Geosci 232 Problem Set 5
Fall Quarter 2009

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5 Problem Set:Thermodynamics, dry and moist

Problem 5.1 Compute the density of CO_2 at the surface of Mars, where the pressure is $6mb$ and the temperature is $220K$. Compute the density of N_2 at the surface of Titan, where the pressure is $1.5bar$ and the temperature is $95K$. Compute the density of a pure CO_2 atmosphere at the surface of Venus, where the pressure is $92bars$ and the temperature is $737K$.

Problem 5.2 A bicycle tire with a mass of $.1kg$ when empty, and a volume of $3liters$ is pumped up with Earth air to a pressure of $4bars$. It is pumped up slowly, so that its temperature remains at the ambient air temperature of $290K$. What is the mass of the tire after it has been pumped up? On Earth,what does the tire *weigh* (in $Newtons$)? What would the mass of the tire be if it were filled to the same pressure with CO_2 instead of air? With He?

Problem 5.3 The aforementioned bicycle tire with a volume of 3 liters is again pumped up with Earth air to a pressure of 4 bars. This time, it is pumped up so rapidly that there is not enough time for it to lose any heat to the surroundings. Assuming the ambient air has a temperature of $290K$, what is the temperature inside the tire immediately after it has been pumped up? What is the pressure after the tire has had time to cool down to the ambient temperature?

Problem 5.4 Referring to the Venus temperature profile in Figure 2.2, discuss the extent to which the Magellan observations of the temperature of the

Venus atmosphere are consistent with a CO_2 dry adiabat. Extrapolate the temperature from the highest observed pressure to the $92bar$ surface of the planet, and compare the result to the observed surface temperature of $737K$. (Note that part of the mismatch is due to the assumption of constant c_p and the inaccuracy of the perfect gas law at high pressures)

$R/c_p \approx .2304$ for CO_2 gas.

Problem 5.5 Consider a glacier of water ice that is $1km$ thick and covers an area of $10^8 km^2$ (about a fifth of the area of the Earth). Suppose that the ice absorbs $50W/m^2$ of solar radiation, and all of this is used to either sublimate (into gas) or melt (into water) the ice. How long does it take for the glacier to disappear in each of these cases? Give your answer in years. The density of ice is about 930 kg/m^3 . What determines whether sublimation or melting occurs?

Problem 5.6 This computer exercise requires that you learn how to write a function in a programming language. It applies this skill to evaluating the Clausius-Clapeyron relation.

The simplified constant L form of the Clausius-Clapeyron relation is adequate for many purposes. To evaluate it, you need to specify a reference temperature and vapor pressure, the latent heat of the phase transition under consideration, and the gas constant (or equivalently the molecular weight). Using your favorite programming language, write a function `psat(T)` which computes the saturation vapor pressure as a function of temperature. The constants needed to evaluate the function can be treated as global constants, hard-wired into the function, or added to the argument list.

Put in the constants appropriate to CO_2 gas in equilibrium with a solid. It is convenient to use the triple point pressure and temperature for the reference pressure and temperature. Use the function to estimate how low temperature would have to be for CO_2 to condense at the Martian surface, assuming the $7mb$ surface pressure typical of the present planet.

Python Tips: Commonly, one makes a function because one wants to evaluate it many times for the same gas but different values of T. If you add the thermodynamic constants to the argument list, you are stuck typing in a long argument list when all you really wanted to change is T. Making the constants into globals solves this problem, but then makes it hard to deal with saturation vapor pressure functions for several different gases in the same program. `phys` provides a better way to deal with the problem, making use of the fact that objects can be made callable. Namely, it provides the class `satvps_function` which takes the needed gas parameters as arguments and creates a function-like object that stores the constants it needs. The triple point temperature and pressure provide a convenient set of base conditions for use in the formula. Thus, to create a constant-L saturation vapor pressure function for CO_2 sublimation, one would write

```
gas = phys.CO2 # get the gas object you want
pCO2 = phys.satvps_function(gas.TriplePointT,
  gas.TriplePointP,gas.MolecularWeight, gas.L_sublimation)
```

This creates a callable object `pCO2`, whereafter you can evaluate the vapor pressure by simply writing `pCO2(T)`. This a simple but very useful technique. Take a look at the class definition in `phys` to see how it is done, since you may want to use the technique yourself sometime.

Actually, it's pretty tedious to have to enter all the thermodynamic constants in the argument list. Why not just give all the constants at once in the form of a `gas` object? In fact, through the power of *polymorphism*, the `satvps_function` class recognizes what kind of thing it's given as an argument, and acts accordingly. Therefore, you could get the same result as above using `phys.satvps_function(phys.CO2)`. By default, this will create a function that uses the latent heat of sublimation for temperatures below the triple point, and the latent heat of vaporization for temperatures above the triple point. But what if you want to force the function to use one latent heat or the other regardless of temperature? For this purpose, the class allows an optional second argument. If it's value is the string `'ice'`, the latent heat of sublimation is assumed. If it is `'liquid'` the latent heat of vaporization is assumed.

Problem 5.7 *Water on the Hadean Earth*

Ancient zircon crystals tell us that liquid water existed somewhere on the surface of the Earth as early as 4.2 billion years ago. What does this tell us about the surface temperature? Does it imply that the temperatures had to be below $100C$ (the "boiling point")? To answer this question, compute the saturation partial pressure of water vapor at an ocean surface as a function of surface temperature. Do this three ways. First, use the exponential form of the Clausius-Clapeyron relation with constant latent heat, based on triple point data. Next, use the exponential form based on "boiling point" data (i.e. that the vapor pressure is $1bar$ at $373.15K$. Finally, use the empirical Antoine equation:

$$p_{sat}(T) = 10^{A-B/(T+C)} \tag{1}$$

where A,B and C are empirical coefficients determined by fits to experimental data. In the range $373–647K$, the coefficients are $(A, B, C) = (5.2594, 1810.94, −28.665)$, if the pressure is given in $bars$ and temperature in K. This fit gets the critical point pressure slightly wrong, but it is adequate for the purposes of the present problem.

Using your results, answer the following. At $100C$ $(373.15K)$ what is the partial pressure of water vapor at the surface? What fraction of the molecules of the atmosphere are water at the surface (assuming that the non-condensing part of the atmosphere has partial pressure of $1bar$)? Does the ocean boil under these conditions? What if the surface temperature is $200C$ instead? At this temperature, what proportion of the mass of water on the planet is in vapor form in the atmosphere, assuming the total water mass to be the same as that of the present ocean $(1.4 \cdot 10^{21} kg)$? How hot would the surface have to be in order for all the inventory of water to go into the atmosphere? Specifically, show that at the critical point temperature for water, almost all of the Earth's oceanic inventory of water has gone into the atmosphere. Do you think that it is a coincidence that the equivalent pressure of the Earth's ocean is close to the critical point pressure?

Hint: At temperatures of $400K$ and above, it is a reasonable approximation that water vapor dominates the mass of the atmosphere, so that by the Hydrostatic Law, the mass of water vapor in the atmosphere can be approximated by $4\pi a^2 \cdot p_{sat}/g$, where p_{sat} is the saturation water vapor pressure at the surface (in Pa) and a is the radius of the Earth.

Remark: Although you will find that the existence of liquid water does not itself limit the surface temperature to below $100C$, the oxygen isotopic content of the zircons does tend to argue for moderate surface temperatures.