

Laboratory investigations of iceberg capsize dynamics, energy dissipation and tsunamigenesis

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[1] We present laboratory experiments designed to quantify the stability and energy budget of buoyancy-driven iceberg capsize. Box-shaped icebergs were constructed out of low-density plastic, hydrostatically placed in an acrylic water tank containing freshwater of uniform density, and allowed (or forced, if necessary) to capsize. The maximum kinetic energy (translational plus rotational) of the icebergs was $\sim 15\%$ of the total energy released during capsize, and radiated surface wave energy was $\sim 1\%$ of the total energy released. The remaining energy was directly transferred into the water via hydrodynamic coupling, viscous drag, and turbulence. The dependence of iceberg capsize instability on iceberg aspect ratio implied by the tank experiments was found to closely agree with analytical predictions based on a simple, hydrostatic treatment of iceberg capsize. This analytical treatment, along with the high Reynolds numbers for the experiments (and considerably higher values for capsizing icebergs in nature), indicates that turbulence is an important mechanism of energy dissipation during iceberg capsize and can contribute a potentially important source of mixing in the stratified ocean proximal to marine ice margins.

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1. Introduction

[2] Iceberg capsize liberates a large amount of gravitational potential energy into the ocean over a relatively short period of time [MacAyeal *et al.*, 2011; Levermann, 2011]. This energy has multiple impacts, including (1) causing horizontal translation, vertical bobbing, and rocking of the iceberg and any adjacent sea ice and icebergs [MacAyeal *et al.*, 2003; Amundson *et al.*, 2010]; (2) generating a wide spectrum of ocean surface gravity waves, including meter-scale short period (<60 s) and several centimeter-scale long-period (100s of s) waves [Amundson *et al.*, 2008, 2010; Nettles *et al.*, 2008; MacAyeal *et al.*, 2009]; (3) stimulating calving or iceberg fragmentation through ice-ice contact or wave-driven ice shelf flexure [MacAyeal *et al.*, 2009]; (4) turbulently mixing stratified proglacial waters, thereby affecting ocean heat flux into and within a fjord (which depends on stratification [see Motyka *et al.*, 2003; Straneo *et al.*, 2010]); (5) generating unique seismic signals when

the icebergs come into contact with the glacier terminus [Tsai *et al.*, 2008]; and (6) dissipating as heat into the water. In addition, the geometry and gravitational stability of an iceberg determines both its sensitivity to capsize and the total amount of potential energy released during capsize [MacAyeal *et al.*, 2011]. Iceberg geometry may therefore be an important parameter for the observed rapid (hours to days) disintegration of Antarctic ice shelves [MacAyeal *et al.*, 2003]. Moreover, the sizable tsunamic waves generated by iceberg capsize may contribute to the violent and rapid nature of these large-scale collapse events [MacAyeal *et al.*, 2011].

[3] In its simplest form, the energy budget of a capsizing iceberg can be expressed as

$$-\Delta U(t) = \Delta K_i(t) + \Delta E_w(t) + \Delta E_s(t) + \Delta E_m(t) + \Delta E_h(t), \quad (1)$$

where ΔU is the change in potential energy, K_i is the kinetic energy of the iceberg and any adjacent sea ice and icebergs (referred to as ice mélange), E_w and E_s are the energy of ocean waves and seismic waves, E_m is energy that is consumed by vertical mixing of the water column (dense parcels move upward), E_h is heat, and t is time. The expression is explicitly written as a function of time to allow for the fact that kinetic energy and wave energy ultimately ends up as heat. The relative importance of (and relationships between) the energy terms in equation (1) is unknown due to inherent dangers and difficulties of instrumenting remote, ice-choked

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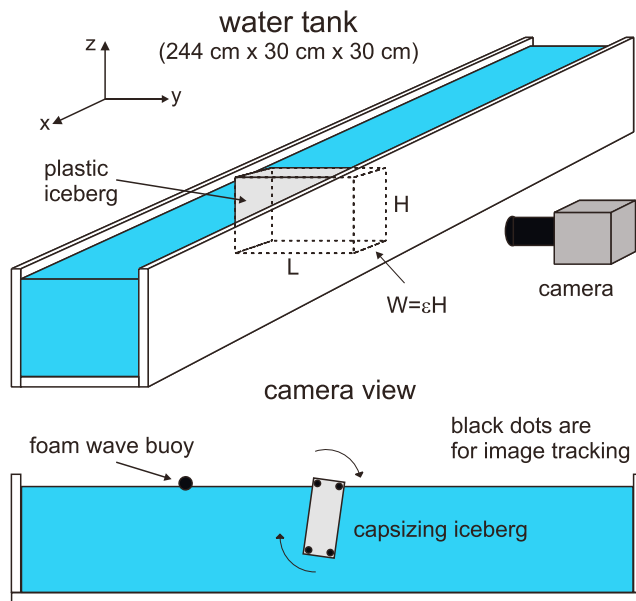


Figure 1. (top) Diagram of the laboratory setup. (bottom) The camera's field of view.

fjords. Such knowledge is desirable for two reasons. First, icebergs that capsize while calving into the ocean contain information (in the form of ocean and seismic waves) that can be used to investigate the processes controlling calving. Second, some processes that affect calving rates (ocean heat flux, ice mélange strength, and ice shelf flexure) may be influenced by the energy released during calving and its immediate aftereffects, i.e., iceberg capsize. It is worth noting some other forms of energy not considered in equation (1), such as elastic energy released by the fracturing of stressed ice, as well as the potential energy gained by the “rafting” of one piece of ice mélange atop another. We assume these forms of energy are insignificant, and will ignore them in our analysis.

[4] As a first step toward quantifying the capsize energy budget, we present a laboratory investigation into the capsize dynamics of single icebergs starting from a state of rest that are in an ‘open ocean’ environment. Although many icebergs capsize in close proximity to the calving terminus and other icebergs, and may do so simultaneously with detachment from the sourceterminus, we began our laboratory investigation with the most simple and tractable end-member: the isolated iceberg in open water. The laboratory apparatus involved synthetic, box-shaped “icebergs” placed hydrostatically into a transparent aquarium tank containing fresh water. The icebergs were allowed (or forced, depending on their aspect ratio) to capsize; the icebergs’ motion and coincident surface gravity waves were recorded with a digital video camera. In this study, our primary motivations were to (1) quantify iceberg stability conditions, (2) quantify the amount and rate of potential energy dissipation via viscous and turbulent motions in the water caused by capsize, and (3) place a bound on the maximum amplitude of and amount of energy contained in surface gravity waves generated by capsizing icebergs. Subsequent studies will consider iceberg

capsize in stratified water bodies and iceberg-iceberg interactions during calving.

2. Experimental Methods

2.1. Laboratory Setting

[5] The laboratory model “fjord” consisted of an optically clear acrylic aquarium tank that is 244 cm long, 30 cm wide, and 30 cm tall (Figure 1), and has walls that are 1.3 cm thick. The tank was filled with fresh tap water at room temperature of density $\rho_w = 997 \text{ kg m}^{-3}$. The depth of the water, D , was varied from 11.4 to 24.3 cm. The icebergs used in the experiments were machined out of low-density polyethylene plastic that had a density of $\rho_i = 920 \pm 1 \text{ kg m}^{-3}$, which was calculated by measuring the mass and dimensions of the icebergs. For all icebergs the precapsize height, H , and length parallel to the axis of rotation during capsize, L , were held constant at 10.3 cm and 26.7 cm, respectively. The length L was chosen to be slightly shorter than the transverse width of the tank (30 cm) in order to minimize contact friction with the tank walls due to small rotations of the icebergs in the x - y plane. The width of the icebergs, W , was varied from 2.5–10.2 cm, so that the aspect ratio $\varepsilon = W/H$ varied from approximately 0.25 to 1.0.

[6] In order to measure the height of the waves produced by a capsizing iceberg, a small, spherical, closed cell styrofoam buoy was placed approximately 25 cm from the iceberg at the beginning of each run. This distance is sufficiently close to the capsize event so that waves reflected off the walls of the tank do not affect our wave amplitude measurements. Due to the very low density of the styrofoam, the buoy was well coupled to the water surface and thus measured the height of the water’s surface at that location as a function of t . The experiments were recorded with a Casio EX-FH20 video camera, which was set to record 30 frames per second. The camera resolution was typically 16 pixels cm^{-1} , although we increased the resolution to 93 pixels cm^{-1} (by zooming in) to measure buoy displacements.

[7] In order to track the positions and angular orientations of the icebergs, black sticker dots were placed in each corner of the rectangular face of the icebergs (Figure 1). The position of each dot was located in each frame of the movie using custom feature-tracking software written in Mathematica (Wolfram Research). Once the positions of the dots were known in every movie frame, the center of mass of the iceberg was calculated as the mean position of the four dots, and the angular orientation was calculated by measuring the vector angle between the vertical (gravitational) direction and the direction of the iceberg parallel to the height H . All angles reported here are measured between the H axis and the vertical, which ranges from -90° to 90° . Although the camera was leveled before each video session, small tilts were still possible within our measurement accuracy. To provide a good measurement of the vertical direction in each video, we used the water level as a perpendicular plane to the vertical. The water level line was extracted from the first frame of each video, fitted to a linear model $z = (\tan \theta_w) x + b$, and the resulting angle θ_w was subtracted from all other angles measured in the video.

2.2. Dynamic and Geometric Similarity

[8] As with any laboratory model of geological phenomena, a discussion of dynamic similitude is warranted [e.g., *Hubbert*, 1937; *Hughes*, 1993]. Our iceberg capsize experiments are defined by four dimensionless numbers. The first two are the aspect ratio, $\varepsilon = W/H$, and the density ratio, ρ_i/ρ_w , where ρ_i and ρ_w are the densities of the icebergs (plastic or ice) and the water. For more complicated iceberg geometries a single parameter such as the aspect ratio will not suffice. Many freshly calved icebergs are roughly box shaped (especially those involving full glacier thickness fracture [e.g., *Amundson et al.*, 2008; *Scambos et al.*, 2009; *Walter et al.*, 2010]), which gives support for the idealized geometries considered here. In the experiments the aspect ratio is varied so as to span a range of observed aspect ratios, and the density ratio differs from field observations by about 3%. We also only consider floating icebergs, so that capsize is not affected by the corners of the iceberg touching the ocean floor. In this regime the main role of water depth D will be to affect the speed of gravity waves in the water.

[9] For our modest-sized laboratory experiments, it is relevant to consider whether surface tension forces could be important for the iceberg dynamics. These effects can be addressed by inspection of the dispersion relation for surface waves in the presence of gravity and surface tension [*Lamb*, 1932]:

$$\omega^2 = \left(gk + \frac{\gamma}{\rho_w} k^3 \right) \tanh(Dk), \quad (2)$$

where ω is the angular frequency, $k = 2\pi/\lambda$ is the wave number, λ is the wavelength, and γ is the surface tension. For a depth $D = 0.15$ m and a wavelength $\lambda = H \approx 0.1$ m, the period of the wave is ≈ 0.25 seconds. This differs by only $\sim 1\%$ from the case where $\gamma = 0$. Thus, we conclude that for the wavelength and period of waves observed in our laboratory model, surface tension should be only a minor effect, and will be ignored in any further analysis.

[10] The other two dimensionless numbers defining the experiments are the Froude (Fr) and Reynolds (Re) numbers, which are the most important numbers for similitude in an overwhelming majority of coastal hydrodynamic phenomena [*Hughes*, 1993]. The Froude number describes how efficiently the iceberg can act as a source of gravitational waves and, is given by the ratio of the characteristic capsize velocity, V_{cap} , to the gravitational wave speed in water: $Fr = V_{cap}/V_{wave}$. The Reynolds number determines whether the fluid flow is turbulent or laminar and is given by a ratio of the inertial to viscous forces during capsize: $Re = (HV_{cap}\rho_w)/\mu$, where $\mu \approx 1.0$ mPa \cdot s is the viscosity of water.

[11] We define V_{cap} by noting that the kinetic energy of a capsizing iceberg is limited by the total amount of potential energy released during capsize, E_{cap} . If we assume that this kinetic energy is in the form of translational motion only, then

$$V_{cap} = \sqrt{\frac{2E_{cap}}{m}}, \quad (3)$$

where $m = \rho_i \varepsilon H^2 L$ is the iceberg's mass. This is simply an upper bound on the translational velocity of the iceberg, and

the actual characteristic translational velocities will be a fraction of V_{cap} due to energy dissipation into the water. (An alternative estimate of V_{cap} based on the kinetic energy of rotational motion is more complicated and yields a result that is comparable to the estimate associated with translational motion.) For the idealized iceberg geometries considered here, E_{cap} is given by

$$E_{cap} = \frac{1}{2} \rho_i g L H^3 \varepsilon (1 - \varepsilon) \left(1 - \frac{\rho_i}{\rho_w} \right), \quad (4)$$

where g is gravitational acceleration [see *MacAyeal et al.*, 2011]. Inserting equation (4) into equation (3) yields

$$V_{cap} = \sqrt{\left(1 - \frac{\rho_i}{\rho_w} \right) g H (1 - \varepsilon)}. \quad (5)$$

[12] To estimate the gravitational wave speed, V_{wave} , we take advantage of the dispersion relation for gravitational surface waves in the absence of surface tension and the definition of group velocity. The dispersion relation is given by setting $\gamma = 0$ in equation (2)

$$\omega^2 = gk \tanh(Dk), \quad (6)$$

We expect the wavelength of the generated waves to be of similar length to the iceberg height, so that $\lambda \approx H$. Furthermore, the water depth must exceed the initial iceberg height, such that $D = \beta H$ where $\beta \geq 1$. Inserting these relations into equation (6) gives

$$\omega^2 = gk \tanh(2\pi\beta) \approx gk. \quad (7)$$

Noting that the group velocity is given by $V_{wave} = \partial\omega/\partial k$, we find that

$$V_{wave} \approx \sqrt{\frac{gH}{8\pi}}. \quad (8)$$

Finally, inserting equations (5) and (8) into the expressions for the Froude and Reynolds numbers gives

$$Fr = \sqrt{8\pi \left(1 - \frac{\rho_i}{\rho_w} \right) (1 - \varepsilon)}, \quad (9)$$

and

$$Re = Fr \frac{\rho_w}{\mu} \sqrt{\frac{gH^3}{8\pi}}. \quad (10)$$

[13] The Froude number is independent of iceberg height and is a simple function of 2 nondimensional numbers: the aspect ratio and the density ratio. As discussed above, the aspect ratios of the synthetic icebergs were varied to cover the range seen in field observations, whereas the density ratio differed from that associated with glacier ice by about 3%. The difference in the density ratio results in a Froude number that is $\sim 10\%$ larger in the experiments than in the field. Additionally, equation (9) predicts that in both cases,

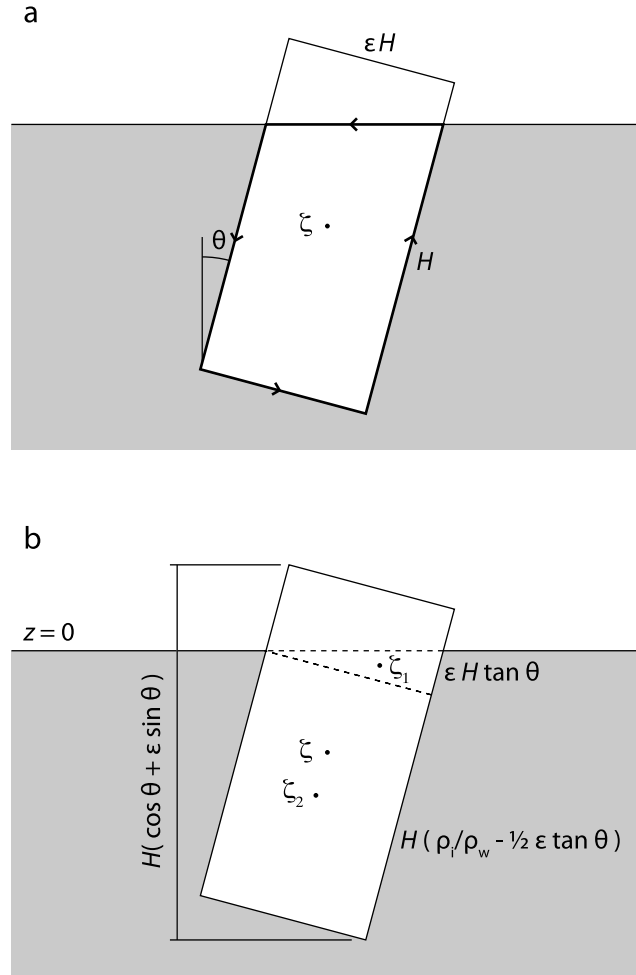


Figure 2. Schematic diagrams of iceberg cross sections. (a) Parameters needed for calculating an iceberg's gravitational potential energy. The contour used for the integral in equation (15) is indicated by the thick black line. (b) Parameters used to calculate the center of mass and center of buoyancy for an iceberg that is held in hydrostatic equilibrium (see section 3.1).

$Fr \approx 1$; however, we will later show that the above treatment overestimates the maximum kinetic energy, and therefore velocity, of the iceberg. The key point, however, is that $Fr < 1$ for both the experiments and the field, and this implies that radiation of energy as surface gravity waves will have a relatively minor contribution to the energy budget in both cases.

[14] The Reynolds number, on the other hand, depends on $H^{3/2}$. All other terms are roughly equivalent between the lab and the field. For an iceberg that is initially 100–1000 m thick, $Re = 10^8 - 10^{10}$. Such large Reynolds numbers indicate that iceberg capsize is a highly turbulent process. The Reynolds number of the experiments is $Re \approx 2 \times 10^4$ or 4–6 orders of magnitude smaller than in the field. Fortunately, the laboratory value for Re is sufficiently large that turbulence should be a dominant mechanism of energy dissipation, thus allowing us to compare our results to field observations and to provide a reasonable estimate of the capsizing iceberg energy budget. To be more specific, we

note that turbulent drag forces on solid bodies have a quadratic dependence on velocity:

$$F_d = \frac{1}{2} C \rho_w A_c v^2, \quad (11)$$

where v is the velocity, A_c is the cross-sectional reference area, and C is the drag coefficient, which will depend on the geometry of the object and surface roughness. If we use V_{cap} as the velocity and H^2 as our area, we see that the drag force $F_d \propto H^3$, so that F_d scales with the mass of the iceberg, as does gravitational forces. For turbulent flow, drag coefficients typically vary by less than an order of magnitude even as the Reynolds number varies by many orders, so that the motion of a large iceberg ($H \approx 1$ km) may be well modeled in our laboratory experiments.

3. Iceberg Energetics

3.1. Gravitational Potential Energy

[15] The gravitational potential energy of an iceberg, U (equation (1)), can be expressed as the potential energy of the submerged portion of the iceberg in a reduced gravity environment plus the potential energy of the subaerial portion of the iceberg, such that

$$U = g(\rho_i - \rho_w)L\zeta_s A_s + g\rho_i L(A - A_s) \left(\frac{\zeta A - \zeta_s A_s}{A - A_s} \right), \quad (12)$$

where ζ is the elevation of the center of mass, ζ_s is the elevation of the center of buoyancy, and A and A_s are the cross-sectional areas of the iceberg and the submerged portion of the iceberg. All elevations are measured with respect to the waterline. Here we have assumed that $L > H$; thus the iceberg can only capsize in the x - z plane (as shown in Figures 1 and 2). The last two terms on the right hand side of equation (12) are equal to the area and center of mass of the subaerial portion of the iceberg. Rearranging equation (12) and noting that $A = \epsilon H^2$ for our idealized icebergs gives

$$U = g\rho_i L \epsilon H^2 \zeta - g\rho_w L \zeta_s A_s. \quad (13)$$

The center of buoyancy, after applying Green's theorem, is given by

$$\zeta_s = -\frac{1}{2A_s} \oint_c z_s^2 dx, \quad (14)$$

where z_s is the vertical position of a submerged boundary element of the iceberg and c indicates that a contour integral is performed around that boundary. Inserting equation (14) into equation (13), we find that

$$U = g\rho_i L \epsilon H^2 \zeta + \frac{1}{2} g\rho_w L \oint_c z_s^2 dx. \quad (15)$$

[16] In this formulation, $U = 0$ if the iceberg disintegrates into infinitesimally small pieces that float on the water surface. The contour integral in equation (15) can be determined explicitly for rectangular icebergs having arbitrary orientation, θ , and elevation, ζ . However, the analytic

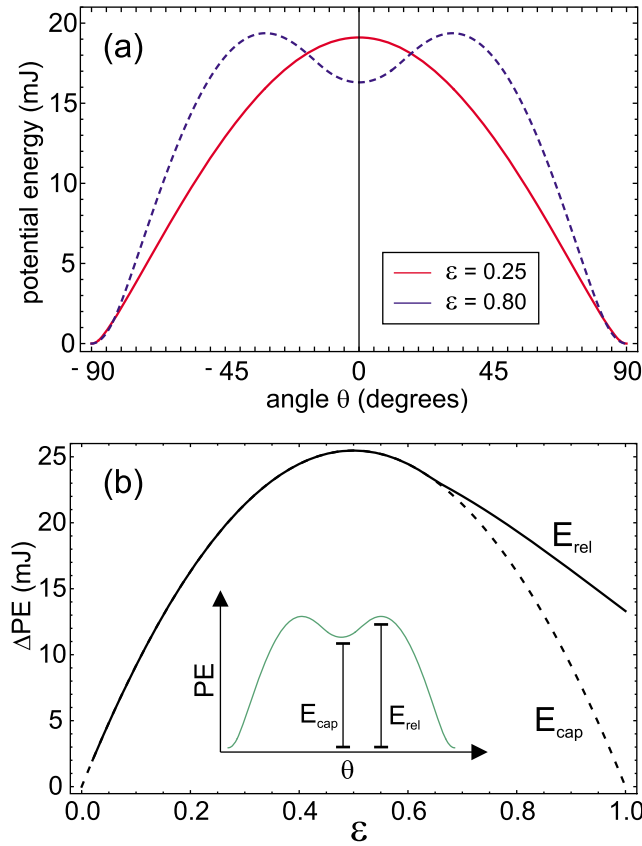


Figure 3. (a) Calculated gravitational potential energy of a plastic iceberg as it is hydrostatically rotated through 90° . The red curve ($\varepsilon = 0.25$) is unstable to small perturbations from an initial orientation of $\theta = 0$. For $\varepsilon = 0.8$, the iceberg cannot spontaneously capsize from its initial orientation because small rotations increase the potential energy. (b) Potential energy difference from capsizing (E_{cap} , equation (4)) and actual energy release (E_{rel}) in our experiments. For $\varepsilon > \varepsilon_c$, the iceberg must be rotated over an energy barrier before it will capsize, so the energy released E_{rel} is greater than E_{cap} .

solution is lengthy and piecewise; we therefore choose to calculate it numerically. We also note that equation (15) yields the same total energy release during capsizing, E_{cap} (equation (4)), as was found by *MacAyeal et al.* [2011].

3.2. Iceberg Capsizing Stability Analysis

[17] One ramification of equation (15) is that rectangular icebergs with aspect ratios larger than some critical value, ε_c , will not capsize unless an energy barrier is overcome by some external agent. Furthermore, by investigating the potential energy curves of icebergs with various aspect ratios (Figure 3a), it becomes evident that $\varepsilon_c < 1$. In other words, an iceberg can be narrower than it is tall yet still not be capable of spontaneously capsizing.

[18] The value of ε_c can be determined by placing an iceberg in hydrostatic equilibrium and investigating the change in torque on the iceberg, which is given by $\tau = -\partial U/\partial\theta$, as the iceberg is rotated away from vertical. For small angular rotations ($\theta \ll 1^\circ$) the center of rotation is assumed to lie on

the waterline ($z = 0$), and thus the submerged area of the iceberg is

$$A_s = \varepsilon H^2 \frac{\rho_i}{\rho_w}. \quad (16)$$

This assumption allows us to simplify equation (13) to

$$U = g\rho_i L \varepsilon H^2 (\zeta - \zeta_s). \quad (17)$$

Thus the torque on an iceberg due to buoyancy is given by

$$\tau = g\rho_i L \varepsilon H^2 \frac{\partial}{\partial\theta} (\zeta_s - \zeta). \quad (18)$$

Due to the idealized geometry used here, ζ and ζ_s can be determined explicitly (for small θ) through investigation of Figure 2. Since the iceberg is held in hydrostatic equilibrium we require that

$$A_s = (\rho_i/\rho_w)\varepsilon H^2. \quad (19)$$

Equation (19) allows us to determine the lengths of the submerged portion of the iceberg (Figure 2b), from which it follows that the vertical elevation of the iceberg's center of mass is

$$\zeta = -H \cos\theta \left(\frac{\rho_i}{\rho_w} - \frac{1}{2} \right). \quad (20)$$

We use geometric decomposition to calculate the center of buoyancy, such that

$$\zeta_s = \frac{\zeta_1 A_1 + \zeta_2 A_2}{A_s}, \quad (21)$$

where ζ_1 and A_1 are the center of mass and area of the small triangle in Figure 2b, and ζ_2 and A_2 are the center of mass and area of the rectangle located immediately below the triangle. The area and center of mass of the triangle are given by

$$A_1 = \frac{1}{2} (\varepsilon H)^2 \tan\theta, \quad (22)$$

and

$$\zeta_1 = -\frac{1}{3} \varepsilon H \sin\theta. \quad (23)$$

The area and center of mass of the rectangle are given by

$$A_2 = \varepsilon H^2 \left(\frac{\rho_i}{\rho_w} - \frac{1}{2} \varepsilon \tan\theta \right), \quad (24)$$

and

$$\zeta_2 = -\frac{H}{2} \cos\theta \left(\frac{\rho_i}{\rho_w} + \frac{1}{2} \varepsilon \tan\theta \right). \quad (25)$$

Inserting equations (19) and (22)–(25) into equation (21) gives

$$\zeta_s = -\frac{1}{2} \frac{\rho_w}{\rho_i} H \left[\frac{\varepsilon^2}{12} \sec\theta + \left(\left(\frac{\rho_i}{\rho_w} \right)^2 - \frac{\varepsilon^2}{12} \right) \cos\theta \right]. \quad (26)$$

Setting equation (18) equal to zero, inserting the equations for ζ and ζ_s (equations (20) and (26)), solving for ε , and letting $\theta \rightarrow 0$ gives

$$\varepsilon_c = \sqrt{6 \frac{\rho_i}{\rho_w} \left(1 - \frac{\rho_i}{\rho_w}\right)}, \quad (27)$$

which is in agreement with the expression derived by *MacAyeal et al.* [2003]. For icebergs in seawater, $\varepsilon_c \approx 0.75$, and for the plastic blocks and fresh water used in our experiments, $\varepsilon_c \approx 0.66$. For $\varepsilon > \varepsilon_c$, icebergs must be rotated to a sufficient angle so that they will capsize. We define E_{rel} as the amount of energy actually released in the capsize (Figure 3b). When $\varepsilon < \varepsilon_c$, then $E_{rel} = E_{cap}$ and the icebergs will spontaneously capsize at $\theta = 0$.

[19] From equation (18) it is clear that the torque exerted on an iceberg depends on the iceberg's aspect ratio. For small rotations from vertical, the torque achieves a maximum value at $\varepsilon = \varepsilon_o$ [see *Amundson et al.*, 2010, Figure 9], which can be evaluated analytically by setting $\partial\tau/\partial\varepsilon = 0$, solving for ε , and letting $\theta \rightarrow 0$. This gives

$$\varepsilon_o = \frac{\varepsilon_c}{\sqrt{3}}. \quad (28)$$

Icebergs with $\varepsilon = \varepsilon_o$ can exert the largest forces on adjacent ice blocks, at least during the initial phases of capsize, and thus may play an important role in initiating large-scale calving events by causing extension of a proglacial ice mélange or highly fractured ice shelf. (Note that the analytic solutions for ε_c and ε_o presented here are in perfect agreement with the values numerically calculated by *Amundson et al.* [2010].)

3.3. Oscillation Frequencies

[20] The equilibrium hydrostatic position of a rectangular iceberg of height H and width εH is

$$z_{eq} = H \left(\frac{1}{2} - \frac{\rho_i}{\rho_w} \right) \quad (29)$$

$$\theta_{eq} = 0.$$

By comparing observed oscillations around this equilibrium with oscillation frequencies predicted from hydrostatic assumptions, we can assess the impact that hydrodynamic forces have on the dynamics of iceberg capsize. The oscillation frequencies can be predicted from Newton's equations of motion [see also *Schwerdtfeger*, 1980] for the 3 degrees of freedom (θ , x , and z):

$$I\ddot{\theta} = -\frac{\partial U}{\partial \theta} + \nu_\theta |\dot{\theta}|^n \text{sign}(\dot{\theta})$$

$$m\ddot{x} = -\frac{\partial U}{\partial x} + \nu_x |\dot{x}|^n \text{sign}(\dot{x}) \quad (30)$$

$$m\ddot{z} = -\frac{\partial U}{\partial z} + \nu_z |\dot{z}|^n \text{sign}(\dot{z}),$$

where $I = \rho_i \varepsilon (1 + \varepsilon^2) H^4 L / 12$ is the iceberg's moment of inertia around its center of mass, ν_θ , ν_x , and ν_z are simple damping coefficients that represent energy dissipation due to drag, and n determines whether energy is dissipated

viscously ($n = 1$) or turbulently ($n = 2$). Since the potential energy does not depend on x (equation (15)), then $\partial U / \partial x = 0$ and the second of equations (30) can be ignored. Bobbing and rocking frequencies are associated with motion in the z and θ directions, respectively. Assuming a hydrostatic, frictionless fluid ($\nu_\theta = \nu_x = \nu_z = 0$) and small rocking oscillations (so that we can again use equation (17)), we find that

$$f_{bob} = \frac{1}{2\pi} \sqrt{\frac{g\rho_w}{\varepsilon\rho_i}} \quad (31)$$

$$f_{rock} = \frac{1}{2\pi} \sqrt{\frac{g(6\rho_i^2 - 6\rho_i\rho_w + \varepsilon^2\rho_w^2)}{H(1 + \varepsilon^2)\rho_i\rho_w}}.$$

[21] The bobbing frequency agrees with the analysis of *Schwerdtfeger* [1980]. However, *Schwerdtfeger* [1980] incorrectly calculated the rocking frequency because the analysis did not allow the iceberg's center of mass to change elevation during rocking. This is a reasonable simplification for very wide icebergs ($\varepsilon > 10$), but greatly overpredicts the natural rocking frequency of narrower icebergs. As an example, for a stable iceberg with $\varepsilon = 0.75$, *Schwerdtfeger* [1980] predicts a rocking frequency 118% higher than our value. Thus, icebergs that are barely stable are much more sensitive to lower-frequency ocean swell than was previously thought. Also, as expected, equation (31) indicates that the rocking period equals zero when $\varepsilon = \varepsilon_c$, and is imaginary when $\varepsilon < \varepsilon_c$ so that small perturbations will grow (leading to capsize) instead of oscillate.

4. Experimental Results

4.1. Iceberg Stability

[22] We performed a series of iceberg stability experiments to verify our derivation of iceberg potential energy (equation (15)). The synthetic icebergs were hydrostatically positioned with the height of the iceberg, H (see Figure 2), oriented vertically. The icebergs were then allowed (or forced) to capsize. The results are summarized in Figure 4. All icebergs with small aspect ratios ($\varepsilon < \varepsilon_c$) spontaneously capsized; the potential energy curves of these icebergs resemble the red curve ($\varepsilon = 0.5$) in Figure 3. Icebergs having aspect ratios close to ε_c have peculiar potential energy curves with shallow minima at $\theta \leq 15^\circ$; these icebergs are unstable at $\theta = 0$ but are able to rest at some other finite angle (see the icebergs just to the right of the dashed line in Figure 4a). This peculiarity is a consequence of the rectangular geometry assumed here (e.g., the minimum disappears for rectangular icebergs with sufficiently rounded corners). Thus we leave it as a demonstration of the subtle relationship between iceberg geometry and stability. Finally, for larger values of ε , the icebergs had to be rotated (by slowly pulling on the icebergs with a thin piece of fishing wire) over a finite energy barrier before they would capsize (see the blue curve, for which $\varepsilon = 0.8$, in Figure 3). The results from stability experiments are in excellent agreement with our analytical derivations of iceberg stability, and thus support the derivation of iceberg potential energy presented above.

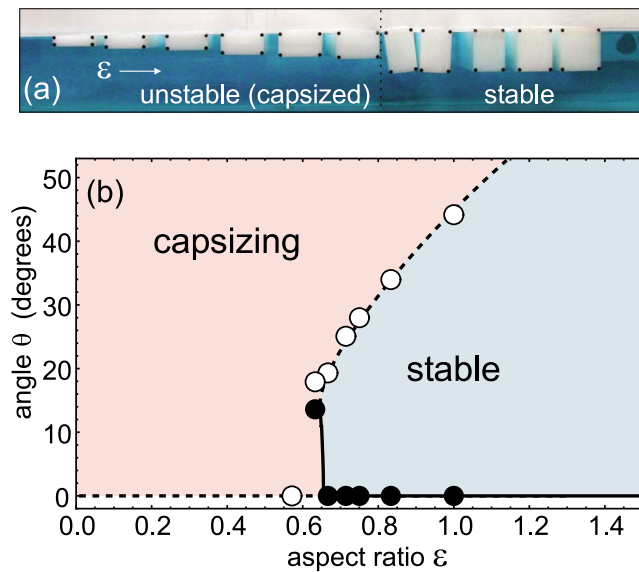


Figure 4. Results from iceberg stability analysis. (a) All plastic icebergs used in the experiment placed in order of increasing ε . When $\varepsilon < \varepsilon_c$, the icebergs are unstable and will spontaneously capsize. In the small area around ε_c they are stable at a finite angle, and at larger values they are stable in the upright ($\theta = 0$) orientation. (b) Stability diagram of box-shaped icebergs used in the experiments. For $\theta = 0$, the stable and unstable phases are separated by the critical aspect ratio, ε_c , given in equation (27). The dotted lines show unstable potential energy maxima, and the solid lines show stable potential energy minima. The solid and open data points are measurements of stable angles (potential energy minima) and unstable equilibria (potential energy maxima).

4.2. Iceberg Dynamics and Kinetic Energy

[23] Some of the potential energy released during iceberg capsize is converted into translational and rotational kinetic energy of the iceberg, which we can directly track in the laboratory through image feature tracking. Typical measurements of the orientation, θ , and horizontal and vertical positions of the center of mass (ξ and ζ , respectively) are shown in Figure 5. As the iceberg rotates, θ goes from zero to 90° (the postcapsize equilibrium position). Iceberg capsize is followed by a short period of vertical and rotational oscillations. The oscillations are rapidly damped due to energy leaving the iceberg through wave generation and viscous and turbulent dissipation.

[24] The kinetic energies associated with iceberg rotation, KE_R , and horizontal and vertical translation, KE_X and KE_Z , are plotted in Figure 6a. The iceberg's kinetic energy is dominated by rotational energy, which peaks shortly before the iceberg has rotated 90° . The most striking result from these experiments is that only a small fraction of potential energy released during iceberg capsize is converted into iceberg kinetic energy (Figure 6b). The maximum kinetic energy of the iceberg is $\sim 15\%$ of the energy it released during capsize, and thus $\sim 85\%$ of the energy must be immediately released into the water (i.e., during the initial rotation of the iceberg from $\theta = 0^\circ$ to $\theta = 90^\circ$).

[25] The maximum power put into the water, P_{\max} , computed by taking the derivative of the total iceberg energy ($PE + KE_R + KE_Z + KE_X$) with respect to time, was found to depend on ε (Figure 7a). The general shape of the curve is likely due in large part to the total energy released during capsize, E_{cap} , which depends quadratically on ε and reaches a maximum at $\varepsilon = 0.5$ [MacAyeal et al., 2011] (see equation (4)). Moreover, the power put into the water does not appear to depend on water depth within our experimental error. Note that the results from the stable region ($\varepsilon > \varepsilon_c$) are not as easily interpreted since the icebergs had to be slowly rotated to overcome a potential energy barrier prior to capsizing (see section 4.1). In Figure 7b, we show P_{\max} in a dimensionless fashion scaled by the actual energy released E_{rel} (see Figure 3b), and a timescale $T_{\text{rel}} = H\sqrt{m/E_{\text{rel}}}$. When plotted in this way, a linear trend emerges which shows that larger icebergs produce more power per unit potential energy released.

[26] To explain this result, we note that a simple drag force on a capsizing iceberg (equation (30)) should be of the form $F_{\text{drag}} \propto \rho_i H L \dot{\theta}^n$, where the product $H \times L$ is the cross-

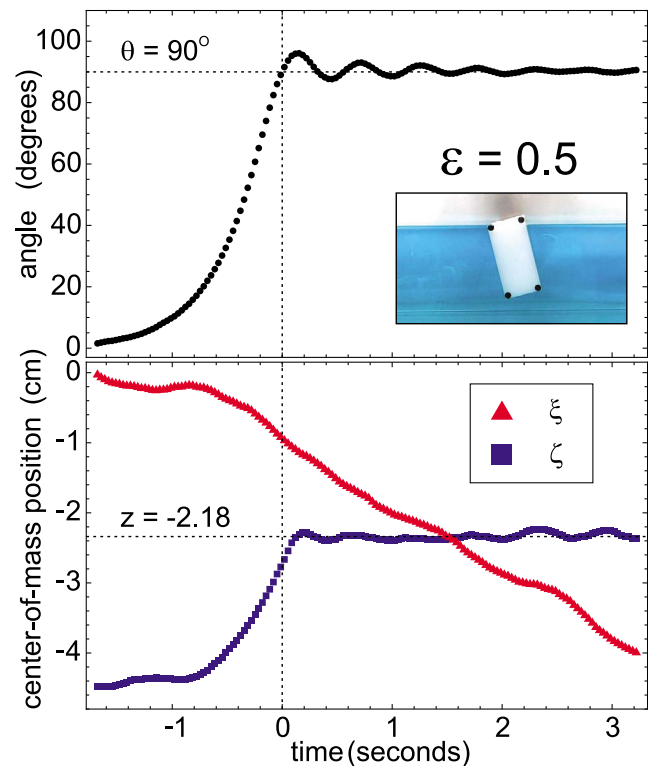


Figure 5. Rotation angle (θ) and center of mass in the x direction (ξ) and z direction (ζ) of an $\varepsilon = 0.5$ capsizing plastic iceberg. All data are shown with $t = 0$ corresponding to the time that the iceberg first rotated to $\theta = 90^\circ$. The horizontal dotted lines indicate the capsized equilibrium angle and vertical position. During the capsize process, ζ increases because this lowers the iceberg's potential energy. The position of ξ changes more slowly, indicating that after capsize the iceberg translates due to momentum transfer to the water. In this example the iceberg translates to the left, indicating that there has been a transfer of momentum to the water in the form of waves traveling to the right.

