GEO 232 2009 HW5 Solutions

Problem 5.1
To get the density use the ideal gas law with pressure in Pa and the appropriate gas constant. For CO_2 on Mars $\rho = 600./(189 \cdot 220) = .014kg/m^3$. For N_2 on Titan find that $\rho = 1.5e5/(297. \cdot 95) = 5.3kg/m^3$ and CO_2 on Venus $\rho = 9.2e6/(189. \cdot 737.) = 66kg/m^3$.

Problem 5.2
To get the density, just use the ideal gas law with pressure in Pa and the gas constant for the gas in use. Then multiply the density by the volume, and add the mass of the empty tire. For air, the density is $1.e5/(287.*290.) = 1.2kg/m^3$. For CO_2 it is $1.e5/(189.*290.) = 1.82kg/m^3$, and for He it is $1.e5/(2079.*290.) = 1.7kg/m^3$. Multiplying by volume and adding empty mass, we get $.114kg$ for air, $.122kg$ for CO_2 and $.102kg$ for He. To get the weight, the masses are multiplied by the acceleration of gravity, which is $9.8m/s^2$ at the Earth's surface. The resulting weight is a force, expressed in $Newtons$.

Problem 5.3
To answer the first problem in this question, we use the fact that potential temperature remains constant for adiabatic processes. Thus if T_1 is the final temperature, $290.(1./1.)^{-\frac{2}{7}} = T_1(4/1.)^{-\frac{2}{7}}$ so $T_1 = 431K$. To determine the pressure after cooling, we use the fact that the mass of air in the tire stays the same as the air cools, and assume the volumes remain fixed (a reasonable assumption if the tire is quite rigid, though one could think of extending the calculation to allow for contraction of the tire as the pressure goes down). If the volume and mass are fixed then the density is fixed, and so one can use the ideal gas law to determine the final pressure p_1. remembering to convert to Pa, the equation is $4.e5/(287. \cdot 431.) = p_1/(287. \cdot 290.)$, so the pressure after cooling is $270000Pa$, or $2.7bar$.

Problem 5.4
To extrapolate the Magellan observation of $440K$ at $7bar$ to the Venusian surface at $92bar$ we use the conservation of potential temperature along the CO_2 dry adiabat. Given $R/C_p \simeq .2304$ for CO_2 find $T_s = 440.(7/92.) ** -.2304 = 796K$ which is fairly consistent with the pioneer measurement of $737K$.

Problem 5.5
To determine the time required to melt or sublimate the ice sheet we must determine the total energy required using the appropriate latent heat and then divide by the total power absorbed $P_{in} = 1e14m^2 \cdot 50W/m^2 = 5e15W$. The mass of the ice sheet is $M = 930kg/m^3 \cdot 1.e14m^2 \cdot 1.e3m = 9.3e19kg$ so the energy required to melt the ice sheet would be $E_{melt} = M \cdot L_{fusion} = 9.3e19 * 3.34e5 = 3.1e25J$ and it would take $197yrs$. To sublimate the ice sheet we use $L_{sublimation} = 2.84e6J/kg$ and find that it would take $1675yrs$. If the surface temperature is below 273.15K sublimation will occur, otherwise the ice sheet will melt.

Problem 5.6

To determine the temperature at which CO_2 would condense from the Martian atmosphere we first construct a function which calculates the saturation vapor pressure of CO_2 as a function of temperature. To do this we use a tool from Ray's *phys* module the satvps_function.

psat = phys.satvps_function(phys.CO2)

Tlist = range(1,161,5)

Plist = [1.e-3*psat(T) for T in Tlist]

Now we can plot the saturation vapor pressure against temperature and see that at the Martian mean surface pressure of 7mb CO_2 would begin to condense below 149K.

Problem 5.7

To determine the partial pressure of an ocean in equilibrium with the atmosphere at 100C we consider three approximations to the Clausius-Clapeyron relation. Assuming a constant latent heat and expanding about the triple point we find a saturation vapor pressure of 1.2bar which is 0.2bar higher than the vapor pressure estimated from boiling point data or using the empirical Antoine equation. Using the 1bar estimate for the partial pressure of water we find that the molar mixing ratio is 1bar/(1bar+1bar) = 0.5. Since the total atmospheric pressure is 2bar which is greater than the partial pressure of the water the ocean will not boil under these circumstances. At 200C the atmosphere would be 94% water, but the saturation vapor pressure would still be 1bar less than the total atmospheric surface pressure so the ocean doesn't boil. The critical point for water occurs at 647K at which point our three approximations to the Clausius-Clapeyron relation yield considerably different results. Expanding about the triple point we estimate that the entire ocean would evaporate below 600K. Expanding about the boiling point indicates that the ocean would go into the atmosphere arounf 610K. The Antoine equation is reasonably accurate near the critical point and gives a saturation vapor pressure of 214bar from which we estimate a mass of water $4\pi a^2 * 214e5/9.8 = 1.1e21kg$ or about 80% of the ocean.