

1 **Scaling Laws for Iceberg Calving: In search of a**  
2 **dynamic stability criterion for calving glaciers**

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1 Motivated by the complexity of the short-time scale behavior of ice frac-  
2 ture, we seek a scaling law that predicts the onset of retreat of calving glaciers  
3 when one or more non-dimensional numbers decreases below a critical value.  
4 In this approach, rather than attempting to model the individual processes  
5 involved in calving, we derive a non-dimensional number that measures the  
6 overall stability of the calving glacier system. Our analysis, using data from  
7 a range of glaciological regimes, including 12 calving glaciers in Alaska, two  
8 of Greenland's outlet glaciers, and four Antarctic ice shelves, indicates that  
9 such a stability criterion exists and may provide a necessary (but perhaps  
10 insufficient) condition for retreat.

## 1. Introduction

11 Calving is an important process in all glaciers that terminate in bodies of water, in-  
12 cluding ice shelves, accounting for as much as half of the mass lost from Antarctica and  
13 40% of the mass lost from Greenland [e.g. *Jacobs et al.*, 1992; *Biggs*, 1999]. The processes  
14 that cause iceberg detachment are complex, involving the initiation and propagation of  
15 fractures over spatial scales ranging from crystal-sized defects to rifts systems that span  
16 hundreds of kilometers. Although the formation and propagation of crevasses and rifts  
17 within the ice are clearly linked to flow dynamics over a wide range of environmental  
18 conditions and flow regimes, we do yet have a well-formulated mathematical model of  
19 iceberg calving [*Van der Veen*, 2002; *Joughin and MacAyeal*, 2005]. This suggests that  
20 success in incorporating iceberg calving into the next-generation ice-sheet models hinges  
21 on formulating (and testing) an empirical parameterization of the calving process in which  
22 time-averaged mass loss from ice shelves and glaciers is related to internal dynamic and  
23 geometric variables.

24 In this study, instead of trying to formulate a calving law, we seek a dynamic crite-  
25 rion that determines whether calving glaciers are stable or will retreat by iceberg calving.  
26 Our approach is motivated by the study of hydrodynamic instabilities. These instabilities  
27 arise in a variety of phenomena, e.g., the onset of turbulence [*Barenblatt*, 1996], that in-  
28 volves a bifurcation, with radically different behavior depending on whether the value of  
29 a particular non-dimensional group falls above or below a critical value. A fully predic-  
30 tive understanding of turbulence has been difficult to develop, yet the Kármán-Prandtl  
31 universal logarithmic law for wall-bounded turbulent shear flow, deduced using scaling ar-

guments, agrees remarkably well with experiments [Barenblatt, 1996]. For this study, we explore the possibility that similar scaling arguments may also apply to iceberg calving. We follow *Van der Veen* [2002] and adopt the term 'calving glacier' to encompass the full range of ice bodies that terminate in water with depth comparable to or greater than the near-terminus ice thickness.

## 2. Calving Laws

Since the pioneering work of *Brown et al.* [1982], researchers have sought to relate a time-averaged calving rate,  $V_c$ , to different geometric or dynamic near-terminus parameters,  $p_i$ :

$$V_c = \psi(p_1, p_2, \dots, p_N), \quad (1)$$

where  $\psi$  is an unknown function. Many researchers have sought linear correlations between the calving rate and near-terminus values of ice thickness, water depth, strain rate and terminus velocity [Brown et al., 1982; Venteris et al., 1996]. But, expanding equation (1) using a standard Taylor series about a state where the terminus position is stable shows that to first order:

$$V_c = V_{term} + \sum_i^N \frac{\partial \psi}{\partial p_i} \Delta p_i + \dots \quad (2)$$

Thus the calving rate, to zeroth-order, is proportional to the terminus velocity; whether the calving glacier will advance or retreat is governed by the higher-order terms. Our interest is in determining whether a calving glacier is quasi-stable or susceptible to rapid retreat. Therefore we re-formulate equation (2) in terms of rate of terminus advance:

$$\frac{dL}{dt} = V_{term} - V_c = \xi(p_1, p_2, \dots, p_N), \quad (3)$$

where  $\xi$  is another unknown function that may depend on a number of internal and external parameters and/or flow history, all of which will vary from glacier to glacier and

possibly over time. Our approach is to use tools from dimensional analysis to reduce the number of degrees of freedom within the system and elucidate physically consistent scaling-laws relating the rate of terminus advance to non-dimensional groups formed from physical parameters [e.g., *Barenblatt*, 1996]. We hypothesize that the transition between a stable ice front and retreat is governed by one or more non-dimensional groups, analogous to the way the transition between laminar and turbulent flow is controlled by the Reynolds number.

### 3. Dimensional Analysis and Stability Criteria

The premise of dimensional analysis is that if the independent variables controlling terminus retreat are known, then any mathematical model that describes iceberg calving must be invariant with respect to a change in units. Dimensional analysis is thereby a useful technique to help guess the correct form for a solution for problems where a mathematical model is not *a priori* known or is too complicated to be useful - conditions that certainly apply to iceberg calving. The first step in this procedure is to determine which parameters are important in controlling the stability of calving glaciers. We anticipate that important geometric parameters are the glacier width  $W$ , length  $T$ , ice thickness  $H$  and water depth  $D$ ; important material parameters are the densities of ice and water ( $\rho_{ice}$ ,  $\rho_w$ ), and the effective viscosity of ice  $\eta$ ; dynamic variables are the near-terminus speed of the glacier  $U$  and acceleration due to gravity  $g$ . To account for both freely floating and grounded termini, we define the effective water depth ( $D$ ) as the distance from the base of the ice to the surface of the water. For a freely floating terminus,  $D$  and  $H$  are related by buoyancy such that  $\frac{D}{H} = \frac{\rho_i}{\rho_w}$ .

74 A non-dimensional basis set could be derived from the null-space of the dimension  
 75 matrix using the Buckingham-pi theorem [see e.g., *Barenblatt, 1996*]. Instead, we exploit  
 76 two observations: (i) thinning has preceded retreat in several cases [*Shepherd et al.,*  
 77 *2003; Pfeffer, 2007*]; and (ii) terminus velocity often increased during retreat [*Van der*  
 78 *Veen, 2002*]. This leads us to postulate that, if all other variables remain constant, a  
 79 larger ice thickness will increase stability, and faster terminus speed will decrease stability.  
 80 Furthermore, since unconfined ice shelves do not exist, we make the further tentative  
 81 hypothesis that an unconfined, freely floating terminus is unstable. (Ice tongues appear  
 82 to be an exception to this rule and we return to this point in section 6.) A potential  
 83 stability parameter that satisfies these assumptions is the Argand Number, relating the  
 84 ratio of depth-averaged buoyancy stress near the terminus to the viscous resistive stresses:

$$S = \frac{\rho_{ice} g H \left(1 - \frac{\rho_w}{\rho_{ice}} \left(\frac{D}{H}\right)^2\right)}{\eta \left(\frac{U}{L}\right)}. \quad (4)$$

86 For the velocity scale, a convenient metric to use is the center-line terminus velocity,  
 87  $U_{term}$ . For the length scale ( $L$ ), we have three choice: the ice thickness, the glacier  
 88 width or the glacier length. The ratio of terminus velocity to ice thickness would be  
 89 appropriate for glaciers that terminate in shallow water or are frozen to their beds, and  
 90 thus are restrained mainly by friction immediately beneath the ice. However, most calving  
 91 glaciers flow rapidly over weak, lubricated beds [*Meier and Post, 1987*]. For glaciers and  
 92 ice shelves that are confined in fjords or embayments, we hypothesize that the most  
 93 reasonable length scale is the width of the glacier. We do not consider the length of the  
 94 glacier to be a suitable length scale because processes occurring far from the terminus are

95 unlikely to control calving behavior at the terminus. Replacing  $U$  with  $U_{term}$  and  $L$  with  
 96  $W$  we obtain:

$$S_{term} = \frac{\rho_{ice} g H \left( 1 - \frac{\rho_w}{\rho_{ice}} \left( \frac{D}{H} \right)^2 \right)}{\eta \left( \frac{U_{term}}{W} \right)}. \quad (5)$$

98 (Since our interest is in forming a non-dimensional number, we omit any factors of two  
 99 that arise from considering such things as the half-width of the glacier.) We call this  
 100 number the ‘terminus stability number’. Intuitively, we expect retreat to occur when  
 101  $S_{term}$  decreases below a critical value while large values will indicate stability. To test this  
 102 hypothesis, we assembled published data from a variety of environments and geometries  
 103 including Alaskan tidewater glaciers, Antarctic ice shelves and Greenland outlet glaciers.

#### 4. Data

104 Our analysis focuses on the Columbia Glacier, Alaska because it contains a continu-  
 105 ous observation record that dates back to the 1950s. Photogrammetric data have been  
 106 collected regularly by USGS since 1974, extending up to the year 2000 [*Krimmel*, 2001],  
 107 along with detailed bathymetry [*Rasmussen and Meier*, 1982, 1985]. This data-set cov-  
 108 ers periods both prior to and during rapid retreat, which began ca.  $\sim$ 1984. We used a  
 109 weighted-nearest-neighbor algorithm with a radius of 500 meters (giving all points within  
 110 500 m of the point an equal weight and a weight of zero to all those points outside the ra-  
 111 dius) to interpolate the velocity, surface elevation and water depth to the terminus [*O’Neel*  
 112 *et al.*, 2006]. Data were then smoothed using a 2-year median filter. In addition, we used  
 113 the historical data published by *Brown et al.* [1982] for 12 Alaskan glaciers (including  
 114 Columbia Glacier, prior to retreat). For Greenland outlet glaciers, we used data from  
 115 *Howat et al.* [2005] for Helheim Glacier and *Joughin et al.* [2004] for Jakobshavn Isbrae.

116 For the Antarctic ice shelves, we focus on the Amery Ice Shelf (AIS) because it is confined  
117 to a relatively narrow embayment (making it easier to define a width), there are historic  
118 data before and after a major calving event ca.  $\sim 1963$  [Budd, 1966], and modern satellite  
119 data that show that Amery is in an intermediate stage of advancement [Fricker *et al.*,  
120 2001]. We also use data for the Ross Ice Shelf [Keys *et al.*, 1998], Pine Island Glacier (D.  
121 Vaughan, personal communication, 2007) and the Larsen A and B ice shelves, prior to  
122 collapse [Scambos *et al.*, 2003].

123 One of the major sources of uncertainty in applying equation (5) to calving glaciers is  
124 what to use for viscosity. Ice is often assumed to deform as a power-law fluid with an  
125 Arrhenius temperature dependence [Paterson, 1994], but this is unlikely to be a valid  
126 approximation to the large-scale rheology in the highly fractured near-terminus region  
127 of rapidly retreating tidewater glaciers. For this study we use a temperature-dependent  
128 Newtonian viscosity with a value of  $1 \times 10^{13}$  Pa·s for temperate tidewater glaciers,  $5 \times 10^{13}$   
129 Pa·s for polar glaciers and  $10^{14}$  Pa·s for ice shelves. This is broadly consistent with the  
130 Arrhenius relationship for ice (with an activation energy of  $54 \text{ kJ mol}^{-1}$  [Paterson, 1994]  
131 and temperature of  $0^\circ\text{C}$ ,  $-10^\circ\text{C}$  and  $-25^\circ\text{C}$ , respectively), but should be regarded as a crude  
132 parameterization of the effect of temperature and introduces a ‘free’ parameter into our  
133 calculations.

134 Defining the width of the Ross and Larsen ice shelves is also problematic. For the  
135 Ross Ice Shelf we use the maximum length of the large tabular bergs that detached  
136 in 2000 ( $\sim 300$  km), assuming that the ‘effective’ width of the ice shelf is the distance  
137 between suture zones. Using the full width, 600 km, would double the stability number.

138 For the irregular geometry of the Larsen A Ice Shelf, we use a value of 50 km (the  
139 approximate distance from Larsen Nunatak to Sobral Peninsula) and 100 km for Larsen  
140 B (the approximate distance between Sobral Peninsula to Robertson Island).

## 5. Results

141 Figure 1 shows the relationship between the terminus stability number and the rate  
142 of terminus advance for each study site (we assumed a constant terminus position for  
143 Pine Island and the other ice shelves). Despite the different environments and geometries,  
144 a consistent pattern is evident. Of the 12 glaciers studied by *Brown et al.* [1982] that  
145 had values of  $S_{term}$  substantially greater than  $\sim 2$  (Grand Pacific, Margerie, Hubbard,  
146 Columbia, Harvard), all were either advancing or close to steady state. In contrast, the  
147 three tidewater glaciers retreating most rapidly (South Sawyer, Muir and Yale) all have  
148 terminus stability numbers greater than two, and have continued to retreat [*Molnia*, 2007].  
149 Although the broad pattern that we see is that glaciers with low values of  $S_{term}$  are rapidly  
150 retreating, there are some exceptions. For example, both Johns Hopkins and McCarty  
151 glaciers had low values of  $S_{term}$ , yet were also slowly advancing. Examining the history of  
152 these glaciers shows that McCarty was retreating rapidly prior to 1978, but subsequently  
153 started to advance [*Molnia*, 2007]. Similarly, Johns Hopkins was retreating slowly during  
154 the measurement period of *Brown et al.* [1982], but has since advanced [*Molnia*, 2007].  
155 Another exception is Tyndall glacier, which was retreating, but had a relatively large value  
156 of  $S_{term}$ . This may be evidence of a transitional regime for those glaciers with  $S_{term}$  close  
157 to unity. However, according to *Molnia* [2007], Tyndall has stabilized at its terrestrial  
158 limit and therefore the assumption that the terminus terminates in deep water may be

159 violated. Values for the Columbia Glacier inferred from the photogrammetric data (red  
160 circles) show the same pattern as the other tidewater glaciers. Columbia Glacier had  
161 high values of  $S_{term}$  when it was stable or slowly advancing.  $S_{term}$  drops to less than 2  
162 once Columbia's rapid retreat phase commenced. Jakobshavn and Helheim glaciers, two  
163 Greenland glaciers that had retreated rapidly, also had very low values of  $S_{term}$  consistent  
164 with the trend for Alaskan tidewater glaciers. (Jakobshavn may have since stabilized).

165 Inspection of the ice shelves in Figure 1 shows a similar pattern as for the glaciers. The  
166 Ross Ice Shelf has the highest stability number. For the AIS we have three measurements  
167 of  $S_{term}$ ; one prior to the major calving event of 1963 (AIS1) with the lowest value of  
168  $S_{term}$ ; one immediately after the calving event (AIS2) with the largest value of  $S_{term}$ ; and  
169 one contemporary measurement which has an intermediate value of  $S_{term}$  (AIS3). The  
170 increase in  $S_{term}$  following a calving event may be a sign of a stable calving regime. On  
171 the other hand, Pine Island Glacier and the Larsen A and B ice shelves have the lowest  
172 values of  $S_{term}$ . (Larsen A and B have similar values of  $S_{term}$  and are denoted by one  
173 symbol in Figure 1). Choosing a different value of the viscosity for the ice shelves would  
174 move these points up or down, but the overall ice shelf trend would remain the same.

## 6. Discussion

175 Despite the widely different environments studied, our results suggest that there exists a  
176 critical terminus stability number that marks the transition from those glaciers that have  
177 stable terminus positions or are retreating/advancing slowly and those that are retreating  
178 rapidly. However, there is no discernible pattern to the rate of retreat; once  $S_{term}$  drops  
179 below approximately 2, retreat occurs at a range of rates. A prominent exception to the

180 pattern is Pine Island Glacier. Although our analysis suggests that Pine Island should be  
181 unstable, there does not yet appear to be any evidence of that Pine Island has entered a  
182 phase of rapid retreat. Thus, the absence of an impending Pine Island collapse may be  
183 evidence that our analysis needs to be modified to accommodate additional parameters.  
184 A likely culprit is surface meltwater, linked by *Scambos et al.* [2003] to the disintegration  
185 of the Larsen A and B ice shelves. Likewise, the presence of unconfined ice tongues is  
186 problematic for our analysis. Here, sea-ice and/or fast-ice may play a mechanical role in  
187 stabilizing ice tongues and ice shelves [*Copland et al.*, 2007]. All of this implies that low  
188 values of  $S_{term}$  may be a necessary, but insufficient condition for rapid retreat. There are  
189 a number of possible variables in addition to sea-ice that we have not considered that  
190 may factor into the stability of calving glaciers. For example, it has been postulated that  
191 the limit of viability of ice shelves is determined by atmospheric temperature [*Vaughan*  
192 *and Doake*, 1996]. However, a single temperature limit alone is insufficient to explain the  
193 stability of all Antarctic and Arctic ice shelves [*Copland et al.*, 2007]. In our view, it is  
194 the combination of dynamic conditions (i.e., a low value of  $S_{term}$ ) in combination with  
195 environmental conditions (e.g., atmospheric temperature, oceanic temperature) that leads  
196 to rapid retreat. In other words, the ‘phase space’ of stability (and instability) of calving  
197 glaciers may depend on more than one variable.

198 Unlike many previously proposed calving laws, our analysis indicates a complex rela-  
199 tionship between ice thickness, velocity and glacier geometry. For instance, because the  
200 velocity depends on ice thickness, an increase in thickness could either increase *or* decrease  
201 stability, depending on how that change affects the ice velocity. To further examine this

relationship, we perform a force balance on a column of ice. The numerator of equation (5) is proportional to the depth averaged horizontal stress,  $\tau_{dr}$ , acting on the ice-water cliff:

$$\tau_{dr} = \rho_{ice} g \frac{H}{2} \left( 1 - \frac{\rho_w}{\rho_{ice}} \left( \frac{D}{H} \right)^2 \right) \quad (6)$$

The denominator of equation (5) is a measure of the viscous resistive stresses within the ice  $\tau_{visc}$ . The stress  $\tau_{dr}$  will be partly balanced by basal resistance to flow from friction along the base  $\tau_{base}$ , by lateral shear along the margins of the embayment or fjord walls  $\tau_{shear}$  and the back-pressure due to any pinning points or terminal moraines near the front of the glacier  $\tau_{pin}$ . Performing a force balance, we find:

$$\tau_{dr} = \tau_{visc} + \tau_{base} + \tau_{shear} + \tau_{pin}. \quad (7)$$

Solving for the viscous stress  $\tau_{visc}$  and then substituting into equation (5) shows that (to within a factor of 2):

$$S_{term} \sim \frac{\tau_{dr}}{\tau_{visc}} = \frac{\tau_{dr}}{\tau_{dr} - \tau_{base} - \tau_{shear} - \tau_{pin}}. \quad (8)$$

Equation (8) indicates that a decrease in stability could be caused by a decrease in either (i) basal shear stress, (ii) lateral shear stress or (iii) buttressing. (In this analysis, we do not speculate how such a decrease might occur, only the consequences). For glaciers that are grounded, a retreat may increase lateral drag thereby triggering a decrease in  $S_{term}$ . If the glacier retreat brings the terminus into deeper water this could further lead to a reduction in basal resistance (due to increasing pore pressure) thereby triggering a greater decrease in  $S_{term}$ , and further increasing the likelihood of retreat. Thus as tidewater glaciers approach buoyancy, more of the driving stress may be balanced by an increase in  $\tau_{visc}$ , promoting instability and increased potential for retreat via iceberg calving. This

223 is similar, at least qualitatively, to the water-depth and height-above buoyancy laws pro-  
224 posed in the literature for tidewater glaciers [*Van der Veen*, 2002] and to the kinetic wave  
225 theory recently proposed by *Pfeffer* [2007]. Applied to ice shelves, our terminus stability  
226 number is similar to the criterion proposed by *Doake et al.* [1998] in which the stability of  
227 ice shelves is related to the presence of a ‘compressive arch’. We predict that ice shelves  
228 are stable provided there is resistance to flow from either the walls or bed. Once that  
229 resistance is removed, i.e., by penetrating the compressive arch, we predict a transition  
230 into an unstable regime where rapid retreat is possible (but additional conditions may  
231 be necessary before retreat for retreat to occur, e.g. surface melt ponds). The terminus  
232 stability number that we propose may provide a heuristic means of calculating how close  
233 a glacier is to violating the compressive arch, without the need for a finite-element model.

## 7. Conclusions

234 Using dimensional analysis, we have found a non-dimensional number,  $S_{term}$ , that de-  
235 fines whether the terminus position of a calving glacier is stable. Measurements for a  
236 variety of temperate and polar tidewater glaciers and Antarctic ice shelves, show that, for  
237 large values of  $S_{term}$ , the terminus is stable and the glacier may even advance. When  $S_{term}$   
238 decreases below a critical value, the glacier appears to be primed for retreat. We speculate  
239 that, like other hydrodynamic instabilities,  $S_{term}$  controls the transition between a stable  
240 and unstable behavior. This relationship appears to hold for a dozen tidewater glaciers  
241 in Alaska, as well as for two Greenland outlet glaciers and several Antarctic ice shelves.  
242 Although a general mathematical model for calving remains elusive, the empirical analysis  
243 that we have presented provides a diagnostic of the health of glaciers and ice shelves that

244 unifies ideas put forward separately to describe the stability of tidewater glaciers and ice  
245 shelves. Moreover, the scaling implicit in the stability criterion presented here may guide  
246 attempts to develop a theoretical framework to describe rapid retreat of glaciers and ice  
247 shelves.

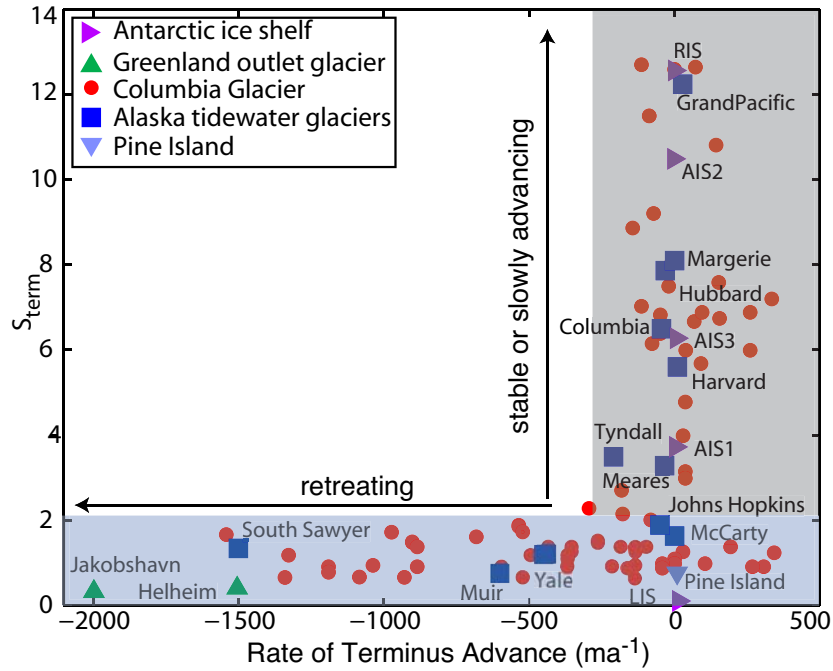
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**Figure 1.** Relationship between terminus stability number,  $S_{term}$ , and rate of terminus advance. Red circles show  $S_{term}$  as a function of time for the Columbia Glacier, computed from the time series of photogrammetric data [Krimmel, 2001]. There is a broad trend in which stable, slowly advancing or slowly retreating glaciers have  $S_{term}$  larger than 2 while those that are retreating rapidly have values of  $S_{term}$  smaller than 2. For all of the Antarctic ice shelves, we assumed the rate of terminus advance to be zero. Larsen A and B ice shelves are denoted by the same symbol because the pre-disintegration values of  $S_{term}$  are similar.