-condensation model in an unbounded domain, for which $q = q_s(y)$ is an exact steady state, and the air remains saturated for all times.

The unbounded case is instructive, but it is very unlike the real atmosphere, because in the real atmosphere the temperature (and hence saturation specific humidity) is strictly bounded above by the maximum values prevailing at the tropical surface. Real air parcels do not have the liberty of wandering off into arbitrarily warm regions. The random walk model can be made more realistic by imposing a reflecting barrier at $y = 0$,

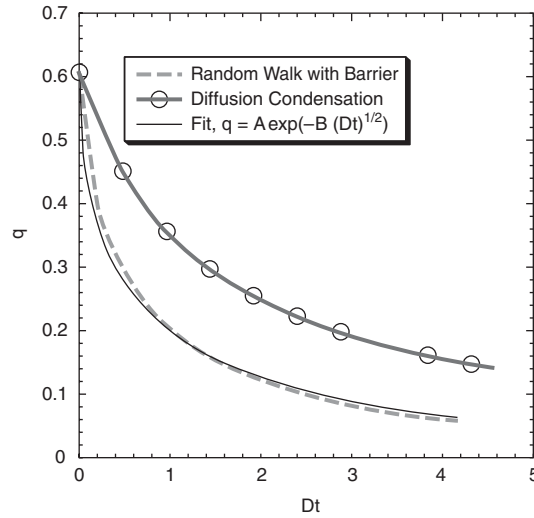


FIGURE 6.8. Decay of ensemble mean specific humidity at $y = 0.5$ for the bounded random walk with a barrier at $y = 0$. The thin black curve gives the fit to a stretched exponential decay of the form $A \exp(-B \sqrt{Dt})$. The upper thick, solid curve gives the moisture decay for the diffusion-condensation model computed in the same domain, and with the same diffusivity. The saturation specific humidity profile is $q_s(y) = \exp(-y)$.

which causes the saturation specific humidity to be bounded above. The maximum excursion statistics for a bounded random walk of this type can also be obtained by application of the reflecting principle, but the answer comes in the form of an infinite sum of shifted Gaussians, which is rather tedious to work with. The results we present here are based instead on Monte Carlo simulation, with ensembles of 10,000 particles or more. The PDF does not sense the presence of the barrier until such time that $(D\tau)^{1/2}$ becomes comparable to y_b and particles have had enough time to frequently reach the barrier. For longer times, the chief effect of the barrier is to shift the most probable maximum excursion to values greater than y_b , which moreover increase without bound as time progresses. It can be shown that the maximum excursion scales with $(D\tau)^{1/2}$ at large τ . The easiest way to see this is to note that the trajectories with a reflecting barrier at $y = 0$ are identical to those in an unbounded domain, transformed by reflecting the negative- y part of the trajectories about $y = 0$. The only trajectories where y_{\max} remains small are those that stay near the barrier, and these become improbable relative to farther-wandering trajectories as time goes on.

The expected behavior can be clearly seen in the upper panel of Fig. 6.7, where we show the time evolution of the PDF of $y_{\max} - y_b$ computed at $y_b = 0.5$. Recall that this PDF is identical to the PDF of $-\ln q/q_s$ if $q_s(y) = \exp(-y)$. Thus, with a barrier the most probable value of $-\ln q$ shifts towards drier values as τ increases. In the lower panel of the figure, we show the PDF of q itself. As in the no-barrier case, the peak shifts

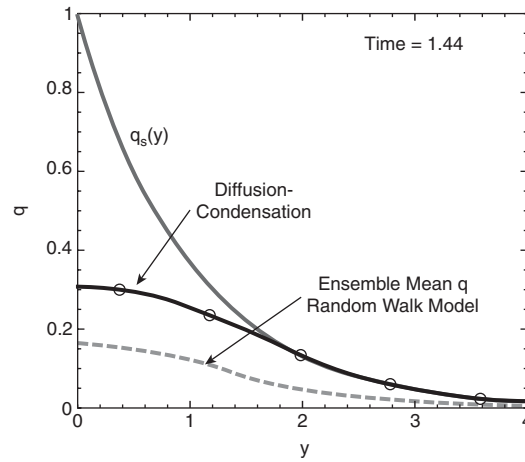


FIGURE 6.9. Profile of ensemble mean q at time $D\tau = 1.44$ for the random walk model with a barrier at $y = 0$. The corresponding profile for the diffusion-condensation model is shown for comparison.

toward dry values as τ increases. However, the pronounced moist tail disappears after a moderate time has passed and nearly-saturated air becomes extremely rare.

The difference with the no-barrier case shows up even more strongly in the decay of ensemble mean humidity, shown in Fig. 6.8. Instead of the slow algebraic decay found produced by the unbounded random walk, the bounded case yields a rapid decay of the form $A \exp(-B\sqrt{Dt})$, in accordance with the fact that the most probable maximum excursion increases with time. In Fig. 6.8 we also compare the stochastic result with the moisture decay yielded by solving the diffusion-condensation equation subject to a no-flux boundary condition at $y = 0$. The diffusivity was chosen so as to correctly reproduce the rate of spread of a cloud of particles in the random walk case. We see that the stochastic process dries air much faster than the corresponding diffusion-condensation process. The roots of this difference lie in the nonlinearity of condensation.

To shed further light on the comparison between diffusion-condensation and the random walk model, we show the profile of the ensemble mean moisture at a fixed time in Fig. 6.9, together with the corresponding moisture profile from the diffusion-condensation calculation. At every y , the stochastic process yields drier air than the diffusion-condensation process. This happens because some of the particles in each ensemble are significantly moister than the mean, and can therefore lose water by condensation upon being slightly displaced. We can also see an important distinction already familiar from the unbounded case: the stochastic process generates subsaturated air in situ even in a region with $d^2q_s/dy^2 > 0$, whereas such regions remain saturated in the diffusion-condensation model until such time as dry air invades from smaller values

of y . The stochastic model thus lacks the sharp moving front separating condensing from noncondensing regions.

To those familiar with the derivation of the diffusion equation in terms of random walk processes, it may come as some surprise that the random walk model of humidity has different behavior from the diffusion-condensation model. If q is a passive tracer, the ensemble average value obtained at each point by running a series of Brownian back trajectories to some initial time does satisfy a diffusion equation. Indeed, this technique has been used to obtain Monte Carlo solutions to the diffusion equation evaluated along aircraft tracks, without the need to solve the diffusion equation globally (Legras et al. 2003). Condensation destroys the means by which the diffusion equation for the ensemble mean field is derived, precisely because the operation of coarse-graining (taking the average of concentrations from many trajectories) does not commute with the operation of condensation. A saturated parcel and dry-air parcel, when averaged together, will not condense until subjected to substantial cooling. The same two air parcels, tracked separately, will yield condensation immediately on the slightest cooling because one is saturated.

The difficulty that emerges from the condensation process is in fact endemic to all systems where the tracer evolves according to some nonlinear process, including most chemical reactions. The difficulty of describing the evolution of the coarse-grained concentration field by means of a partial differential equation calls into question the very notion of an “eddy diffusivity” when nonlinear processes and unresolved small-scale fluctuations are involved.

We suggest that the random walk model with a barrier provides a minimal conceptual model for the generation of dry air in the atmosphere. It generates a rate and a profile; it generates unsaturated air in the interior of regions with exponentially decaying saturation specific humidity, without waiting for dry air to invade from the boundary; it predicts the probability distribution of subsaturation, which is a necessary input to radiation and other nonlinear processes; it has much in common with the way the real atmosphere generates dry air. Thus, it is a much better conceptual model than diffusion-condensation. As we saw in section 6.5.1, when the diffusion-condensation model is supplied with moisture from a boundary layer that (like the observed one) is nearly saturated, it saturates the entire atmosphere except for a thin strip near the ground. In the next section, we will see that the random walk model is free from this shortcoming.

6.5.3. Forced Equilibria in the Stochastic Problem

We now examine the statistical equilibria obtained by balancing the drying processes of the preceding section against some simple models of the moisture source. As a first example, we introduce a moisture source into the bounded random walk model by supposing that parcels are reset to saturation when they encounter the boundary at

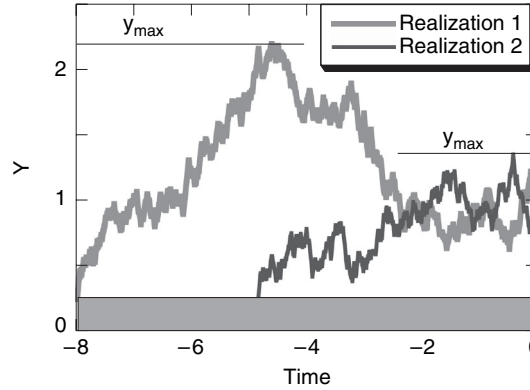


FIGURE 6.10. Two realizations of a random walk between the time of most recent encounter with the moisture source at the boundary and the time of arrival at the target position.

$y = 0$. At large times, the humidity at the target point is no longer determined by the maximum y (minimum q_s) encountered over an arbitrarily long time τ . The statistic of interest is now the maximum y attained since the *most recent* encounter with the boundary. This statistic is illustrated in Fig. 6.10. Back trajectories that take a long time to hit the boundary have a higher probability of significantly overshooting the target position, and therefore yield drier values.

The equilibrium specific humidity PDF is given by equation (6.4), where the moistening probability $P_{sat}(\tau)$ is the probability that a trajectory starting at y first encounters the boundary after time τ . This encounter-time probability is also a classic statistic studied in the theory of Brownian motion, where it is sometimes referred to as a “stopping time problem.” It can be derived from a maximum *negative* excursion statistic analogous to equation (6.16), giving the probability that the *smallest* value visited was y_{min} , which necessarily is less than y_b . There is a characteristic time $\tau_D = y_b^2 / D$ in the problem. For $t \ll \tau_D$, few particles have had time to encounter the boundary because the width of spread of the cloud starting at y_b is smaller than the distance to the boundary. For $\tau \gg \tau_D$, most particles have already encountered the boundary, and the probability of a new first encounter decays like $\tau^{-3/2}$. When plugged into the convolution (6.4), these results imply that the equilibrium moisture PDF looks something like $Q_{min}(q|q(y_b), \tau_D)$, that is, like a freely decaying drying process that has only been allowed to run for a time τ_D . However, because of the fat $\tau^{-3/2}$ tail of the encounter-time PDF, there is also a considerable additional population of dry air, arising from trajectories that have taken much longer than τ_D to encounter the boundary.

Results of an equilibrium Monte Carlo simulation are shown in Fig. 6.11. In this simulation, 20,000 particles started at various y_b are random-walked while keeping track of their moisture, until almost all have reached the boundary. The moisture PDFs were computed assuming $q_s = \exp(-y)$, as before. Recall that in this case, the PDF of $-\ln(q/q_s)$ is also the PDF of the maximum excursion relative to y_b . As seen in

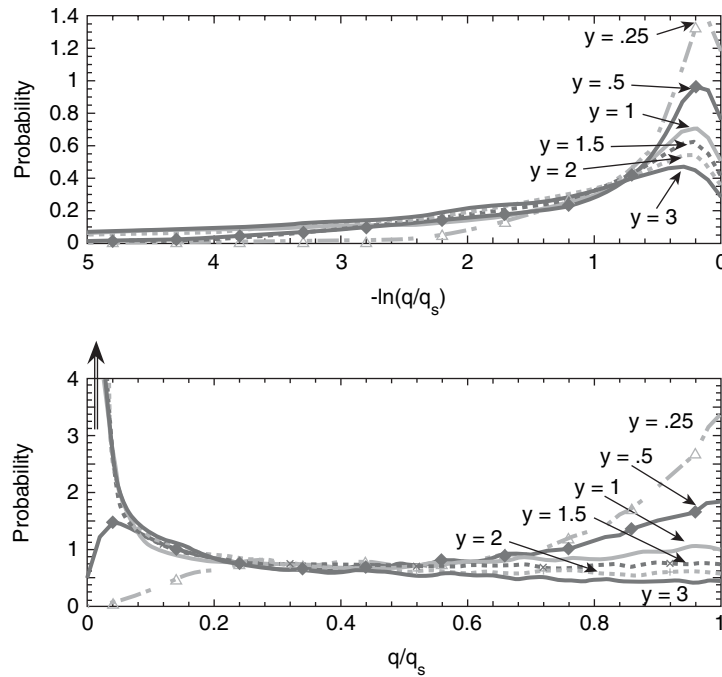


FIGURE 6.11. Equilibrium probability distributions for $-\ln(q/q_s)$ (top panel) and q/q_s (bottom panel) at the various positions indicated on the curves. The PDFs are shown at $y = 0.25, 0.5, 1, 1.5, 2$, and 3 . Particles execute a random walk in the domain $[0, \infty]$, and the moisture tagged to each particle is reset to saturation when they encounter the boundary at $y = 0$. Calculations were carried out with $q_s(y) = \exp(-y)$. Plotting of the full height of the dry spike in the lower panel has been suppressed to make the remainder of the behavior more visible.

the top panel, as we increase the distance from the source, the proportion of particles whose overshoot is near zero reduces, and the most probable overshoot moves slightly toward larger values. The most prominent feature at larger distances, though, is that the distribution develops a fat tail, indicating a fairly high probability of large overshoots (large values of $-\ln q/q_s$). If q/q_s were lognormal, the tails in the distribution shown would be Gaussian. Replotting the data with a logarithmic ordinate (not shown) reveals that the tails in the equilibrium distribution are exponential rather than Gaussian.

In the PDFs of q itself (lower panel), the fat tails in the overshoot probability create a pronounced dry spike near $q = 0$. The overall evolution of the q PDF as y is increased can be described as a shift of probability from nearly saturated values to very dry values. For positions moderately close to the source, the PDF of q is distinctly bimodal, with a moist peak and a dry peak; the moist peak disappears at larger distances. At all distances, there is a broad shoulder of intermediate humidities, over which the probability is nearly uniform.

The cold-trap configuration provides a very clear-cut example of the contrast between mean field theories like diffusion that model everything in terms of the field

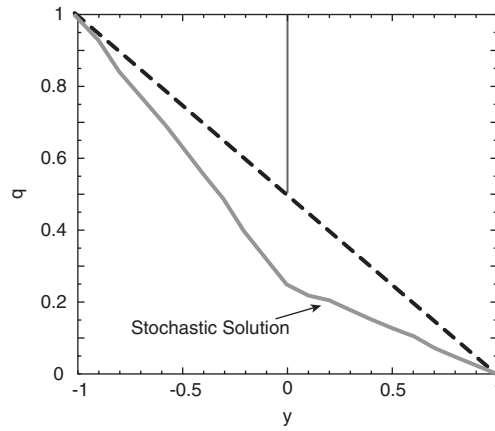


FIGURE 6.12. The equilibrium profile of mean humidity for a cold-trap configuration subjected to boundary conditions $q = 1$ at the left-hand boundary and $q = 0$ at the right. The cold trap is at the center of the domain, and resets the moisture to $q = \frac{1}{2}$. The dashed line gives the result for the diffusion-condensation equation, whereas the solid line gives the result of a Monte Carlo simulation of the random walk model.

of an ensemble average quantity and models like the random walk model that retain information about fluctuations. The specific configuration we consider is subjected to boundary conditions $q = 1$ at $y = -1$ and $q = 0$ at $y = 1$. The saturation specific humidity is unity everywhere, except for a narrow region near $y = 0$ where $q_s = \frac{1}{2}$. For the diffusion-condensation model, the equilibrium profile of q is the straight-line pure conduction profile joining the boundary values, indicated by the dashed line in Fig. 6.12. The cold trap causes no condensation, and has no effect on the moisture profile in the diffusion-condensation model. We also solved this problem using the random walk model, by random-walking 5000 particles, resetting their moisture values to 1 or 0 upon encounters with the left or right boundary, and resetting the moisture to $\frac{1}{2}$ upon encounter with the cold trap. The ensemble mean moisture profile for this model is given by the solid line in Fig. 6.12, and has a kink indicating dehydration by the cold trap. It is as if the cold trap exerts an “action at a distance” caused by the fluctuations about the mean moisture, allowing it to have an effect on moisture that cannot be determined on the basis of the knowledge of the mean alone. The origins of this behavior are no great mystery. In the stochastic model, there are only three possible values for q : 1, $\frac{1}{2}$, and 0. A particle with $q = 1$ will always lose moisture upon crossing the cold trap, so there are no such particles to the right of the cold trap. Moreover, particles with $q = 1$ to the left of the cold trap but not too far from it have a high probability of crossing over and back again, reducing the population of such particles to the left of the cold trap. This behavior is illustrated in Fig. 6.13, which shows how the population of particles depends on y . A population consisting of half $q = 1$ particles and half $q = 0$ particles

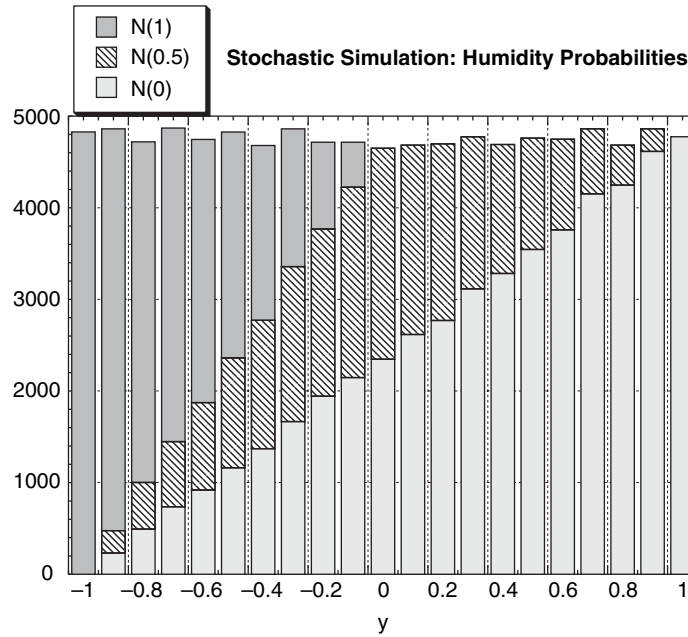


FIGURE 6.13. Spatial dependence of the number of particles of each type in the random walk simulation of the cold-trap configuration shown in Fig. 6.12.

would not condense on the basis of the average q for the ensemble, but does nonetheless lose moisture since each individual $q = 1$ particle will condense when crossing the cold trap.

What we have just presented is but one instance of a large class of possible stochastic water vapor models kept in equilibrium by a moisture supply. Many refinements or variations are possible. For example, one can include a second reflecting barrier, at $y_b > 0$; this would represent some of the effects of the confinement of trajectories within the troposphere. Another variant is the random-walk/steady-subsidence model, which approximates the tropical situation. In this case, we think of y as latitude; particles execute a random walk in y through a horizontally homogeneous q_s field, but q_s is allowed to vary with pressure p and the trajectories undergo a steady subsidence in p as they random-walk in y . Parcels are reset to saturation when they encounter the boundary at $y = 0$, which is thought of as the region of the Tropics that is maintained near saturation by deep convection. This model is essentially equivalent to the subsidence model described by equation (6.5), with $P_{sat}(\tau|y_o)$ taken to be the probability distribution of time required for particles starting at $y = y_o$ to encounter the boundary. Since the mean waiting time gets longer as distance from the boundary increases, the air at any given p becomes drier with distance from the boundary because it has subsided more in the time it takes to get there. Still a further refinement of this model would be to replace the assumption of resaturation at $y = 0$ with an assumption that the parcels are resaturated when they encounter a more general, spatially complex

set Σ in (x, y) space. One would correspondingly replace the one-dimensional random walk with either a two-dimensional random walk in (x, y) or a random walk in y and steady sheared advection in x . This model now begins to approach the model in Pierrehumbert (1998), where trajectories were modeled using observed winds on isentropic surfaces, but with a steady subsidence across isentropic surfaces, and were resaturated upon encounters with the actual tropical convective region.

6.6. How Will Water Vapor Change in Reaction to a Changing Climate?

It would be fruitful to tinker with the stochastic water vapor model in search of improved, non-Brownian statistical descriptions of the trajectories that might better reproduce the salient properties of real trajectories. The key characteristic of mixing to compute is the maximum excursion probability distribution. Time-correlated random walks can still be treated using the reflection principle so long as they are Markov processes. However, there are few general methods for computing such path statistics for non-Markov random walk processes, such as Levy flights, and it is likely that one would need to resort to Monte Carlo simulations to get the needed PDF, if such processes turn out to be needed to capture the salient characteristics of atmospheric trajectories. We believe this is the correct path to take in the quest for water vapor parameterizations suitable for incorporation in idealized climate models. In this section, however, we leap ahead to the direct use of trajectories computed from observed or simulated wind fields, without any further attempt at a reduced statistical description. Our goal in this section is to study how temperature affects the relative humidity of the free troposphere, taking into account the Lagrangian nature of the problem. In particular, we shall attempt to provide some precise justification for the expectation that free tropospheric humidity will increase as the climate becomes warmer.

The procedure we employ here is identical to that used in the equilibrium stochastic model, except that the back trajectories are computed using simulated or observed/analyzed three-dimensional global wind fields. In particular, the effect of internal mixing and moisture sources are neglected; trajectories are tracked back to the boundary layer, and assumed to be saturated at that point. The moisture at the terminal point of the trajectory is then the minimum saturation specific humidity experienced since the encounter with the boundary layer. This approach was employed in Pierrehumbert and Roca (1998), who found excellent agreement with the observed tropical dry-zone humidity. The same advection-condensation model has since been applied to much larger datasets in Roca et al. (2005) and Brogniez (2004). We will concentrate our efforts on the midlatitude moisture distribution, since this region has received less attention than the subtropical problem. The region of study is the midlatitude Atlantic region bounded by 100° E to 0° E longitude and 30° N to 60° N latitude.

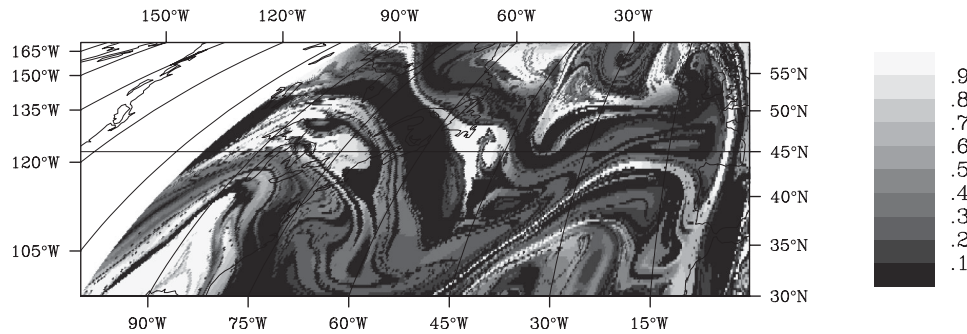


FIGURE 6.14. 500 mb relative humidity at 12Z on December 3, 1994, reconstructed using the back trajectory method driven by NCEP winds and temperature.

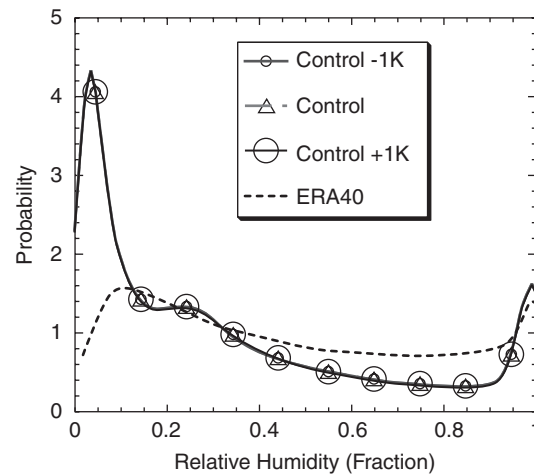


FIGURE 6.15. Probability distribution of relative humidity for December 1994 over the region shown in Fig. 6.14, computed 4 times daily using NCEP winds and temperatures. Results for experiments with temperature uniformly increased or decreased by 1 K are also shown, but the curves are barely visible because they lie almost exactly on the control case. For comparison, the relative humidity PDF over the same time and region for the ERA-40 analysis is also shown.

First, we computed the back trajectory reconstructed relative humidity fields using NCEP winds and temperatures for December 1994. The reconstruction was performed four times daily at a spatial resolution of 0.25 degrees in latitude and longitude. To provide a general idea of what the fields look like, a map of relative humidity at 12Z on 3 December 1994 is shown in Fig. 6.14. It shows the filamentary structure familiar from earlier work, with small-scale alternations of moist and dry air.

The PDF computed over the region of study for the entire month of data is shown in Fig. 6.15. It has a dry spike and a moist spike, with a broad region of nearly uniform

but weakly decreasing probability in between. In this sense the computed PDF looks qualitatively like the PDF for the equilibrium random walk model forced by a saturated boundary layer, at the point $\gamma = 0.5$ (Fig. 6.10). For the sake of comparison with an observationally based moisture estimate, we also show the relative humidity PDF calculated from 500 mb ERA-40 analyses covering the same time period and region.⁵ As compared to the trajectory reconstruction, the ERA-40 PDF has a much less pronounced dry spike, but a greater population of air with intermediate saturation. A pattern like this could be obtained by allowing some mixing between moist and dry filaments in the trajectory model, possibly representing a physical effect left out of the trajectory model. It could also result from moistening of dry air by evaporation of precipitation falling through it. On the other hand, the lack of a dry spike in the ERA-40 result may be an unphysical artifact of excessive numerical diffusion in the model used to do the analysis/assimilation cycle. Is reality more like ERA-40 or more like the trajectory-based reconstruction? We do not know of any humidity dataset that can unambiguously answer this question at present.

Now we pose the question of what happens if the trajectories are kept fixed but the temperature at all points in the atmosphere is increased by a uniform amount ΔT . In a real climate change, such as caused by the doubling of CO_2 , one can expect the statistics of trajectories to change somewhat, and it is well known also that the warming is not uniformly distributed in latitude and altitude. In posing the simplified form of the problem, we are supposing for the moment that such effects are of only secondary importance in determining the changes in relative humidity. The problem stated this way forms a kind of null hypothesis about the behavior of water vapor, which can serve as a launching point for more sophisticated extensions.

The effect of uniform warming on the relative humidity PDF can be determined by straightforward reasoning. Consider a point with temperature and pressure (p, T) . The humidity here is determined by the point (p_m, T_m) where the most recent minimum saturation specific humidity following a resaturation event occurred (i.e., the position of the “time of last saturation”). Using the fact that the specific humidity is conserved in the absence of condensation, we find that the relative humidity is $(e_s(T_m)/p_m)/(e_s(T)/p) = (p/p_m)(e_s(T_m)/e_s(T))$. If we increase temperatures uniformly and keep trajectories fixed, p_m stays the same and the new relative humidity is $(p/p_m)(e_s(T_m + \Delta T)/e_s(T + \Delta T))$. Now, if ΔT is small compared to T and T_m , as is usually the case, the Clausius-Clapeyron relation implies that the new relative humidity is approximately

$$(\Delta T) \approx \left(\frac{p}{p_m} \right) \frac{e_s(T_m) + e_s(T_m) \frac{L}{R_v T_m^2} \Delta T}{e_s(T) + e_s(T) \frac{L}{R_v T^2} \Delta T} \approx r h(0) \left(1 + \frac{L}{R_v T} \left(\frac{T^2}{T_m^2} - 1 \right) \frac{\Delta T}{T} \right). \quad [6.19]$$

This expression assumes the perfect gas law, but otherwise proceeds directly from Clausius-Clapeyron without the need to assume the approximate exponential form

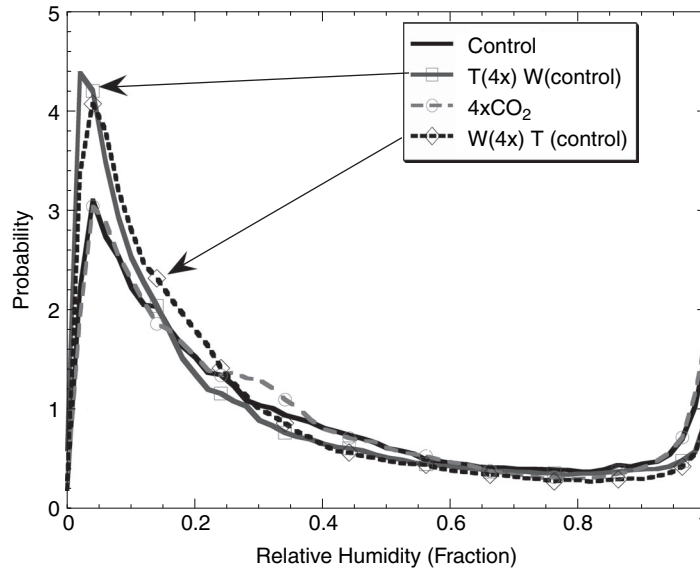


FIGURE 6.16. Relative humidity PDFs reconstructed by the back trajectory technique driven by various combinations of winds and temperature fields from a control and a $4 \times \text{CO}_2$ GCM simulation. See text for definition of the cases.

given in equation 6.1. This result implies that the relative humidity increases with warming, but the coefficients are such that the changes are quite small for moderate values of ΔT . For example, with $T = 260 \text{ K}$, $T_m = 240 \text{ K}$ and $\Delta T = 1 \text{ K}$, the increase in relative humidity at each point is only $0.014rh(0)$. Hence, it is expected that moderate uniform warming or cooling should leave the PDF of relative humidity essentially unchanged. To confirm this reasoning, we recomputed the NCEP back trajectory PDFs with temperature uniformly increased by 1 K and also with temperature uniformly decreased by 1 K . These curves are plotted in Fig. 6.15, and overlay the control back trajectory PDF almost exactly.

We now apply the same technique to diagnosis of humidity changes in a GCM simulation of climate change. This enables us to probe the effects of changes in the temperature structure, and changes in the statistics of the trajectories. We consider two equilibrium simulations with the FOAM GCM coupled to a mixed-layer ocean model. Two simulations were carried out, both with realistic geography: the first with 300 ppmv CO_2 (the “control” case) and the second with 1200 ppmv CO_2 (the “ $4 \times \text{CO}_2$ ” case). Figure 6.16 shows the relative humidity PDFs constructed using the back trajectory method applied to the December GCM wind and temperature fields for both the control case and the $4 \times \text{CO}_2$ case; the latter is labeled $W(4\times) T(4\times)$. Despite the changes in temperature pattern and wind fields, the PDFs have almost the same shape for both simulations. The warm simulation has a slightly greater population of air with relative humidity in the vicinity of 30%, at the expense of a reduction in more saturated

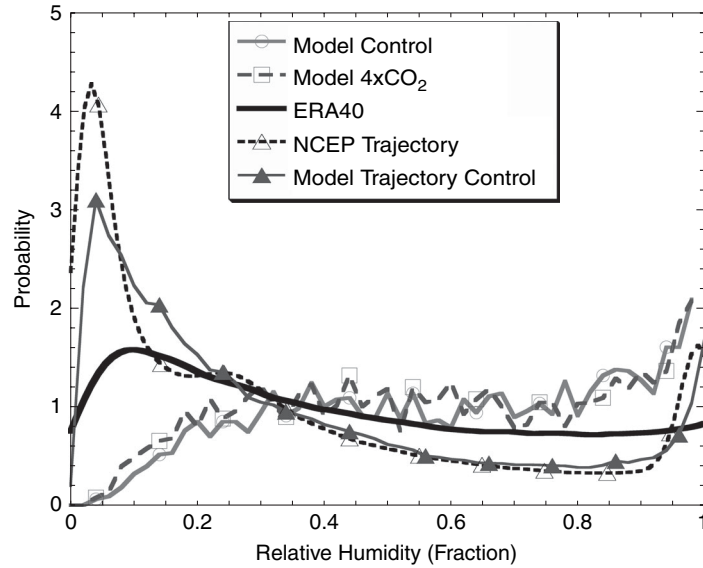


FIGURE 6.17. Relative humidity PDFs for the internally computed GCM moisture fields (model control and model $4 \times \text{CO}_2$) compared with results from the ERA-40 analysis, the NCEP trajectory reconstruction, and the trajectory reconstruction driven by GCM-simulated winds and temperature for the control case (model trajectory control).

air. We performed two additional back trajectory reconstructions in an attempt to isolate the importance of changes in temperature structure and trajectory statistics. The reconstruction labeled “ $T(4 \times) W(\text{control})$ ” was carried out with the temperature field from the $4 \times \text{CO}_2$ run but wind fields from the control run. The reconstruction labeled “ $T(\text{control}) W(4 \times)$ ” was carried out with the wind field from the $4 \times \text{CO}_2$ run but temperature fields taken from the control run. Both reconstructions generate more dry air and less saturated air than the previous two reconstructions. Changes in the temperature structure in a warmer climate, taken in isolation, enhance the production of dry air. However, the trajectories in the warmer climate adjust so as to avoid the colder regions, leaving the PDF invariant when both effects are applied in conjunction. Why this cancellation of effects should take place is at present wholly mysterious. The cancellation is particularly difficult to understand in light of the fact that trajectory changes, taken in isolation, also enhance dry air production. It will be very interesting to see whether other GCMs exhibit similar behavior.

How does the relative humidity PDF reconstructed from back trajectories compare with that based on the humidity computed internally to the model? These may be expected to be different, since the former reconstructs what the humidity would be in the absence of mixing and internal moisture sources, whereas the low resolution GCM allows a considerable degree of mixing. As seen in Fig. 6.17, the dry spike seen in both NCEP and model Lagrangian reconstructions is completely absent from the

internally computed PDF. This is a result of probably excessive mixing in the GCM. Significantly, the wind statistics in the GCM do not seem to be a problem, for the Lagrangian reconstruction based on model winds is reasonably similar to that based on NCEP winds. The PDF from the ERA-40 analysis is also shown in the figure to underscore that the GCM is deficient in dry air. This deficiency is not surprising, since the leakage of a small proportion of moisture from saturated air is enough to eliminate a considerable population of dry air. We do not wish to imply that this deficiency is characteristic of all GCMs, though it may indeed be endemic to low resolution GCMs. It is interesting that, despite the considerable effect of mixing on the internally computed humidity, the GCM humidity PDF is nonetheless invariant between the control and $4 \times \text{CO}_2$ case.

In contrast with the GCM and trajectory-based diagnostic results presented above, the idealized humidity model of Minschwaner and Dessler (2004) predicts a modest decrease of free tropospheric relative humidity as the climate warms; this slightly reduces the positive water vapor feedback, but does not eliminate it. Since Minschwaner and Dessler (2004) is a tropical model, whereas the analyses above were applied to a midlatitude case, a more meaningful comparison must await the application of the trajectory-based diagnostics to tropical regions. This is straightforward, but will be deferred to future work. If the disagreement persists, it might be due to the neglect of lateral mixing in Minschwaner and Dessler (2004), or it might be due to the low vertical resolution of the simulated and analyzed tropical climate as compared to the resolution employed in Minschwaner and Dessler (2004).

6.7. Conclusions

The behavior of water vapor in the climate system is complex and multifold, and there will probably never be any one simple theoretical framework that accounts for everything one would like to understand about water vapor. The problem in its full complexity, as manifest in either atmospheric observations or comprehensive GCMs, defies human comprehension. Understanding, as opposed to mere simulation, requires simple models whose behaviors can be grasped in their entirety, even if they are wrong or incomplete in some particulars. This would be true even if GCM simulations were perfect, and the necessity is even more pressing in the face of simulations that are both imperfect and in certain regards suspect. We have presented some simple ideas pertinent to the proportion of subsaturated air in the atmosphere, and the degree of its subsaturation. The emphasis on highly subsaturated (“dry”) air arises from its importance in determining the radiative feedback of water vapor.

In particular, based on an analysis of the way large-scale atmospheric trajectories influence subsaturation, we have provided a concise and defensible statement of why one should expect atmospheric water vapor to increase as climate gets warmer: *The*

specific humidity at a given point in the atmosphere is determined by the saturation specific humidity at the point of minimum temperature encountered along the trajectory extending backward in time from this point until it encounters a moisture source sufficiently strong to saturate it. If the statistics of the trajectories do not change too much as the atmosphere warms, this minimum temperature increases, leading to an increase in the water vapor content of the target point. This is a somewhat cartoonish statement that is modified in its details by closer study, but one that survives a fair amount of scrutiny. It is intended to replace wholly indefensible statements which simply invoke the Clausius-Clapeyron relation. Clausius-Clapeyron is indeed at the root of the behavior of water vapor, but the proper use of the relation hinges on identifying the temperature to which the relation should be applied; it's not the surface temperature, and the effect of the relation on evaporation is of little relevance to water vapor feedback. If one takes the cartoon picture to its idealized extreme, it predicts that if trajectories are held fixed while the atmosphere warms uniformly in space and time, the relative humidity probability density function (PDF) will remain nearly invariant for small or moderate warming. This behavior was confirmed in a midlatitude case, using trajectories and unperturbed temperatures from NCEP analyses.

The Lagrangian viewpoint lends itself to the formulation of diagnostics that can be applied to atmospheric data and GCM simulations, and used to compare the operation of processes in GCM simulations to that in atmospheric data by applying the same diagnostics to both. In essence, one uses three-dimensional wind and temperature fields to study the pattern of an idealized moisture substance that does not diffuse from one air parcel to another, and that is maintained by an idealized source (typically in the boundary layer). Applied to midlatitude NCEP data, the diagnostic yields a relative humidity PDF whose dominant features are a dry spike at 5% relative humidity, a lesser spike at saturation, and a broad tail of intermediate degrees of saturation in between. The corresponding PDF computed from ERA-40 data has a much broader and less pronounced dry peak, centered on 10% relative humidity, and a greater probability of air with intermediate humidity. A key unresolved question at this point is whether reality looks more like the ERA-40 analysis, or the nondiffusive trajectory reconstruction. The ERA-40 analysis, which incorporates assimilated satellite data, could be indicative of the importance of mixing processes in the real atmosphere; likely candidates include vertical turbulent diffusion at small scales (Haynes and Anglade 1997) and vertical redistribution of moisture owing to evaporation of precipitation falling through very dry air. On the other hand, the broad dry maximum in the ERA-40 PDF may be symptomatic of excessive numerical diffusion in the assimilation process.

The Lagrangian moisture reconstruction diagnostic is an instance of the general idea of studying the behavior of an idealized water-like substance that isolates some key feature of the problem one would like to understand and which is easier to understand than the real thing. The isentropic study of Yang and Pierrehumbert (1994) was one of the earlier examples of this approach. That study examined, in effect, a water-like

tracer that condensed, but had zero latent heat so that condensation would not cause a trajectory to leave a fixed dry isentropic surface. The recent work by Galewsky et al. (2005) also makes use of an idealized water substance that doesn't release latent heat, but this time in an Eulerian framework that allows for the effects of mixing amongst air parcels. In the same spirit, Frierson et al. (2006) have used a synthetic water vapor with adjustable latent heat but no radiative impact, in order to focus on the influence of moist processes on energy transport.

The Lagrangian moisture diagnostics were also applied to a GCM simulation of warming induced by a quadrupling of CO_2 . The midlatitude relative humidity PDF reconstructed on the basis of trajectories driven by model winds was found to be invariant between the control and $4 \times \text{CO}_2$ cases, even though the model warming is not spatially uniform and the trajectory statistics are allowed to change. However, the invariance of the PDF was found to result from a mysterious cancellation between the effects of nonuniform warming and changes in the excursion statistics of the trajectories, each of which individually causes a moderate increase in the population of subsaturated air. The humidity PDF reconstructed on the basis of the model wind and temperature closely resembles that computed on the basis of observed winds and temperatures. However, the PDF of relative humidity computed internally to the GCM is very different from the Lagrangian diagnostic, and is completely missing the dry peak. We believe this is probably due to excessive mixing in the low-resolution model. Nonetheless, the PDF of the GCM internally computed humidity is still invariant under warming, indicating that the basic invariance inferred from the nondiffusive Lagrangian calculation survives the addition of a considerable degree of mixing.

In order to illustrate the manner in which certain trajectory statistics govern the probability distribution of humidity, we formulated and analyzed a family of idealized models of water vapor in which trajectories are modeled as random walk processes. The calculation predicts the shape of the PDF and the way the PDF depends on distance from the humidity source. With some further refinements, it is even possible that models of this class could be made suitable for use in idealized climate models, enabling such models to treat a broader range of questions concerning water vapor feedback. Without a moisture source, the stochastic model predicts that, in midlatitude conditions, the mean humidity at any given point decays like $\exp(-\sqrt{Dt})$ for some constant D if the random walk is bounded above in temperature space. The humidity PDF quickly develops a peak at dry values, with an approximately exponential moist tail. In the presence of a moisture source at the boundary, the equilibrium PDF develops a dry spike at sufficiently large distances from the source, with a "fat tail" extending to more saturated values. At moderate distances from the source, the PDF is bimodal, with a secondary peak at saturation. This shape is similar to that found in midlatitudes using realistic trajectories.

We have compared the predictions of the stochastic model with those of a more conventional approach based on diffusion of moisture. Diffusion creates monolithic

regions of saturated air and dries the atmosphere in a manner very different from the idealized stochastic model, which more closely reproduces the mechanisms operating in the real atmosphere. Diffusion is not a suitable approach to representation of water vapor in simplified climate models, at least not if one's aim is to treat water vapor feedback. The comparison between the stochastic model and the diffusion model also highlights the mathematical novelty of the former. Viewed as a stochastic problem, the distribution of subsaturation depends on probability distributions on the space of *paths*, rather than just on the space of endpoints of paths encountered in conventional linear diffusion problems. Lagrangian history probability problems of this type cannot be adequately modeled with a mean field theory like the partial differential equation for diffusion. One needs a Monte Carlo approach, or some other approach that retains information about fluctuations as well as ensemble averages. This calls into question the whole utility of eddy diffusivity as a means of parameterizing mixing of tracers subject to a nonlinear removal process.

The trajectory approach, as usually formulated, does not treat regions of deep tropical convection explicitly. These are detected through the large-scale wind field as regions of persistent upward motion, and are almost always saturated with moisture. This has the virtue of not requiring information about convection that is not implicit in the wind field, but it does amount to an assumption of saturated detrainment from convective regions. This leaves out all the mechanisms of the sort discussed by Tompkins and Emanuel (2000), which could keep convective regions significantly subsaturated. The trajectory approach could be improved by making explicit use of information about convection, and allowing for subsaturated detrainment. The logarithmic dependence of outgoing longwave radiation on specific humidity means, however, that the subsaturation would have to be very substantial before it had much effect on water vapor feedback.

We have sidestepped the issue of cloud radiative effects by speaking throughout of “clear sky” radiation when translating water vapor into outgoing longwave radiation. However, the minute a separation between clear-sky and cloudy-sky radiation is invoked, one is implicitly assuming that such a distinction is meaningful. The concept of “clear sky” outgoing long-wave radiation makes sense when applied to large coherent regions free of mid- or high-level clouds, and one should in fact understand the term to allow the inclusion of parts of the scene including boundary-layer clouds (which do not affect the outgoing longwave radiation). In contrast, tropical convective regions and the midlatitude storm tracks are typically characterized by a more-or-less continuous and spatially complex distribution of cloudy and clear air, and all types in between. Since condensed water is so opaque to infrared radiation, the escape of infrared radiation to space is more controlled by the size of the clear-sky holes between clouds than by fluctuations in their humidity. This engages the whole subject of fractional cloud cover, which may well be the most problematic aspect of cloud representation in climate models. Learning to characterize and predict the radiative effect of intermingled,

coupled assemblages of cloudy and clear air is one of the greatest challenges to the understanding of climate.

Acknowledgments

The research upon which this chapter is based was funded by the National Science Foundation under grants ATM-0121028 and ATM-0123999. We are grateful to the editors of this book for numerous valuable suggestions.

Notes

1. q/q_s is not precisely equal to the relative humidity, which is defined as the ratio of the mixing ratio of water vapor to the saturation mixing ratio (equivalently, the ratio of the partial pressure of water vapor to the saturation partial pressure). However, for the purposes of this chapter, where water vapor is assumed to be a minor constituent of the atmosphere, mixing ratio and specific humidity can be regarded as practically interchangeable.

2. When coupled to a mixed-layer ocean, the FOAM GCM is essentially a portable, Beowulf-oriented re-implementation of CCM3 (Kiehl et al. 1998). All simulations reported in this chapter were carried out at R15($4.5^\circ \times 7.5^\circ$) resolution. Further details on FOAM can be found at www.mcs.anl.gov/foam.

3. The recent work of Sherwood and Meyer (personal communication) on relative humidity PDFs employs a special case of this class of solutions.

4. In the regularized case with small but finite τ , the removal process is less singular and the steady solution has q very slightly greater than q_s and nonzero removal rate S .

5. We used ERA-40 analyses in preference to NCEP analyses because we found that the ERA-40 analyses seem to give better agreement with patterns seen in satellite retrievals. We used the analyses in preference to satellite data itself because the latter represent averages over a fairly deep atmospheric layer, whereas the analyses are available at an individual model level.

References

- Brogniez, H., 2004: *Humidité de la Troposphere Libre Africaine: Elaboration d'une Archive Meteosat, Analyse Climatique et Evaluation de Modeles*. PhD thesis, Univ. Paris VI, 250 pp.
- Colman R.A., and McAvaney B.J., 1997: A study of general circulation model climate feedbacks determined from perturbed sea surface temperature experiments. *J. Geophys. Res. — Atmospheres*, **102 (D16)**, 19383–19402.
- Emanuel, K., and Pierrehumbert, R.T., 1996: Microphysical and dynamical control of tropospheric water vapor. In *Clouds, Chemistry and Climate*, NATO ASI Series **35**. Springer: Berlin, 260 pp.
- Frierson D., Held H., and Zurita-Gotor P., 2006: A gray-radiation aquaplanet moist GCM. Part I: Static Stability and Eddy Scales. *J. Atmos. Sci.*, in press.

- Galewsky J., Sobel A., and Held I.M., 2005: Diagnosis of subtropical humidity dynamics using tracers of last saturation. *J. Atmos. Sci.*, **62**, 3353–3367 (doi: 10.1175/JAS3533.1).
- Hall A., Manabe S., 1999: The role of water vapor feedback in unperturbed climate variability and global warming. *J. Climate*, **12**, 2327–2346.
- Haynes P., and Anglade J., 1997: The vertical-scale cascade in atmospheric tracers due to large-scale differential advection. *J. Atmos. Sci.*, **54**, 1121–1136.
- Held I.M., and Soden B.J., 2000: Water vapor feedback and global warming. *Annu. Rev. Energ. Env.*, **25**, 441–475.
- Karatzas, I., and Shreve S.E., 1991: *Brownian Motion and Stochastic Calculus*, Springer: New York, 469 pp.
- Kelly K.K., Tuck A.F., and Davies T. 1991: Wintertime asymmetry of upper tropospheric water between the Northern and Southern Hemispheres. *Nature*, **353**, 244–247.
- Kiehl J.T., Hack J.J., Bonan G.B., Boville B.A., Williamson D.L., and Rasch P.J., 1998: The National Center for Atmospheric Research Community Climate Model: CCM3. *J. Climate*, **11**, 1131–1149.
- Legras B., Joseph B., and Lefevre F., 2003: Vertical diffusivity in the lower stratosphere from Lagrangian back-trajectory reconstructions of ozone profiles. *J. Geophys. Res. — Atmospheres*, **108** (D18), 4562, doi:10.1029/2002JD003045.
- Lesigne E., 2005: *Heads or Tails: An Introduction to Limit Theorems in Probability* (A. Pierrehumbert, translator), American Mathematical Society, 149 pp.
- Lindzen R.S., 1990: Some coolness concerning global warming. *B. Am. Meteorol. Soc.*, **71**, 288–299.
- Manabe S., and Wetherald R.R., 1967: Thermal equilibrium of atmosphere with a given distribution of relative humidity. *J. Atmos. Sci.*, **24**, 241–259.
- Minschwaner K., and Dessler A.E., 2004: Water vapor feedback in the tropical upper troposphere: Model results and observations. *J. Climate*, **17**, 1272–1282.
- Pauluis O., and Held I.M., 2002: Entropy budget of an atmosphere in radiative-convective equilibrium. Part II: Latent heat transport and moist processes. *J. Atmos. Sci.*, **59**, 140–149.
- Petoukhov V., Ganopolski A., Brovkin V., Claussen M., Eliseev A., Kubatzki C., and Rahmstorf S., 2000: CLIMBER-2: A climate system model of intermediate complexity. Part I: Model description and performance for present climate. *Climate Dynamics*, **16**, 1–17.
- Pierrehumbert R.T., 1998: Lateral mixing as a source of subtropical water vapor. *Geophys. Res. Lett.*, **25**, 151–154.
- Pierrehumbert R.T., 1999: Subtropical water vapor as a mediator of rapid global climate change. In Clark P.U., Webb R.S., and Keigwin L.D., eds., *Mechanisms of global change at millennial time scales*. American Geophysical Union: Washington, D.C. Geophysical Monograph Series **112**, 394 pp.
- Pierrehumbert R.T., 2002: The hydrologic cycle in deep time climate problems. *Nature*, **419**, 191–198.

- Pierrehumbert, R.T., and Roca R., 1998: Evidence for control of Atlantic subtropical humidity by large scale advection. *Geophys. Res. Lett.*, **25**, 4537–4540.
- Roca R., Lafore J.-Ph., Piriou C., and Redelsperger J.L., 2005: Extra-tropical dry air intrusions into the West African Monsoon mid-troposphere: An important factor for the convective activity over the Sahel. *J. Atmos. Sci.*, **62**, 390–407.
- Salathe E.P., and Hartmann D.L., 1997: A trajectory analysis of tropical upper-tropospheric moisture and convection. *J. Climate*, **10**, 2533–2547.
- Sherwood S.C., and Dessler A.E., 2000: On the control of stratospheric humidity. *Geophys. Res. Lett.*, **27**, 2513–2516.
- Shine K.P., and Sinha A., 1991: Sensitivity of the Earth's climate to height-dependent changes in the water-vapor mixing-ratio. *Nature*, **354** (6352), 382–384.
- Soden B.J., 1998: Tracking upper tropospheric water vapor. *J. Geophys. Res.*, **103** (D14), 17069–17081.
- Soden B.J., and Bretherton F.P., 1993: Upper-tropospheric relative-humidity from the Goes 6.7 μ channel—method and climatology for July 1987. *J. Geophys. Res.—Atmospheres*, **98** (D9), 16669–16688.
- Spencer R.W., and Braswell W.D., 1997: How dry is the tropical free troposphere? Implications for global warming theory. *B. Am. Meteorol. Soc.*, **78**, 1097–1106.
- Tompkins A.M., and Emanuel K.A., 2000: The vertical resolution sensitivity of simulated equilibrium temperature and water-vapour profiles. *Quart. J. Roy. Meteorol. Soc.*, **126B**, 1219–1238.
- Weaver A.J., Eby M., Wiebe E.C., et al. 2001: The UVic Earth System Climate Model: Model description, climatology, and applications to past, present and future climates. *Atmosphere-Ocean*, **39**, 361–428.
- Yang, H., and Pierrehumbert, R. T., 1994: Production of dry air by isentropic mixing. *J. Atmos. Sci.*, **51**, 3437–3454.
- Zhang C.D., Mapes B.E., and Soden B.J., 2003: Bimodality in tropical water vapour. *Quart. J. Roy. Meteorol. Soc.*, **129** (594), 2847–2866.
- Kalnay, E., and coauthors, 1996: The NCEP/NCAR 40-year reanalysis project. *Bull. Amer. Meteor. Soc.*, **77**, 437–471.