Distinguishing meanders of the Kuroshio using machine learning

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Abstract The Kuroshio south of Japan is often described as being bimodal, with abrupt transitions between a straight path state that stays near the coast (small meander) and a meandering state that deviates from the coast (large meander). Despite evidence of the existence of two or more states of the Kuroshio, previous data-driven studies have shown only high variability of the current; they have not, however, demonstrated bimodality in the sense of two states of relatively high probability separated by a region of relatively low probability. We use singular value decomposition (SVD), a standard time series analysis method for characterizing variability, and diffusion maps and spectral clustering (DMSC), a machine learning algorithm that seeks modality, to investigate Kuroshio reanalysis output. By applying these methods to a time series of velocity fields, we find that (1) the Kuroshio is bimodal, with high inflow and low path variability in the small meander and low inflow and high path variability in the large meander, (2) the state of the system correlates highly with the location of the recirculation gyre south of Japan, and (3) the meanders are better characterized by path variability than by mean path. Because these results are consistent with satellite sea surface height data, they are not an artifact of the model used for reanalysis. Further, our results provide evidence for a previously proposed transition mechanism based on the strengthening, migration, and weakening of the recirculation gyre south of Japan and can therefore help direct future modeling studies.

1. Introduction

The Kuroshio is a western boundary current that runs along the southeastern coast of Japan, transporting heat and salinity poleward and thereby influencing large-scale weather, industry, and biology [Zhang et al., 2012]. The Kuroshio south of Japan is widely claimed to exist in one of two persistent states: a small-meander state in which it does not separate from the coast and a large-meander state in which the current axis is about 2° south of the coast at about 137° E [Taft, 1972]. The residence time in each state is reported as 5–10 years [Masuda, 1982; Nitani, 1975], with transitions between the two states taking several months.

The two states of the Kuroshio were noted by Taft [1972] in a study where the current axis was determined as the maximum surface velocity along North-South ship transects spaced 1° latitude apart. Representative paths were obtained by interpolating the locations of the current axis; transects where the maximum surface velocity was below 1 knot were not used. Figure 1 shows paths calculated from the Simple Ocean Data Assimilation (SODA) surface velocity reanalysis data set [Carton and Giese, 2008] using a similar algorithm to that used in Taft [1972]. The paths show that the Kuroshio was in the small-meander state from March 1956 to March 1959 and that it was in the large-meander state from July 1959 to November 1962. While these paths provide evidence of two distinct states, there is no consensus on either a rigorous characterization of each state or on the mechanism responsible for transitions between states [Hasumi et al., 2010].

Several studies have proposed that the Kuroshio is bimodal, with the current residing almost exclusively in either the large or small-meander state. Schmit and Dijkstra [2001] use a high-resolution ocean general circulation model to support the hypothesis that multiple mean paths of the Kuroshio are possible. Qiu and Miao [2000] use a two-layer primitive-equation model to propose that Kuroshio bimodality is the result of a self-sustained internal oscillation of the recirculation gyre south of Japan. Pierini [2006] and Pierini et al. [2009] find that the downstream Kuroshio Extension is an important bimodal feature of the system. Further, there are many proposed mechanisms for the transition between the small and large meanders. Several modeling studies conclude that the large-meander state can exist only when the Kuroshio volume...
transport is high [White and McCreary, 1976; Chao, 1984; Akitomo et al., 1991; Sekine, 1990] and that the transition occurs at a critical level of inflow. Other models implicate the formation of eddies as driving the transition between states [Ebuchi and Hanawa, 2003; Maltrud and McClean, 2005; Nakano and Hasumi, 2005; Hurlburt et al., 1996]. A third set of models relies on wind forcing to trigger the transition between meanders [Kurogi and Akitomo, 2006; Seager et al., 2001]. Qiu and Miao [2000] find an internal oscillation based on the strengthening and subsequent destabilization of the recirculation gyre south of the current.

There are likewise a number of potential indices for the state of the Kuroshio. Based on the volume transport criterion, an index can be constructed from the inflow into the current. Kawabe [1980] used the difference in sea surface height between two points as a proxy for the state of the Kuroshio. An index of the offshore distance of the Kuroshio was presented by Qiu and Miao [2000], who used the location of the 16°C isotherm at a depth of 200 m as the Kuroshio current axis. Further, Kawabe [1995] used sea surface height data to show that there are at least three possible paths of the Kuroshio (nearshore nonlarge meander, offshore nonlarge meander, and large meander). As we show in section 3, the existence of many possible paths is consistent with a fundamentally bimodal system because path shape does not define statistical modes; instead, multiple paths can fall into the same mode.

Previous data analyses argue for the existence of multiple states for the Kuroshio and thus support the claim that the Kuroshio is highly variable. However, these studies do not address bimodality in the sense of two states of relatively high probability separated by states of relatively low probability. According to this definition of bimodality, a system can have two distinct states without being bimodal if it usually resides in between the states. An example of this is the El Niño Southern Oscillation (ENSO), which has two distinct end-member states (El Niño and La Niña) but reaches these extremes only rarely. Previous studies of the Kuroshio have produced indices that show high variability between two accepted states, but these indices do not exhibit bimodal probability distributions. In this study, we show rigorous bimodality by testing separately for bimodality and variability.

Further, the progress we make in characterizing the states of the Kuroshio is crucial for better evaluation of the proposed models and mechanisms. Due to the difficulty in separating the two meanders using simple physical measures, we use two distinct machine learning algorithms to determine the two states and confirm the bimodal behavior of the Kuroshio. The first is singular value decomposition (SVD), which we use to characterize the dominant aseasonal mode of variability present in the Kuroshio system.

The second algorithm is diffusion maps and spectral clustering (DMSC), a method that reduces the dimensionality of the velocity field time series by treating the data as a random walk. The advantages of DMSC are twofold: first, it allows us to specifically check for bimodality of the Kuroshio. Second, if bimodality is found, DMSC automatically identifies the modes and returns an index of the state of the system. Using SVD...
and DMSC in tandem allows us to establish both the dominant mode of variability of the Kuroshio, as well as its dominant bimodal feature. Agreement between SVD and DMSC results indicates that bimodality is not only present, but that the variability associated with this bimodality is the dominant form of variability in the system dynamics.

The remainder of this paper is organized as follows: in section 2, we describe DMSC and SVD and discuss the advantages of each algorithm. In section 3, we apply the algorithms to the Kuroshio system and show that they successfully find two distinct states. In section 4, we give a physical interpretation of the results. In section 5, we conclude, and in Appendix A we give a detailed description of the DMSC algorithm.

2. Methods

2.1. Data

We use surface velocity fields from January 1950 to December 2008 from the SODA database [Carton and Giese, 2008] as the input for both SVD and DMSC. The database contains monthly reanalysis velocity fields on a 5° (longitude and latitude) mesh. The reanalysis uses raw data sourced from various satellite, drifter, and bathythermograph observations. These are summarized in Table 1.

The domain we use is 131°E–140°E and 29.25°N–35.25°N, although results presented in section 3.4 show that the algorithm is robust to the choice of domain. The velocity field at time t is a point $y_t \in \mathbb{R}^{234}$. To construct the input used for both algorithms, we divide the reanalysis time series into individual years and concatenate all of the fields within each year to create 58 points $x_t \in \mathbb{R}^{234 \times 12}$. This approach has two advantages over using individual monthly velocity fields: (1) it prevents the algorithms from simply identifying seasonal modes and (2) it encodes dynamical information in the data. The qualitative features of the results presented in section 3 still hold when the algorithms are used with monthly input, but the seasonal cycle does introduce some noise that is eliminated by the use of yearly data.

To ensure that the results in sections 3.1 and 3.2 are not artifacts of the model used in the SODA reanalysis, we check them in section 3.5 using AVISO satellite sea surface height (SSH) data. The AVISO data set contains dynamic topography (SSH with respect to the geoid) in 10 day intervals and a 1°×1° grid for the years 1993–2009 [NCAR, 2013].

2.2. Singular Value Decomposition

Singular value decomposition is a standard method used in oceanography and other fields, which seeks to explain the variance of a high-dimensional time series $\{x_t\}_{t=1}^N$ by projecting each point onto a set of spatially orthonormal patterns [Bretherton et al., 1992]. These patterns are called empirical orthogonal functions (EOFs); the EOFs are in one-to-one correspondence with Principal Components (PCs) [Hannachi, 2004]. Each

<table>
<thead>
<tr>
<th>Variable</th>
<th>Name</th>
<th>Dates</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ocean temp</td>
<td>Mechanical bathythermograph</td>
<td>&lt;1968</td>
<td>Temperature sensors lowered by winches to a depth of 285 m.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>The number of observations is under 5000 yr$^{-1}$</td>
</tr>
<tr>
<td>Ocean temp</td>
<td>Expendable bathythermograph</td>
<td>&gt;1968</td>
<td>Probes dropped from &quot;ships of opportunity,&quot; measure the thermal profile of</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>the top kilometer of the ocean</td>
</tr>
<tr>
<td>Ocean temp, salinity</td>
<td>Argo floats</td>
<td>&gt;2000</td>
<td>Floats that drift at a depth of 1000 m for 9 days, then descend to 2000 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>and get temperature and salinity profiles as they rise to the surface</td>
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<tr>
<td>Sea surface temp (SST)</td>
<td>AVHRR/2 AVHRR/3</td>
<td>&gt;1981</td>
<td>Satellites, respectively, using 5 and 6 channel radiometers to report SST</td>
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<td></td>
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<td>at daily, 1°×1° resolution. Channels used for SST are 3B, 5, and 6, with</td>
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<td></td>
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<td>bands at 3.55–3.93, 10.30–11.30, and 11.50–12.50 μm, respectively</td>
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<tr>
<td>Sea surface height (SSH)</td>
<td>Topex/PoseidonJason 1</td>
<td>&gt;1991</td>
<td>Satellites that get SSH at 10 day resolution, using C and Ku-band radio</td>
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<td></td>
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<td>altimeters</td>
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<tr>
<td>Surface velocity, SST</td>
<td>Global drifter program</td>
<td>&gt;1979</td>
<td>Drifters spaced approximately 500 km from each other that get SST and</td>
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<td>velocity data</td>
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principal component is a one-dimensional time series of projection coefficients for the corresponding EOF. The EOFs and PCs are ordered in descending order of the magnitude of variance each mode explains. Importantly, even though SVD projects the original time series onto spatial modes, none of the modes need be independently observed. Each EOF represents a variable feature of the system; the state of the system at any time is a linear combination of all of the variable features. The original time series can therefore be reconstructed via

\[ x_i = \sum_{j=1}^{M} \text{EOF}_j \times PC_i. \]  

### 2.3. Diffusion Maps and Spectral Clustering

The diffusion maps and spectral clustering algorithm (DMSC), introduced by Belkin and Niyogi [2003] and further developed by Coifman et al. [2008], seeks to reduce the dimension of a high-dimensional time series by finding multimodal features of the underlying process. A more detailed description of DMSC can be found in Appendix A. DMSC constructs a random walk in phase space among the points of the original time series \( \{x_t\}_{t=1}^{N} \). The transition probability between two points \( x_i \) and \( x_j \) falls off exponentially with the Euclidean distance between them as:

\[ p_{ij} \propto e^{-\frac{|x_i - x_j|^2}{2\epsilon_{ij}}}. \]  

The exponential decay of transition probability with distance ensures that the random walk is likely to persist on sets of points with small pairwise separations. If the process that produced the time series is multimodal, these sets correspond to individual modes and determine the number of large eigenvalues of the random walk transition matrix \( P \). Thus, a large dropoff (spectral gap) in the magnitude of eigenvalues from \( \lambda_i \) to \( \lambda_{i+1} \) indicates that the underlying process is likely multimodal, while the lack of a spectral gap suggests a lack of multimodality.

Specifically, we expect a bimodal system to have one large eigenvalue so that \( \lambda_1 \gg \lambda_2 \) (we ignore the first eigenvalue \( \lambda_0 \) since it is always true that \( \lambda_0 = 1 \)). In this case, the first eigenvector can be used as a one-dimensional index for the state of the system. For an application of DMSC in the geophysical sciences, see Giannakis and Majda [2012].

### 2.4. A Comparison of SVD and DMSC

To illustrate the differences between SVD and DMSC, we apply both algorithms to El Niño Southern Oscillation (ENSO). The sea surface temperature (SST) in the eastern equatorial Pacific is highly variable: large positive anomalies in this region occur during El Niño and large negative anomalies occur during La Niña. The Oceanic Nino Index (ONI: a 3 month running mean SST anomaly in the region \( 5^\circ S-5^\circ N, 120^\circ W -170^\circ W \)) describes the oscillation between these states [Trenberth, 1997]. Importantly, strong El Niño or La Niña states are end-members of the smooth ONI time series. Because this time series is not bimodal, we expect SVD to find ENSO variability and DMSC not to identify bimodal behavior.

Figure 2 summarizes the results of both algorithms when tested on a time series of equatorial Pacific temperature fields from the SODA database. EOF1 (Figure 2a) shows a positive temperature anomaly in the eastern Pacific; we therefore expect PC1 to correlate strongly with ONI. Figure 2b shows that this is indeed the case, so we conclude that SVD successfully finds ENSO variability. In contrast, Figure 2c shows that the DMSC eigenvalues do not exhibit a spectral gap, reflecting the fact that ENSO is not bimodal.

### 3. Results

#### 3.1. DSMC: States Characterized by Path Variability and Gyre Location

Applying DMSC with constant \( \epsilon_i = 3 \sqrt{i} \) to the time series of yearly velocity fields yields a spectral gap between \( \lambda_1 \) and \( \lambda_2 \) with \( \lambda_2 = 42 \lambda_1 \). This suggests bimodal behavior. We thus consider a new, one-dimensional time series comprising the entries \( v_{1,i} \), of the first nonstationary eigenvector, \( v_1 \), of the transition matrix \( P \). To find the persistent features that correspond to this eigenvector, we then run k-means 2 clustering [Xu and Wunsch, 2005] on the entries of \( v_1 \), which yields a threshold entry value \( H \). The association rule is then
where $C_0$ and $C_1$ are the two clusters.

To visualize the difference between the two clusters, we seed paths in the southwestern region of the Kurashio; this is the portion of the domain where the current is most localized. Each path is a streamline emanating from $31^\circ$N and at a random longitude between $131.4^\circ$E and $132.4^\circ$E; this choice of longitude corresponds to the strongest region of the Kuroshio near the southwestern coast of Japan and excludes most paths that “escape” into the recirculation gyre centered at $136^\circ$E, $30^\circ$N. We first seed a random number of paths, typically $n \in \{0, ..., 8\}$ paths in each monthly velocity field in each year that falls into $C_0$; we then repeat this in a separate plot for all years that fall into $C_1$. The two clusters, with paths seeded in each and the average velocity fields in the background, are given in Figure 3. The seeded paths are used only for visualization and are not part of the DMSC algorithm.

Figure 3 suggests that $C_1$ corresponds to the small-meander state and that $C_0$ corresponds to the large-meander state. The time series of entries of the first eigenvector $v_1$ is used to assign years to $C_0$ and $C_1$;
each entry can thus be viewed as an index of the state of the system during the corresponding year. We call this the diffusion maps and spectral clustering (DMSC) index. The analysis in section 3.3 suggests that the DMSC index is bimodal; further, the switching between clusters occurs on an approximately decadal timescale.

Figure 3 also shows that the small and large meanders, respectively, do not correspond to states where paths either always remain close to the coast or always meander southward around 136\degree{}E; this is in contrast with previous work [e.g., Yoshida, 1961; Maximenko, 2002]. Instead, the small-meander state contains only paths that remain near the coast, while the large-meander state contains both paths that remain near the coast and those that exhibit traditional large-meander behavior. In fact, most paths seeded in the mean of \( C_0 \)—that is, the average velocity field in the large-meander cluster—remain near the coast and do not look like large-meander paths.

A striking difference between the average velocity fields in the two clusters is the location of the recirculation gyre, which is located at \( (136.25^\circ E, 31.25^\circ N) \) in the small meander and shifts \( \sim 5 \) S and \( \sim 2 \) W during the large meander. The importance of the location of the recirculation gyre is confirmed by the sensitivity analysis in section 3.4: in order to identify the states shown in Figure 3, the algorithm domain must include either the gyre or the region immediately downstream.

3.2. SVD: Gyre Location Is Key

Applying SVD to the yearly velocity data shows that the location of the recirculation gyre is the dominant form of variability in the system. The first EOF (Figure 4) shows an anticyclonic gyre centered at 136.25\degree{}E, 31.25\degree{}N and a cyclonic gyre centered at 133.75\degree{}E, 30.25\degree{}N. These actually correspond to different locations of the recirculation gyre in different years: when the first principal component is positive, the gyre is nearer to 136.25\degree{}E, 31.25\degree{}N. When the first principal component is negative, the gyre is nearer to 133.75\degree{}E, 30.25\degree{}N. We propose that these states correspond to the small and large meanders, respectively, as described by DMSC.

To test the notion that DMSC and SVD find the same phenomenon, we compare the DMSC index with the first principal component PC\(_1\). The two indices are very highly correlated, with a Pearson product-moment correlation coefficient of \( r = 0.98 \) [Rodgers and Nicewander, 1988]. These indices, along with others discussed in section 3.3, are shown in Figure 5. The near-perfect alignment of the DMSC index and PC\(_1\) shows that SVD and DMSC find the same feature of the Kuroshio system; we therefore conclude that the changing location of the gyre is both the dominant bimodal feature and the dominant variable feature in the system. This suggests that the bimodality of the gyre is a dominant dynamical feature of the system; the implications of this result for the meander transition mechanism are discussed in section 4.

3.3. Comparison With Other Indices

To further test the validity of the diffusion maps results, we compare the DMSC index to several physically intuitive indices. We use a path index, which is constructed from the average distance from the coast of the farthest 10% of paths. We also construct an index based on the gyre location (gyre index). This is done by calculating the displacement of the gyre center from its small-meander location along a line parallel to the coast of Japan. All indices are then normalized to have zero mean and equal variance.

Figure 5 shows a comparison of the DMSC (blue), gyre (blue), SVD (purple), path (red), and inflow (green) indices. There is close agreement among the indices; the gyre and DMSC indices are most closely matched
with a correlation coefficient of $r = 0.93$. The paths in Figure 3, as well as the correlation of the path index and the DMSC index, suggest the importance of the gyre in defining the state of the Kuroshio. The DMSC index is also well correlated with both the inflow ($r = 0.65$) and path ($r = 0.68$) indices, which suggests that the current and the recirculation gyre are strong during the small meander and weak during the large meander. This agrees with Qiu and Miao [2000], who present a model of the Kuroshio system in which the strengthening, migration, and weakening of the gyre is responsible for the transition between meanders.

Figure 6 shows histograms of the DMSC index (blue), the gyre index (green), and the path index (red). As expected, the DMSC and gyre indices agree well and show strong bimodality, with the majority of each time series being spent in either the small or large meander; the path index is qualitatively similar and show slightly weaker bimodality. The histograms show a slight preference for the small meander, with 54% of time spent nearer to this state than to the large-meander state.

The major discrepancy between the DMSC and the path index is that DMSC shows excursions into the large meander in 1958, 1970, and 1983. Figure 7 (top) shows the velocity fields with seeded paths in January of 1960 and 1978. Figure 7 (bottom) shows paths in January 1967, which is a strongly small-meander year according to both indices. Figure 7 (bottom) shows paths seeded in the velocity field of June 1970, a year classified as large meander by the DMSC index but small meander by the path index. As with other such years, the center of the recirculation gyre (135°E, 30°25’ N) is about 2° to the west and 1° to the south of where it is during small-meander years.

The proposed importance of the recirculation gyre also explains the discrepancy between the DMSC and path indices in 1970. As Figure 7 (bottom, right) shows the recirculation gyre is approximately 2° to the west and 1° to the south of where it is during small-meander years; however, there is an anomalous second gyre centered at 139°E, 31°N. This eastern gyre pushes paths back toward the coast and prevents the vast majority of paths from forming a large-meander pattern.

### 3.4. Sensitivity Analysis

We test the sensitivity of the diffusion maps and spectral clustering algorithm to domain size by simultaneously varying the minimum and maximum latitude, $\text{lat}_{\text{min}}$ and $\text{lat}_{\text{max}}$, as well as the minimum and maximum longitude, $\text{lon}_{\text{min}}$ and $\text{lon}_{\text{max}}$. The results indicate that the algorithm is robust to changes in domain size, with minimal impact on the identified clusters and their temporal evolution.
long$_{\text{min}}$ and long$_{\text{max}}$ of the domain in which we perform the analysis. For each domain size, we compute the correlation coefficient of the DMSC index with the new domain and the DMSC index with the original domain.

For any fixed values of (long$_{\text{min}}$ $\leq$ 133.5° E, lat$_{\text{min}}$ $\leq$ 30° N), the correlation coefficient between the DMSC indices for the old and new domains is $r > 0.9$ as long as (long$_{\text{max}}$ $\geq$ 137° E and lat$_{\text{max}}$ $\geq$ 32° N). This is consistent with our previous discussion in section 3.1 of the importance of both the recirculation gyre and the region immediately downstream of the gyre (where the current is localized in the small meander and highly variable in the large meander). The algorithm breaks down if the boundaries are chosen such this region is excluded in many months. When this region is included, the algorithm is robust to enlargement of the domain to include most of the Pacific basin.

To test whether DMSC simply finds the location of the gyre without identifying the Kuroshio, we ran the algorithm on a modified domain that excludes the region occupied by the gyre but includes the region just downstream of the gyre; this is the region where the current is localized in the small meander and highly variable in the large meander. The resulting modified DMSC index has a correlation of $r = 0.94$ with the original DMSC index; this suggests that DMSC identifies variability in the Kuroshio and not simply the location of the gyre.

Finally, we check the sensitivity of DMSC to the local-scale parameter, $\epsilon$, which is roughly the radius around each data point $x_i$ such that the nearby data are well approximated with a hyperplane. In the previous

Figure 7. Velocity fields and paths seeded in several years of interest. (a) 1960 and (b) 1978 are large-meander years according to both the diffusion maps and spectral clustering (DMSC) and the path indices; (c) 1967 is small meander according to both; (d) 1970 is large meander according to the DMSC index but not the path index.
analysis, we used $\epsilon = 3$; this is comparable with the standard deviation of the distances of velocity fields, which is 3.12 (corresponding to $\sim 0.2 \text{ m/s}$ at each grid point). The results are robust to using $\epsilon \in (2, 6)$. This is consistent with the fact that if $\epsilon$ is chosen too small (significantly less than one standard deviation), the transition probability away from any point $x_i$ becomes negligible; if $\epsilon$ is chosen too large (greater than two standard deviations), the random walk is approximately uniform over the data and the underlying structure is lost.

3.5. Comparison With AVISO SSH Data

Given that the results presented in sections 3.1 and 3.2 implicate the recirculation gyre in the bimodal nature of the Kuroshio, we must confirm that the bimodality of the system is not an artifact of the model used in the SODA reanalysis. To do this, we compare SODA-based indices to ones calculated from AVISO satellite sea surface height (SSH) data for the years 1993–2009.

First, we calculate the DMSC index for the years in which AVISO data are available and compare it with a DMSC index calculated using the AVISO data. The indices agree well, but neither shows bimodality for this period. This is a result of the AVISO time series being too short to contain any significant excursions into the large meander, rather than a failure either of the data or of our methods. This is supported by the fact that the DMSC, gyre, and path indices calculated using the full SODA time series show no significant excursions into the large meander between 1993 and 2009.

To show that the SODA data are consistent with AVISO data (and that the bimodality we find is therefore not an artifact of the SODA model), we calculate the gyre index using the AVISO data. A comparison of a gyre index calculated using AVISO data with the SODA DMSC index for the years 1993–2009 yields a correlation coefficient of $r = 0.83$. We therefore conclude that the features found using SVD and DMSC on the SODA reanalysis are not model artifacts.

4. Discussion

As noted in section 3.1, the small-meander state contains only paths that remain near the coast, while the large-meander state contains both paths that remain near the coast and those that exhibit traditional large-meander behavior. From this observation, we conclude that large and small meanders are separated by variation in path distance from the coast more than by the average path distance from the coast: the small meander has low variance in path distance from the coast while the large meander has high variance in path distance. Accurate models of the Kuroshio must explain this change in variability in addition to the previously observed change in the current axis.

Importantly, the gyre moves toward the southwest of Japan during the large meander. This is a region where the Kuroshio is highly localized at all times; thus, the gyre moves closer to the incoming current and spins a portion of it off to the southeast. Because only the easternmost portion of the current is affected by the gyre, most paths that originate at the western portion of the current continue to follow small-meander trajectories. This accounts for the increased variance of path location during large-meander years despite a relatively small shift in the mean path location.

While this study does not explicitly yield information about the transitions between the two states, it provides qualitative evidence for the transition mechanism proposed by Qiu and Miao (2000):

1. Low-potential vorticity (PV) water is carried northward by the Kuroshio.
2. Accumulation of a low PV anomaly strengthens the recirculation gyre to the south of the Kuroshio and presses the current to the coast, leading to a small meander.
3. The strengthening gyre increases shear, causing instability that leads to the large meander.
4. The large-meander path leads to cyclonic eddies that mix PV, weakening the gyre.
5. The system restarts when low-PV water is once more transported from the south.

Our results show that, as in the above mechanism, the small meander is prevalent when inflow (and therefore current velocity) is high, the gyre velocities are strongest, and the gyre is near the northeasternmost part of its range (step 2). The gyre then moves to the southwest and weakens (steps 3 and 4). When this
happens, a portion of the current is spun off to the southeast; this leads to the formation of the large meander. As we have shown, the large-meander state corresponds to lower inflow and a southwestern gyre location, as predicted by steps 3 and 4 of the mechanism above. Thus, while our analysis does not directly yield information about the dynamics of the system, it shows that the salient features of the small and large meanders agree well with those that would result from the dynamics proposed by Qiu and Miao [2000].

5. Conclusions

By using SVD in tandem with DMSC, we are able to characterize both the variability and the bimodality of the Kuroshio. We find that the recirculation gyre south of Japan is both the dominant form of variability and the dominant bimodal feature in the system. With DMSC, we automatically create a one-dimensional index that characterizes the state of the Kuroshio; this index is highly correlated with the location of the gyre. Further, we find that small and large meanders are well characterized by small and large path variation, respectively, but are poorly characterized by mean path location. We thus conclude that:

1. The Kuroshio is bimodal, with high inflow and low path variability in the small meander and low inflow and high path variability in the large meander.
2. The dominant form of variability and the dominant bimodal feature in the system is the location of the recirculation gyre.
3. The large and small meanders are better characterized by path variability than by mean path location.

More work is needed to definitively characterize the dynamics of the Kuroshio and the mechanism by which it transitions between the small and large meanders. Further, Pierini [2006] proposes that the Kuroshio Extension east of Japan displays bimodality that is driven by an internal oscillation related to the mechanism responsible for bimodality of the Kuroshio South of Japan. A similar study of the Kuroshio Extension and of the entire Kuroshio system could shed light on Kuroshio Extension variability and its connection of bimodality of the Kuroshio south of Japan.

By rigorously characterizing the states of the Kuroshio, we hope to facilitate further work on the dynamics of the Kuroshio. Information about the mechanism of transition can help in developing better models as well as in further data analysis of the transition itself [Miller et al., 1994; Vanden-Eijnden and Weare, 2012, 2013].

Appendix A: Diffusion Maps and Spectral Clustering

DMSC is a method that reduces the dimensionality of a high-dimensional time series by considering the data as a random walk. Specifically, each point in the time series is stripped of the time dimension and is then plotted in state space. The data are then treated as a random walk, with the transition probability between any two points $\mathbf{x}_i$ and $\mathbf{x}_j$ given by

$$
 p_{ij} \propto e^{-\frac{||\mathbf{x}_i - \mathbf{x}_j||^2}{2\sigma^2}},
$$

(A1)

where $\mathbf{x}_i$ and $\mathbf{x}_j$ are local-scale parameters. The transition probability between any two points decays exponentially with the distance between the points. In this study, we normalize the variance of the list of pairwise distances between data points, $d_{ij}$, to unity in order to allow the DMSC parameter to scale well with domain size for a constant choice of the local-scale parameter $\epsilon$.

The exponential decay of transition probability with distance ensures that the random walk is likely to persist for a relatively long time on sets of points with small pairwise separations. These sets need not correspond to obvious clusters—groups of points close to one of several centroids—but can be spatially separated sets with similar centroids, e.g., concentric rings in $\mathbb{R}^2$. Note that, in this case, standard clustering algorithms, such as k-means, would fail to identify the concentric rings because they are centered at the same point. The random walk constructed in diffusion maps would persist within each ring for a long time before transitioning to the other ring.

To exploit this feature of the random walk, we construct the transition matrix $P$ with entries $p_{ij}$ and compute its eigenvalues. By the Perron-Frobenius theorem, the largest eigenvalue of $P$ is $\lambda_0 = 1$; the associated eigenvector $\mathbf{v}_0$ corresponds to the stationary distribution [Seneta, 1973]. The next largest eigenvalues and their...
associated eigenvectors correspond to the most persistent features (those with long relaxation times to the stationary distribution) of the random walk. The presence of a spectral gap among the first few eigenvalues allows us to estimate the amount of persistent features. That is, a large dropoff from \( \lambda_i \) to \( \lambda_{i+1} \) indicates that there are \( i \) relatively persistent features of the random walk [Rohrdanz et al., 2011]. These features correspond to clusters in state space.

The spectral clustering portion of the algorithm associates points in the original time series with the clusters present in the random walk. We construct a low-dimensional data set comprising the first \( i \) "important" eigenvectors of \( \mathbf{P} \). Each point in this data set is \( (\mathbf{v}_j, \mathbf{v}_{j+1}, \ldots, \mathbf{v}_N) \), where \( j \in \{1, \ldots, N\} \) and \( N \) is the number of points in the original data. The new data encode information about the spatial separation between points, so running a clustering algorithm on it finds the persistent features of the random walk. We then assign points in the original data to corresponding clusters in the high-dimensional state space to find the physical significance of the persistent features of the original time series; in this study, we compute the average of each physical cluster in order to illustrate the qualitative difference between the two states.

The full diffusion maps and spectral clustering algorithm is:

1. Compute the transition matrix \( \mathbf{P} \) according to equation (A1) using the original time series \( \{x_i\}_{i=1}^N \). Normalize so that the entries in each row and column sum to 1.
2. Compute the eigenvalues of \( \mathbf{P} \). Determine whether there is a spectral gap to isolate persistent features.
3. Construct a low-dimensional dataset \( \{x_i\}_{i=1}^N \) using the first \( j \) eigenvectors, where \( j \) is the number of large eigenvalues preceding the spectral gap.
4. Run a clustering algorithm on the low-dimensional data to obtain clusters \( \{\hat{C}_k\}_{k=1}^j \).
5. Map the original time series to clusters in the high-dimensional state space using the rule
   \[ x_i \in C_k \iff \hat{x}_i \in \hat{C}_k. \]  
   \[ \text{(A2)} \]
6. Compute the averages and/or centroids of the physical clusters \( C_k \).

While the advantage of DMSC is its specificity in picking up on multimodality, its results must be interpreted carefully for several reasons. First, unlike SVD, DMSC does not necessarily find features explaining a significant amount of the variance in a system; it is therefore imperative to verify the dynamic relevance of DMSC results (as we do in this study by comparing to SVD results). Second, while DMSC can be used to characterize the modes of a system, it does not yield explicit dynamical information about the pathway(s) the system takes among modes. Finally, DMSC can be sensitive to the choice of \( \epsilon \), so robustness to this parameter should be checked prior to drawing conclusions.

References


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