Geosci 232 Problem Set 2
Fall Quarter 2009
Due Wednesday, Oct 28.

October 21, 2009

2 Problem Set:Blackbody radiation and planetary temperature

Problem 2.1 What is the total power radiated by a blackbody sphere of radius $1m$ having uniform temperature $300K$, in the wavenumber range between $500cm^{-1}$ and $750cm^{-1}$. You may assume the Planck density to be approximately constant over this range. Compare this to the total power radiated over all wavenumbers. *Note:* Blackbody radiation is emitted with equal intensity in all directions, so you must remember to take this into account in computing the total energy flux coming from the surface.

Problem 2.2 Compute the *wavenumber* of maximum emission of an object with temperature $200K$. What is the rate of energy emission from each square meter of the object's surface, in the wavenumber band extending $50cm^{-1}$ on either side of the maximum? You may assume for the purposes of this calculation that the Planck function is constant over this range of wavenumbers. How good is this assumption?

Problem 2.3 *Color index and stellar temperature*

Suppose we observe the flux density from a star at two different frequencies ν_1 and ν_2, with $\nu_2 > \nu_1$. Let the measured flux densities be F_1 and F_2. Suppose further that the star radiates like an ideal blackbody with temperature $T_{\circledast}$. The two flux densities will then be proportional to the Planck function at the corresponding frequencies, multiplied by the inverse-square attenuation of the starlight. Write the flux densities as astronomical magnitudes using $M_j = C_j - 2.5 \log_{10} F_j$, where C_j the constant defining zero magnitude for each band. Show that $M_2 - M_1$ is independent of the distance from the star. This difference is called a *color index*. Plot the color index as a function of $T_\circledast$ for ν_2 corresponding to the astronomer's blue ("B") band at $0.44\mu m$ and ν_1 corresponding to the visual ("V") band at $0.55\mu m$, and show that the magnitude difference is a monotonically decreasing function of temperature. Find analytic expressions in the low-frequency and high frequency limit, and use these to get an approximate expression for $T_\circledast$ as a function of the color index.

Astronomers define color indices for specific standard filter bands. The B-V color index is very commonly reported as a proxy for the temperature of an object. Various empirical fits are available, which take into account the actual standard filter characteristics. For example, for temperatures below $9000K$, the empirical relation is $M_B - M_V = -3.684 \log_{10} T + 14.551$. Compare this fit to the results you got in your calculation above, which in essence assume infinitely narrow filters. In making this comparison, choose $C_1 - C_2$ so that the agreement is exact at $6000K$.

Problem 2.4 A cylindrical space station with length h and radius r is in an orbit about the Sun at a distance where the solar constant is $L_\circledast$. The space station has zero albedo in the visible range and radiates as a perfect blackbody. The flow of air inside keeps the entire station at the same temperature, and the skin is a good conductor of heat, so that its temperature is the same as that of the interior. The orientation of the station is such that the axis of the cylinder is always perpendicular to the line joining the center of the station to the center of the Sun. Find an expression for the temperature of the station. Put in numbers corresponding to the mean Solar constant at Earth's orbit, assuming $r = h$.

Now suppose that the equipment in the interior of the space station consumes 1 megawatt of solar-generated electrical power, which is dissipated as heat. How much warmer would this make the station once the equipment was turned on? To get rid of this excessive heat, you are to design a radiator, which is a large, thin flat plate heated by pumped water from the space station so that its temperature is the same as the interior of the space station. The radiator is perfectly reflective in the visible range, but acts as a perfect blackbody in the infrared range. How large should the radiator plate be in order to get rid of the excess heat? For the purposes of this part of the problem you may assume $r = h = 50m$.

Problem 2.5 Consider a planet covered in water ice with a uniform albedo of .7 . The planet is tide-locked, so that the same face always points to the Sun; the other side is in perpetual night. The atmosphere has negligable greenhouse effect. Compute the solar constant needed to begin melting the ice under the following three alternate scenarios: (a) The atmosphere is so efficient at transporting heat that the entire surface of the planet (dayside and nightside) has the same temperature, (b) The atmosphere is only moderately efficient, so that the dayside temperature is uniform but essentially no energy is carried away to the nightside, (c) There is no atmosphere, so that each bit of the planet's surface is in equilibrium with the solar radiation it absorbs.

Problem 2.6 The discovery of a planetary system orbiting the red-dwarf star Gliese 581 was announced in 2007. This system is of particular interest, because two of the planets (581c and 581d) have masses just a few times that of Earth, and so are presumably solid (ice or rocky) bodies rather than gas giants. The star has a luminosity .013 times that of the Earth's Sun. Gliese 581c orbits at a mere .073 A.U. (Astronomical Units) from its star, and 581d orbits at .25 A.U. The temperature of the photosphere of Gliese 581 is $3480K$.

Compute the equilibrium temperatures of these two planets, assuming that they are isothermal spherical bodies with an albedo of .3 , and assuming that the atmosphere has no greenhouse effect. What is the gap between the median-emission wavenumber of the star and that of each of the planets? Compare this to the situation of Mercury in our own solar system. Based on the data given, estimate the radius of the star in the Gliese system.