CHAPTER 6

Glacial Hydrology

> "If you follow the glacier down to the river (at no slight risk of breaking your neck) you see a large torrent issuing from beneath the glacier itself to join the Chundra River. . . ."
>
> ***Journal of a Tour Through Spiti*, P.H. Egerton (1864)**[1]

6.1 Introduction

The flow of water through glaciers is a topic with extraordinary mystique. Meltwater channels form on the glacier surface and then disappear into chasms and caverns, while roaring rivers emerge from tunnels at the glacier front. Until very recently, some of the world's largest lakes remained hidden and unknown beneath the Antarctic Ice Sheet. Iceland's ice caps periodically release immense floods; the largest ones convey more water than the Mississippi and Nile combined.

Glacier hydrology is the study of water storage and transport in glaciers – and how glaciers release this water to river systems. Glacier hydrology has important practical applications:

1. Rivers fed, in part, by melt from glaciers provide much of the water supply for agriculture in regions such as the Canadian prairies and central Asia, and for hydroelectric power production in France, Switzerland, and Norway. In comparison with other types of streamflow, glacier runoff has unusual features such as large diurnal fluctuations and maximum flow during summer. Glaciers act as reservoirs that store water in solid form in cool summers and release large amounts in hot dry summers when water from other sources is in short supply. Also important is long-term storage; the mean annual flow of a glacier-fed river measured during a period of glacier retreat overestimates the amount of water available when the glaciers stabilize or start to advance. On the other hand, sustained retreat – a likely consequence of climate warming – will eventually reduce or even eliminate this source of water. To forecast the flow of glacier-fed streams requires an understanding of accumulation and ablation processes (Chapters 4 and 5), of glacier hydraulics, and of adjustments of glacier geometry.

[1] Quoted in *The Little Ice Age* by Jean M. Grove

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2. In the Alps and in Norway, tunnels have been drilled underneath glaciers to capture water to feed hydroelectric plants. Site selection depends on a knowledge of subglacial water flow. Is the water dispersed widely over the bed or concentrated in a few large channels?

3. The sudden drainage of glacier-dammed lakes and of water stored within glaciers – and mudflows associated with these events – has caused extensive damage in Iceland, Peru, and several other countries. Prediction of such hazards becomes increasingly important as glaciers respond to a warming climate, because many of the ice dams that currently impound lakes are thinning, and new lakes are forming.

We must also know how water flows at the glacier bed in order to understand two of the major problems in glaciology: the mechanisms by which glaciers slip over their substrates (Chapter 7) and the causes and mechanics of glacier surges (Chapter 12). Results from glacier hydraulics, which deals with the downward flow of water through its own solid, may also have application to the upward flow of melted rock through the solid rock of the Earth's mantle and crust.

In this chapter we first review the main features of the hydrological system on, within, and beneath glaciers. We then discuss the characteristics of glacier runoff, the subject most relevant to studies of water resources. This is followed by an extended discussion of glacier hydraulics, in particular the mechanics of different types of drainage systems. Finally, we briefly examine glacier floods and subglacial lakes, two of the extraordinary aspects of water in glaciers.

6.1.1 Permeability of Glacier Ice

In temperate glacier ice, the boundaries of the ice grains enclose a network of narrow water lenses and veins (Nye and Frank 1973). The diameters of these voids are controlled by salinity and *capillary effects:* changes of pressure and temperature related to curvature of the void walls (Lliboutry 1996). Tube-like voids with diameters of a few millimeters have also been observed (Raymond and Harrison 1975). Water may percolate through the ice along such features but the quantity must be very small; meltwater is often trapped in depressions on glacier surfaces to form ponds and lakes that persist for months. Furthermore, the flow of water through the ice is insufficient to flush it clean of impurities; precise measurements of temperatures show depressed values reflecting the presence of solutes (Section 9.4.1). In a study of Blue Glacier, Washington State, Raymond and Harrison estimated the flux of water draining through the ice to be about $0.1\,\mathrm{m^3\,yr^{-1}}$ per square meter. This is trivial compared to the quantity of water moving through glaciers in larger voids like fractures and pipes. For most purposes the ice between such features can be regarded as impermeable. The effective permeability of a glacier thus depends on the sizes, spacing, and connectivity of fractures and other large voids. The upper horizons in the accumulation zone, however, are permeable firn.

6.1.2 Effective Pressure

Effective pressure, usually symbolized N, refers to the difference between water pressure P_w and pressure in the surrounding ice P_i:

$$N = P_i - P_w. \tag{6.1}$$

This quantity appears frequently in analyses of glacier hydraulics.

6.2 Features of the Hydrologic System

In Sections 6.2.1 through 6.2.4 our discussion follows the water from the glacier surface, through the ice, along the bed, and into the foreland. Figure 6.1 provides a schematic view of the system.

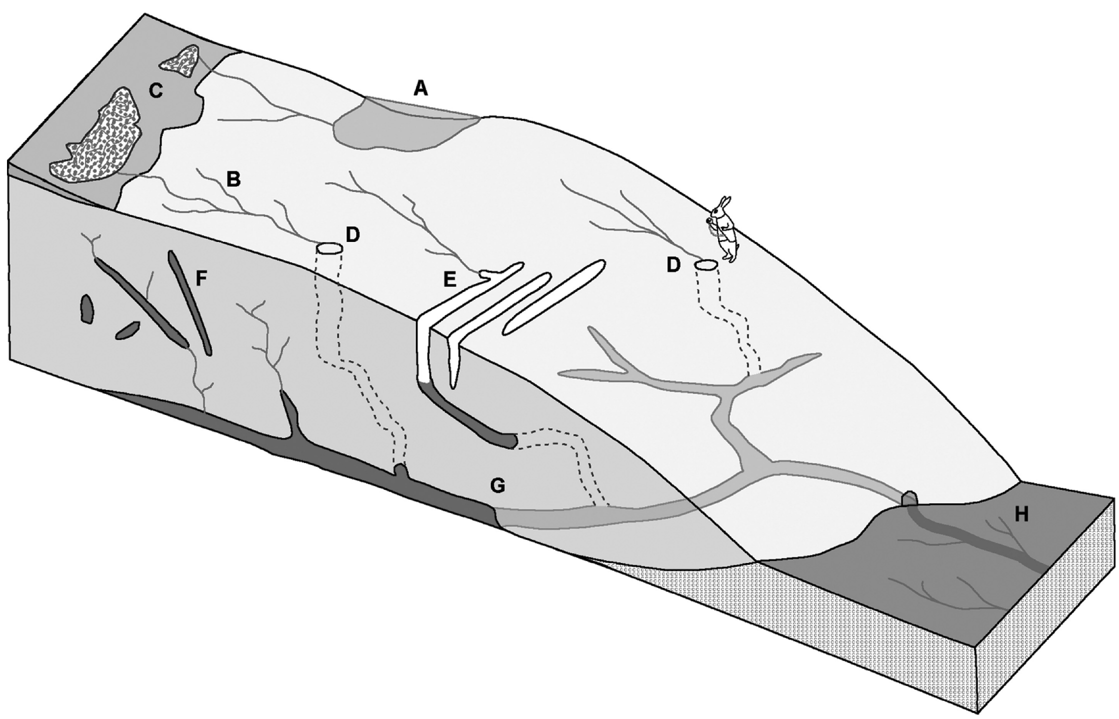

Figure 6.1: Some elements of the glacier water system: (A) Supraglacial lake. (B) Surface streams. (C) Swamp zones near the edge of the firn. (D) Moulins, draining into subglacial tunnels (for scale, white rabbit is about 10 m tall). (E) Crevasses receiving water. (F) Water-filled fractures. (G) Subglacial tunnels, which coalesce and emerge at the front. (H) Runoff in the glacier foreland, originating from tunnels and also from upwelling groundwater. Though not depicted here, water is also widely distributed on the bed in cavities, films, and sediment layers. Sediment and bedrock beneath the glacier contain groundwater. (Refer to the insert for a color version of this figure)

6.2.1 Surface (Supraglacial) Hydrology

In summer, meltwater flows on the ice surface of the ablation zone in networks of channels resembling an ordinary river system. The channels are smooth-walled and often sinuous, and convey water at speeds of a few meters per second. Some of this water flows off the sides and terminus, but most of it disappears into the glacier through cracks and vertical passageways called *moulins*. Some water returns to the surface in artesian upwellings.

Where firn mantles the surface, water percolates downward and refreezes (early in the season) or ponds on the impermeable ice or on dense firn. Runoff begins when a saturated layer forms, a process well studied in terrestrial watersheds (e.g., Colbeck 1972; Colbeck and Anderson 1982; Gray and Male 1981). In contrast to flow in the channel networks on bare ice, drainage along firn- and snow-covered regions is inefficient. Trapped water forms a saturated layer, whose upper surface defines a water table. Fountain (1989), for example, identified a water table in the firn on South Cascade Glacier (Washington State) in late summer; it was typically within about 1 m of the surface. Schommer (1976) measured summertime water levels in boreholes from 12 to 25 m below the surface of Aletschgletscher, Switzerland. He also observed that the water drained from the boreholes at the end of summer, suggesting that firn does not store significant amounts of water during the winter.

Thin firn often saturates all the way to the surface, forming a "swamp zone" with abundant puddles of standing water. As the melt season progresses, the swamp zone moves up-glacier ahead of the boundary between firn and ice. Overall, the surface drains more efficiently as more ice is exposed and the remaining firn fills with water.

Meltwater lakes are sometimes abundant in regions of gentle surface slope. Many such lakes form each summer in the ablation zone of the southern Greenland Ice Sheet; Figure 6.2 shows an example. Because they are darker than the surrounding snow and ice, these lakes increase the total absorption of solar energy by the surface, enhancing further melt. The largest reported lake was about $9\,\mathrm{km}^2$ in area, and the greatest depth was about 12 m (Box and Ski 2007). Some of these lakes drain rapidly; Box and Ski reported flood volumes as great as $31\times10^6\,\mathrm{m}^3$ in one day, while Das et al. (2008) observed a $44\times10^6\,\mathrm{m}^3$ lake drain in about 17 hours. Much of the latter event occurred rapidly in one 90-minute period with an average discharge rate of $8700\,\mathrm{m}^3\,\mathrm{s}^{-1}$ – a larger flow than the Niagara Falls. Once a lake begins to drain, melt can rapidly enlarge the passageways conveying water, whether surface channels or conduits and fractures in the ice. Such enlargement, in turn, increases the rate of drainage. We should therefore expect some ice-bound lakes to drain rapidly, leading to floods in the glacier foreland or on the bed beneath. Whether rapid drainage occurs or not depends on how effectively the rate of melt increases when drainage begins – cold water and small lake volumes both inhibit melting and can allow stable drainage (Raymond and Nolan 2000).

Figure 6.2: A meltwater lake on the surface of the Greenland Ice Sheet, 30 km from the western margin, August 2005. The lake's diameter is 1.4 km on its long axis; the volume is about 30×10^6 m^3 (Box and Ski 2007). (Refer to the insert for a color version of this figure)

6.2.2 Englacial Hydrology

Fractures produced by tension (Section 10.9), including open crevasses, provide pathways for surface water to penetrate into a glacier. Water entering crevasses may be stored in them or may drain into fracture networks or channels deeper in the glacier. Water also enters a glacier in deep vertical shafts known as *moulins*. Many surface streams flow into moulins and cascade out of sight, a visual and auditory spectacle akin to waterfalls. Melt maintains moulins and allows them to grow; as water descends through a glacier, frictional dissipation converts potential energy to heat, which melts the ice walls. Moulins initially form where surface streams intersect crevasses, and where fractures occur beneath lakes. Dye and other tracers put into moulins usually flow quickly through the glacier, especially in mid- to late summer, indicating a connection to a well-developed plumbing system (e.g., Lang et al. 1979; Nienow et al. 1998).

Inspection of boreholes using video cameras shows that numerous water pockets, fractures, and channels exist within temperate glaciers. Fountain et al. (2005) saw several dozen englacial fractures in Storglaciären, a 250-m-thick mountain glacier in Sweden. Many of these openings were elongate and appeared to be continuous across adjacent boreholes; such voids resemble subhorizontal fractures rather than conduits. Moving bubbles indicated water flow through many of them. Others showed no sign of water flow and were interpreted as blind fractures. Fountain et al. concluded that fracture networks, possibly originating as surface crevasses, could account for much of the water storage and transport within temperate glaciers. It is likely that fractures and widely spaced channels connected to moulins both carry significant flows of water.

Polythermal Glaciers Observations show that ice flow speeds up in summertime in the ablation zones of some polar glaciers, including the Greenland Ice Sheet (Battle 1951; Müller and Iken 1973; Zwally et al. 2002; Copland et al. 2003b; Joughin et al. 2008; van de Wal et al. 2008; see our discussion in Chapter 11). Except near tidewater margins – where the balance of forces fluctuates seasonally – the only plausible explanation is that surface water penetrates the glacier in moulins or fractures and then spreads out over the bed, facilitating basal slip (Iken 1972). In western Greenland, these events correspond not only to periods of rapid melt but also to drainage of large lakes on the glacier surface.

Water-filled crevasses penetrate to the bottom of a glacier if a strong supply of meltwater maintains pressures high enough to prevent closure by ice flow. In polar glaciers, the water flow and associated heat dissipation must also be strong enough to prevent closure by freezing. Observations demonstrate that surface water can indeed penetrate through a great thickness of subfreezing ice, as much as 1 km in the case of the Greenland Ice Sheet. The large lake-drainage event observed by Das et al. (2008) provides a good example from Greenland. The lake drained slowly at first, probably through tensional fractures located along one shore. Slow drainage continued for about 16 hours, during which time fractures were presumably growing downward. Then the lake level dropped dramatically, at up to 12 m hr^{-1}. The entire lake drained in about 90 minutes, while the ice sheet surface simultaneously rose and shifted sideways; clearly, the fractures had reached the bed and water was rapidly escaping along the interface, some 980 meters below the surface. In the weeks following this event, new surface melt continued to flow into the glacier through the moulins formed during lake drainage.

Studies on John Evans Glacier, Ellesmere Island, have outlined a seasonal pattern of fracture penetration (Boon and Sharp 2003). This glacier is about 15 km long, and its thickness typically ranges between 100 and 200 m in the lower 4 km, the region of interest. Internal temperatures are well below freezing. Radar reflections indicate a thawed bed in the central ablation zone, but a subfreezing bed in the accumulation zone and beneath the thin ice of the margins and terminus (Copland and Sharp 2001; Copland et al. 2003b). The mean annual temperature at the equilibrium line is only about −15 °C, but the surface melts extensively in summer. Surface water forms a mosaic of disconnected lakes and streams early in the melt season. No water escapes to the glacier foreland at this time. After a few weeks, lakes on the glacier expand and grow to a depth of several meters in a flat region about 4 km from the terminus. Lakes partially drain into new moulins in sporadic events, presumably when englacial passageways extend downward but do not yet connect to the bed. Some or all of the water entering the glacier refreezes, releasing latent heat and warming the surrounding ice. Eventually, through a combination of warming and accumulation of surface melt, the passageways connect through to the glacier bed, the surface lakes fully drain, and the glacier releases water to its foreland.

6.2.3 Subglacial Hydrology

Observing water flow at the glacier bed is difficult. Most information derives from borehole measurements of a few parameters like pressure, conductivity, and turbidity, and from measurements of water and impurities in streams emerging at the glacier front. In a few places, tunnels built for hydroelectric projects access glacier beds. The paucity of observational constraints has not hindered the development of elaborate theories, which we discuss later in this chapter. The theories yield valuable insights but should be viewed with a skeptical eye.

Several major aspects of subglacial hydrology have been learned:

1. Diverse passageways store and convey water at the glacier bed. There are ice-walled conduits; there are channels incised into rock or sediment but roofed with ice; there is water distributed widely but irregularly in thin films, small channels, and meter-scale cavities; there is water in the pore spaces of sediments and bedrock; and there are giant subglacial lakes. Channels cut upward into the ice are referred to as *Röthlisberger-* or *R-channels*. Channels incised in bedrock are called *Nye-* or *N-channels*. Visitors to glaciers commonly see R-channels where they emerge at the glacier terminus (Figure 6.3).

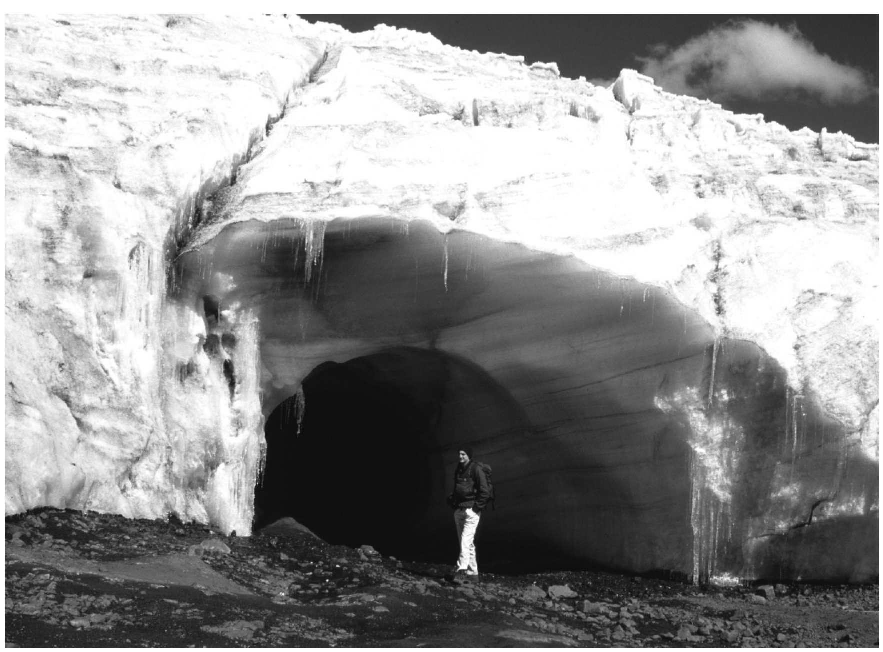

Figure 6.3: A tunnel (R-channel) emerging at the terminus of Pastaruri, Peru. It formed during drainage of a lake. Photo courtesy of M. Hambrey. (Refer to the insert for a color version of this figure)

2. Water supply competes with drainage; understanding the competition is key to understanding variations of the water system. Primary water sources are rain and melt from the surface, melt along the bed and within the ice, and, in mountain landscapes, runoff from valley sides. Annual melt of the surface typically amounts to a few meters, whereas annual basal melt from geothermal and frictional heat typically yields one centimeter. Surface melt provides by far the largest source for many glaciers, but is generally weak in the upper parts of accumulation zones, and is entirely absent from most of Antarctica and the center of Greenland. When drainage cannot remove all the water arriving at the bed, the volume of stored water increases and water pressures build up. The water pressure partly counteracts the weight of the glacier, and so reduces forces of contact between the ice and underlying rock.

3. Most drainage occurs along the interface between the ice and its substrate. Such drainge is fundamentally difficult, however, because of the great weight of the overlying ice. (We refer to the weight per unit area as the *overburden pressure*.) Ice tends to flow into voids and close them. For drainage to occur, basal water pressures must therefore usually rise to nearly match the overburden pressure; otherwise passageways cannot be open enough, and pressure gradients strong enough, to accommodate and drive the drainage. For example, basal water pressures measured in boreholes through West Antarctic ice streams are within 0.2 MPa of the approximately 9 MPa ice-overburden pressure – and, at many sites, within 0.05 MPa (Kamb 2001; Engelhardt et al. 1990). No surface melting occurs in this region, and basal water originates entirely from melt at the bed.

4. In one common situation, however, drainage does not require high water pressures. Where abundant surface water reaches the bed, concentrated flows melt ice, enlarging passageways and creating a plumbing system of efficient pipes. The pressure in such features is significantly less than the overburden and can even temporarily fall to the atmospheric value if the water source abruptly shuts off and conduits drain.

5. The basal water system affects a glacier's basal slip motion, and vice versa. Water on the bed lubricates the interface and facilitates slip. Lubrication arises from high values of water pressure and from separation of the ice from its substrate. Both increase sliding at the interface. High water pressures also reduce the shear strength of basal sediments, a prerequisite for their deformation. But sliding over an irregular rock bed also opens cavities that store and convey water, and bed deformation may close channels incised into sediments. Thus the connections go both ways; hydrology influences slip but slip influences hydrology. Such feedbacks remain poorly understood but play an important role in phenomena like glacier surges and seasonal variations of glacier flow.

6. In many places, the basal water system changes significantly as a function of time. Such variations partly arise from the feedbacks noted in the previous paragraph; surges, for example, are episodic phenomena involving coupled changes of basal drainage and glacier motion. The most prevalent source of variability, however, is the fluctuating water supply

from surface melt and rain. Beneath glaciers with abundant summertime surface melt, water pressures and fluxes vary greatly in daily cycles and over seasons. Thus, there must exist a system of large passageways that provides a free hydraulic connection between the glacier surface and the bed (Mathews 1964; Nienow et al. 1998). Mathews (1964) was the first to obtain a time series of basal water pressures. He measured pressures at the end of a shaft that reached the base of South Leduc Glacier, Canada, from a mine underneath the glacier. His record showed periods of stable water pressure interrupted by large and rapid rises associated with rapid snow melt or heavy rain.

Marked spatial inhomogeneity at the scale of meters also characterizes the basal water system. Water pressure at one location on the bed often differs greatly from pressure only a few tens of meters away (Murray and Clarke 1995; Fudge et al. 2008). The corresponding variations over time differ in magnitude, phase, and sign. For large differences of water pressure to prevail over short distances, parts of the glacier bed must conduct water poorly.

Figure 6.4 shows the variations in basal water pressure measured in three boreholes through Trapridge Glacier, Canada (Murray and Clarke 1995). Basal water pressure displays large variability over spatial scales of several meters and time scales of hours to days. In some boreholes, said to be *connected* to the glacier drainage system, water pressures vary greatly in direct correspondence to the daily influx of meltwater from the surface. Other boreholes are *unconnected* or isolated; these tap into a patch of the bed that responds, not directly to the diurnal influx, but to elastic deformations of the ice forced by pressure variations in the connected regions nearby. In some cases the unconnected regions are so isolated that their pressures stay nearly constant.

Measurements at other glaciers have shown that diurnal pressure fluctuations originate in conduits but also propagate laterally for tens of meters through permeable sediments (Hubbard et al. 1995).

Figure 6.4 also illustrates that water pressures frequently exceed the local overburden pressure, a situation referred to as *super-flotation conditions* or *excess water pressures*. Such conditions have long been known from glacier drilling programs; water sometimes jets out of boreholes when they connect to the basal system.

The efficiency of drainage through conduits increases greatly over the melt season. Figure 6.5 shows the changes over one month in early summer of mean velocities and transit times for water entering a single moulin and emerging at the terminus of Haut Glacier d'Arolla in Switzerland (Nienow et al. 1998). Over time, the large water fluxes from surface melt increased the size and connectivity of conduits. Moreover, the onset of such efficient drainage migrated up-glacier following the retreating snowline. Exposure of bare ice increased melt rates and eliminated water storage in snow; both factors increased the flow into moulins and, until the conduit system developed fully, increased water pressures within and beneath the glacier (Gordon et al. 1998).

Available data show that average basal water pressures are higher in winter than in summer, an observation first made by Mathews (1964). The seasonal variation is thus opposite

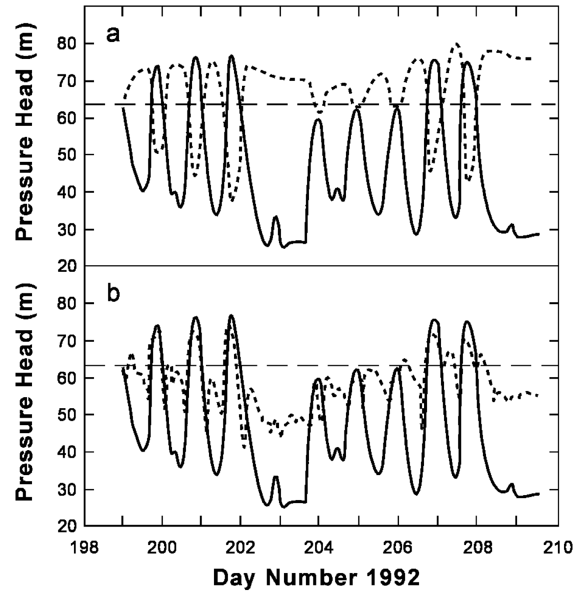

Figure 6.4: Day-to-day variation of basal water pressure measured in three boreholes, Trapridge Glacier. The solid curve in both panels shows the same borehole, repeated for comparison. Pressure variations in different boreholes may be correlated, anticorrelated, or uncorrelated. Horizontal dashed lines indicate overburden pressure. Adapted from Murray and Clarke (1995).

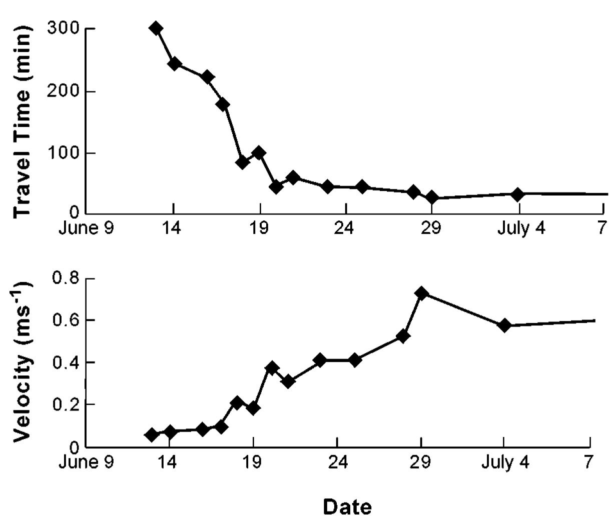

Figure 6.5: Evolution of mean velocity and travel time for water entering a single moulin and appearing at the glacier front, Haut Glacier d'Arolla. Data acquired by tracer injection. Adapted from Nienow et al. (1998).

to the diurnal one, with high pressures in periods of minimal surface melt. High pressures in winter must arise from restricted drainage.

7. Very little is known about the mechanics of water drainage along beds of sediment. A deforming sediment passively transports water in its pore spaces. Most of the water flow may concentrate in broad channels incised in the sediment, referred to as *canals*, or in a *macroporous horizon* of permeable layers at the top of the sediment. Ordinary R-channels should form if the sediment does not deform rapidly or erode easily.

The volume of water present at the glacier bed obeys a conservation relation. Define the volume per unit area as H_w, the average thickness of the water layer. H_w changes over time if the outflow of water from a given region of the bed (Q_{out}, volume per unit time) does not keep pace with the inflow (Q_{in}). For a region of bed with area $\mathcal{A}$, conservation requires that $\mathcal{A}\, dH_w/dt = Q_{in} - Q_{out}$. The inflow includes water from the surface, which arrives at the bed in conduits and fractures, and water draining along the bed from up-glacier. One of the major goals of glacier hydrology is to learn, in essence, how Q_{in} and Q_{out} relate to the size and shape of passageways along the bed, and to dynamic variables like the water pressure and hydraulic gradients. In this chapter, Sections 6.3.2 through 6.3.4 examine this problem. Generally, the fluxes of water increase with the slope and pressure gradients that drive flow. Water flow also increases with the efficiency of the drainage system. A system of large, connected tunnels conveys water much more readily than a system of cavities and thin layers dispersed widely on the bed. As we will see, this difference of efficiency greatly affects water conveyance along the bed.

The formal statement of water-volume conservation needed for models of the basal water system is

$$\frac{\partial H_w}{\partial t} = \dot{S}_w - \nabla \cdot \vec{q}_w, \tag{6.2}$$

where $\dot{S}_w$ denotes the source flux (water arriving at the bed, volume per unit time per unit area), and $\vec{q}_w$ the water flux along the bed (volume per unit time per unit length). Two separate mechanisms lead to accumulation of water at the bed: prolific sources of water (e.g., surface melt on warm days) and downstream decreases of drainage (e.g., a constriction in the drainage system that ponds water upstream).

6.2.4 Runoff from Glaciers

Water typically emerges at the glacier terminus in a few large streams; this indicates that the water coalesces into master channels while flowing along the bed. The pattern of flow in these streams provides information about the plumbing inside the glacier (e.g., Meier and Tangborn 1961; Mathews 1963; Elliston 1973; Nienow et al. 1998). Moreover, the stream flow – the runoff – is an important water resource for inhabitants and ecosystems downstream.

6.2.4.1 Observations

Figure 6.6 shows typical records for fine summer weather. The discharge (volume flowing in unit time) has a marked diurnal variation superimposed on a *base flow* whose volume changes more slowly. The maximum discharge may be roughly twice the minimum; less for large glaciers, more for small ones. The peak discharge comes a few hours after the peak in melt but, as summer advances, the daily rise and fall in discharge becomes more rapid and the time lag decreases. For example, at Mittivakkat Glacier in southeast Greenland the lag between peak melting and discharge decreases from 5 to 7 hours in May to 3 to 4 hours in August (Mernild et al. 2006).

In the northern hemisphere, total daily discharge usually reaches its maximum in late July or early August, when warm weather and extensive exposures of ice lead to the strongest ablation on the glacier surface (Chapter 5). When summer snowfalls shut down melting, the diurnal variation in discharge ceases and the base flow declines (Figure 6.7). When melting begins again, the base flow takes several days to reach its former level. Water flows throughout the winter in many places, but with no diurnal variation and greatly reduced base flow. Base flow generally consists of meltwater from snow and firn, water that travels slowly through the glacier, and water released from temporary storage. Groundwater and runoff from slopes above the glacier may

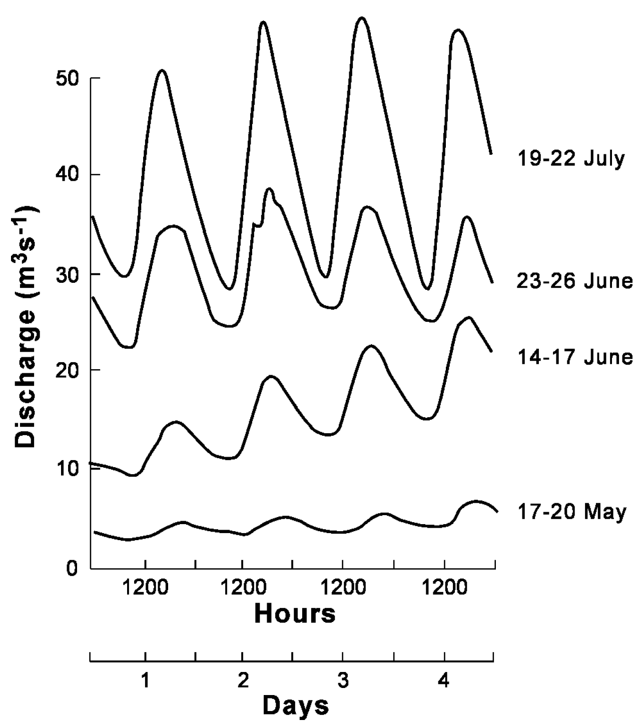

Figure 6.6: Discharge from a glacierized basin near Zermatt, Switzerland, in four periods of fine weather in summer 1959 (dates indicated beside each curve). Adapted from Elliston (1973).

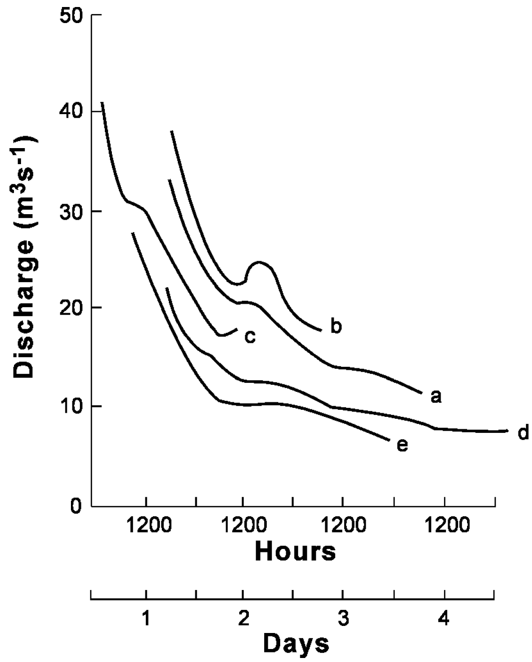

Figure 6.7: Discharge from the same basin as in Figure 6.6 after summer snowfalls. (a) 29 June–2 July 1959, (b) 30 July–1 Aug. 1959, (c) 26–28 June 1960, (d) 4–8 Sept. 1960, (e) 17–19 Sept. 1960. Redrawn from Elliston (1973). Used with permission from the International Association of Hydrological Sciences, Elliston, G.R., Water movement through the Gornergletscher, IAHS Press, International Association of Scientific Hydrology Publications, vol. 95, Fig. 2/pp. 79–84.

also contribute. The diurnal cycles largely involve meltwater originating in the ablation area and draining rapidly to the bed through moulins.

Although the diurnal cycle is strong (Figure 6.6), the runoff should not be regarded as a simple reflection of the melt production. Some water is stored within the glacier and later released. Water may take a considerable time to travel from its site of origin to the glacier terminus. From an analysis of discharge curves, Elliston (1973) inferred that at least half the meltwater spent at least one day within Gornergletscher. Lang (1973) found the lag needed to give maximum correlation between daily discharge and air temperature. He concluded that water spent, on average, two to three days in Aletschgletscher in early summer but only one day in August and September. These analyses give mean residence times for the whole drainage basin; they do not tell where most of the delay occurs. This can be determined by observing when dye, spread on the surface of the accumulation zone and thrown down moulins in the ablation zone, appears in the stream at the terminus. Ambach et al. (1974) and Behrens et al. (1976) did this on Hintereisferner. Dye from the upper part of the accumulation zone first appeared at the terminus 10 days later, with the maximum concentration after 17 days. In a similar experiment shortly before bad weather the dye was never detected. The travel time from the lower part of the accumulation zone, in contrast, was only about 20 hours, whereas times from the ablation zone varied from 0.5 to 3 hours. Thus the major delay was in the snow and firn; in the ice, water moved at rates similar to those for open channels. The reduction in the time difference between the daily peaks in discharge and melt as the season progresses (Figure 6.6) results partly from development of the subglacial and englacial drainage systems (Nienow et al. 1998) and partly from the reduction in the thickness and extent of snow cover (Willis et al. 2002).

Seasonal Variation and Storage Of the total annual runoff from mid-latitude glaciers, the vast majority occurs in the summer melt season. Østrem (1973) estimated that 85% of the runoff from Scandinavian glaciers occurs from June to August. Escher-Vetter and Reinwarth (1994) reported that 90% of the runoff from Vernagtferner, Austria, occurs from June to October. Figure 6.8 shows the runoff pattern over a melt season from a glacier in the Canadian Rockies – the same glacier whose energy budget we used as an example in Chapter 5 (Shea et al. 2005). The foreland stream flows freely by late June and shuts down in mid-September. Runoff reaches a maximum in late summer when melt production is most rapid. It diminishes when temperatures temporarily fall below freezing, and declines rapidly at the end of the melt season. One brief but intense storm precipitated 11 mm of rain in one hour on August 19 (day 232 in the figure). This generated a runoff spike during the following hour, with a maximum discharge more than twice the size of any of the other daily maxima.

Storage and release of water are important factors in the seasonal runoff pattern. Stenborg (1970) studied these processes in Mikkaglaciären. He compared the total discharge from the

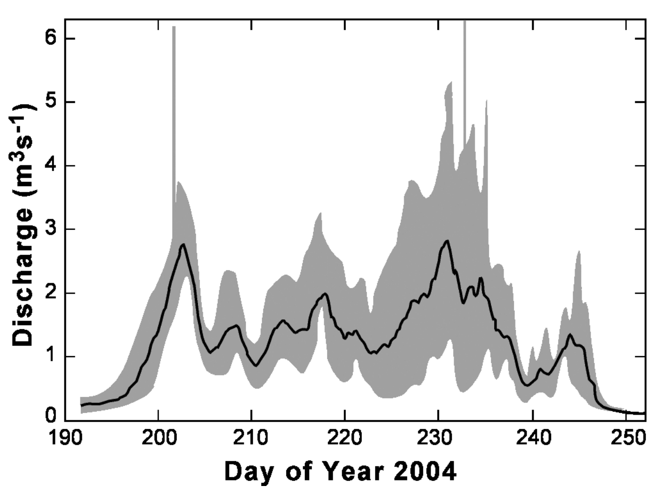

Figure 6.8: Variation of runoff from Haig Glacier (Canadian Rockies) over an entire summer season. The solid curve shows the center-weighted running mean over two days. The gray band shows the range of daily fluctuations. Two brief, exceptionally large discharge events occurred. The first peaked at 6.1 m^3 s^{-1}, the second at 12.2 m^3 s^{-1}. Adapted from Shea et al. (2005). Data courtesy of Shawn Marshall.

stream at the terminus with the melt estimated from meteorological parameters using regression equations. Until mid-July, melt exceeded run-off; the surplus was as much as 25% of the total summer discharge. From mid-July to early August, run-off exceeded melting, as the glacier released water stored earlier in the summer. Again, Tangborn et al. (1975) found that summer run-off from the basin of South Cascade Glacier exceeded the measured ablation by 38%. Storage occurs at different times on different glaciers, but generally corresponds to the early part of the melt season – late spring to mid-summer (Jansson et al. 2003). Although much of the water is stored in firn and slush, some is contained in cavities in the ice or at the bed. Sudden floods sometimes seen in streamflow records provide further evidence for release from cavities. Mathews (1963) identified such events in the runoff from Athabasca Glacier, Canada. The floods interrupted the normal diurnal cycle and appeared to be unrelated to weather conditions. In some cases discharge took one or two days to attain a maximum, and remained above average for up to a week. Records covering thirteen summers showed ten floods exceeding $25 \times 10^4\,\mathrm{m}^3$ and numerous smaller ones. There are no ice-dammed lakes around Athabasca Glacier so the water must have been stored within the ice. Walder and Driedger (1995) analyzed the occurrence of similar floods from South Tahoma Glacier, on Mount Rainier. They concluded that, although the water is stored within the glacier, the probability of a flood increases in periods of abundant rain and surface melt.

In the tropical Andes and the Himalayas, the seasonal cycle of runoff from glaciers differs from the mid-latitude case shown in Figure 6.8 (Willis 2005, p. 8). Alternation of wet and dry seasons, rather than warm and cool ones, dominate the seasonal cycle. High precipitation and runoff occur in the wet season. During the dry season, ablation continues to feed streamflow. The total amount is reduced but the portion originating as glacier melt increases (Mark and Seltzer 2003).

6.2.4.2 Relation to Internal Water System

In combination, observations of runoff suggest that during the melt season the glacier acts like a reservoir that is drained continuously by the streams emerging at its terminus and refilled daily by meltwater. The glacier contains a considerable amount of free water. While much of the water is in snow and firn, the ice also contains appreciable quantities in crevasses, moulins, and cavities. Some of these are isolated, at least temporarily; others are connected to the main drainage system. As long as the channels and cavities contain enough water, ice flow cannot close them. They close during the winter and are reopened by meltwater at the beginning of the following summer. This can happen in a few weeks or even days. The glacier bed seems to become flooded, fresh crevasses open, old moulins are reactivated, and new ones form. Glacier streams carry heavy loads of sediment at this time. During the initial period, some water is stored because the channels are too small to carry it all away. The channel system develops further during the summer as increasing amounts of meltwater enlarge old channels and open new ones. Thus the time that water spends in the glacier and the lag between maximum melt and maximum runoff are reduced. Initially isolated cavities connect to the main drainage system so that, for a time, runoff exceeds melting. Floods seen in streamflow records probably result from the draining of a series of previously isolated cavities; for example, when the barriers between a series of water-filled crevasses are breached successively.

6.2.4.3 Linear Reservoir Models

Picturing a glacier as a reservoir leads to the "linear reservoir model" approach for characterizing glacier runoff, a technique used in water resource analyses. Suppose that a glacier contains a volume of water V_w. For conservation of mass,

$$\frac{dV_w}{dt} = S_w - Q_w, \tag{6.3}$$

where the source term S_w denotes the flux into the system from melt and precipitation, and Q_w signifies the runoff (here both S_w and Q_w are volume per unit time). The key assumption of the linear reservoir approach is that the glacier releases water at a rate proportional to the stored amount:

$$Q_w = k_r\, V_w = V_w/\tau_r. \tag{6.4}$$

Here k_r defines the *reservoir storage constant* and its inverse τ_r is the *residence time*. Substitution of Eq. 6.4 into Eq. 6.3 gives the linear equation

$$\frac{dQ_w}{dt} + \frac{Q_w}{\tau_r} = \frac{S_w}{\tau_r}. \tag{6.5}$$

The solutions of this equation are broadly consistent with the observations discussed previously. A cyclic input S_w generates a cyclic variation of runoff, with peak values delayed relative to the time of maximum input. After the input shuts off, the runoff declines exponentially with time. Equation 6.4 does not allow for sudden releases of stored water and so the approach cannot account for all aspects of observed runoff.

Glaciers are often represented as a series of parallel snow, firn, and ice reservoirs, with distinct residence times τ_s, τ_f, and τ_i (Baker et al. 1982; Hock and Noetzli 1997). The runoff Q_w then equals the sum of the runoffs predicted from three separate equations like Eq. 6.5, one for each reservoir. Table 6.1 lists example residence times from studies that calibrated the parameters using hourly and daily runoff observations. In each case, runoff was measured at gauging stations within several kilometers of the glacier terminus and includes some flow from nonglacial sources.

Observations of diurnal variability, discussed in Section 6.2.4.1, demonstrate that the pathway taken by water through a glacier changes over the course of a melt season. A single set of reservoir parameters therefore should not apply at all times. For example, consider the small cirque glacier in the French Pyrenees studied by Hannah and Gurnell (2001). The runoff here was best simulated by using only two parallel reservoirs: "slow" and "fast" systems. The appropriate values for τ_r decreased over the melt season, from 45 to 18 hours for the slow system, and from

Table 6.1: Storage constants (mean residence times) adopted in applications of linear reservoir models in glacierized catchments.

Catchment	Glacierized area (km^2)	τ_f (hr)	τ_s (hr)	τ_i (hr)	Reference
Vernagtferner, Austria	9	350	30	16	Baker et al. (1982)
Storglaciären, Sweden	3.1	430	30	4	Hock and Noetzli (1997)
Various catchments, Switzerland		350	120	40	Verbunt et al. (2003)
Rhône, Switzerland	20.3		125	113	Schaefli et al. (2005)
Lonza, Switzerland	28.4		96	41	Schaefli et al. (2005)
Drance, Switzerland	70.1		142	110	Schaefli et al. (2005)
Hofsjökull, Iceland	880	400	60	20	de Woul et al. (2006)

13 to 5 hours for the fast one. The relative contribution from the "slow" system declined over time and only one "fast" system was active late in the melt season.

At Black Rapids Glacier, a 20-km-long valley glacier in Alaska, 10% to 20% of the summer runoff travels through a "fast" system that responds to diurnal inputs (Raymond et al. 1995). The remainder moves through a "slow" system that varies little over weeks. Raymond et al. found that the slow-system waters contained high concentrates of solutes, indicating a long time of contact with the bed. The fast and slow systems here, then, do not reflect the difference between ice and firn, but rather the difference between surface melt from the lower ablation zone and waters moving slowly along the glacier bed.

A crude representation of the water system underlies the linear reservoir approach. Differences between the surface, englacial, and subglacial water systems are not represented. There is no accounting for exchange of water between the reservoirs. The primary assumption, Eq. 6.4, is sometimes invalid. Nonetheless, comparisons of simulated and measured runoffs show that the strategy provides a useful phenomenological description, especially for small glaciers. For water resource applications, predicting the source term – the ablation and rain – is generally a more important challenge than correcting the crude representation of drainage.

A considerably more thorough method for calculating runoff accounts for flow through different parts of a glacier, using numerical models for each part and the connections between them (Section 6.3.6.1). No runoff model accounts, however, for the occasional floods of internally stored water, which occur randomly and defy prediction (Walder and Driedger 1995).

6.2.4.4 Water Resource Contribution to a Drainage Basin

Glaciers can contribute significantly to the runoff from a drainage basin, but their role is often falsely characterized; enhanced runoff caused by glacier retreat must be distinguished from the "normal" runoff related to the annual hydrologic cycle. Each year, a glacier releases a total

Table 6.2: Comparison of annual precipitation with variations of mass balance.

Glacier	Precipitation ($m\,yr^{-1}$)	Standard deviation of average balance ($m\,yr^{-1}$)	Ratio of std. dev. to precip.
Nigardsbreen	3.8	1.06	0.3
Hintereisferner	2.3	0.54	0.2
Peyto	2.2	0.55	0.3
Storglaciären	1.5	0.75	0.5
White	0.4	0.26	0.7

Data from Oerlemans (2001), pp. 54 and 100.

runoff, Q_g, equivalent to

$$Q_g = A_g\left[P_g - E_g\right] - B_n\,, \tag{6.6}$$

where A_g denotes glacier area, and P_g and E_g are the mean precipitation and evaporation rates on the glacier. The annual change of ice volume, B_n, depends on the accumulation minus ablation, summed over the glacier (Section 4.2.4; B_n is the annual glacier balance). In Eq. 6.6, changes of stored water within the glacier are assumed negligible for a year, as are groundwater fluxes.

If a glacier is equilibrated to the climate, the through-flow of water, $A_g\left[P_g - E_g\right]$, normally contributes more to the runoff than do yearly volume changes. Table 6.2 lists estimated annual precipitation rates for several glaciers and compares them to typical annual volume changes per area of glacier. The latter are given by the standard deviation of the glacier-average specific mass balance, the thickness of ice removed or added to the whole glacier in a year (Chapter 4).

Thus, in a temperate region, in a typical year, the change of ice volume appears to modify the runoff by about 30% or less. Averaged over several years, the contribution would be smaller. The tabulated numbers also suggest that ice-volume changes cause a larger fraction of runoff variability in dry climates than wet ones.

If the glacier disappeared entirely, its basin would still release a runoff $A[P - E]$. Whether this represents an increase or decrease depends on how precipitation and evaporation change. Precipitation might increase with climate warming, but the establishment of vegetation can significantly increase E, which, in general, includes transpiration as well as direct evaporation.

Regardless, three factors give glaciers a critical role in runoff generation:

1. During periods of sustained retreat, the shrinking ice volume ($B_n < 0$) feeds a significant flow of water to the river system. During advance, the rivers are deprived.

2. During years of drought, melt continues to supply the rivers with water; B_n often turns strongly negative in years of low precipitation. Thus glaciers buffer the water supply against drought.

3. Over the seasonal cycle, the glacier gains mass in winter and loses mass in summer. Seasonal storage and release of surface water occur in all landscapes where snowpacks accumulate, but only in glacial landscapes does the release persist for the duration of the warm season.

In a typical alpine watershed, glaciers occupy headwater basins and high summits but much of the landscape is free of permanent ice. The annual runoff from the watershed has both glacial and nonglacial components. The fraction of runoff originating from glaciers typically decreases downstream as the fraction of total basin area covered by ice decreases. The glacial component is nonetheless significant in some populated regions, especially in the Himalayas and the Andes; for example, glaciers contribute 12% of the annual runoff in the 5000 km^2 Rio Santa watershed in the Cordillera Blanca, Peru (Mark and Seltzer 2003). But in many regions, even where glaciers are a vivid element of the landscape, the glaciers contribute little to the annual total. On the eastern slopes of the Canadian Rockies, for example, glacier melt contributes only 2% of the annual flow in the Bow River at Banff (Hopkinson and Young 1998). The situation changes drastically in periods of drought, however. Glacier inputs to the Bow River can exceed 50% in late summer of a dry year.

How might the glacial contribution to runoff change when climate warms? Consider two hypothetical climate scenarios (Figure 6.9): an abrupt warming and a progressive warming.

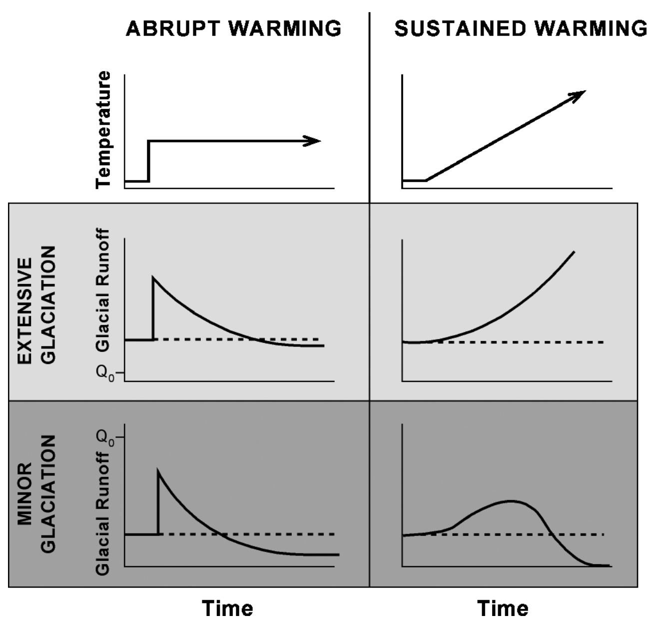

Figure 6.9: Schematic diagram showing plausible variations of glacial runoff in two climate warming scenarios, on two landscapes. "Extensive glaciation" means that glaciers cover a large fraction of the land surface and a wide range of altitudes. "Minor glaciation" means that glaciers are present only in high basins. The value Q_o indicates the relative scale of the vertical axes on the discharge plots.

Suppose that each acts on two landscapes, one with extensive glaciers covering a large area and large altitude range, the other with a small glacial cover confined to the highest basins and slopes. On the former, abrupt warming leads to a pulse of runoff production followed by reequilibration to a slightly lower value; the glacial cover has shrunk, but ablation zones have merely shifted up-slope. Under progressive warming, equilibrium lines move continuously up-slope, and the remnants of glacier tongues at low elevations rapidly ablate. On the landscape with a small glacial cover, progressive warming initially increases the glacial runoff, but runoff subsequently declines to zero as the glaciers expire. This pattern – initial increase followed by decline – is predicted for glacier runoff in the Alps over the next century as the climate warms (Braun et al. 2000).

6.3 The Water System within Temperate Glaciers

The following sections discuss theoretical analyses that provide a more detailed view of some elements of the water system introduced in Section 6.2.

6.3.1 Direction of Flow

Water flows from high to low elevations and also toward regions of reduced pressure. Thus water flows not strictly "downhill" but down the gradient of a *hydraulic potential* (Domenico and Schwartz 1998),

$$\phi_h = P_{\mathrm{w}} + \rho_{\mathrm{w}}\, g\, z, \tag{6.7}$$

with P_{w} the water pressure and z the elevation. ϕ_h equals the potential energy per unit volume of water. (Dividing ϕ_h by $\rho_{\mathrm{w}}\, g$ gives a quantity with units of length, called the *hydraulic head*.) The force per unit volume driving flow, the *potential gradient*, is

$$-\nabla\phi_h = -\nabla P_{\mathrm{w}} - \rho_{\mathrm{w}} g \nabla z, \tag{6.8}$$

and the direction of this vector gives the general direction of flow. Shreve (1972) applied this relation to analyze water movement in glaciers; much of the following discussion uses his analysis.

6.3.1.1 Englacial Flow

Strictly, ϕ_h is defined only within conduits containing water, but for convenience we regard it as defined throughout the glacier. As a first approximation, assume that the water pressure everywhere equals the pressure of the overlying ice. (Later we show this is not true; conduits open and close because these pressures are unequal.) Thus

$$P_{\mathrm{w}} = \rho_i\, g\, [\mathrm{S} - z], \tag{6.9}$$

where ρ_i is the density of ice and S the elevation of the glacier surface. It follows that

$$\phi_h = \rho_i\, g\, \mathrm{S} + [\rho_w - \rho_i]\, g\, z. \tag{6.10}$$

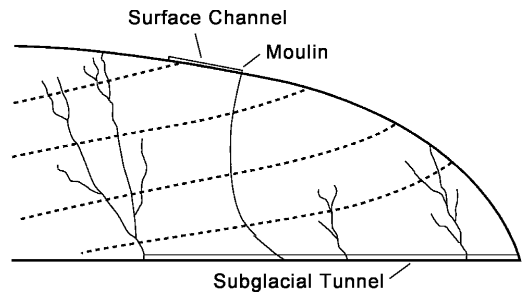

Figure 6.10: Schematic diagram of equipotential surfaces (broken lines) and related theoretical drainage pattern in a glacier. Small channels within the ice should tend to be perpendicular to the equipotential surfaces, but the inclination of a moulin is determined by that of the crevasse from which it formed. Water also flows along fractures of various orientations, in whichever direction the potential decreases. (See Figure 6.1 for a more complete depiction of features.)

Equation 6.8 shows that water flows in the direction perpendicular to the surfaces defined by constant ϕ_h. On such a surface, the gradient of ϕ_h is zero. The elevation of the equipotential surfaces z_ϕ thus varies along the glacier (direction x) as

$$\frac{dz_\phi}{dx} = \left[\frac{-\rho_i}{\rho_w - \rho_i}\right]\frac{d\mathrm{S}}{dx} \approx -11\,\frac{d\mathrm{S}}{dx}; \tag{6.11}$$

the equipotential surfaces slope about eleven times more steeply than the surface and in the opposite direction. Figure 6.10 depicts the drainage system predicted by this analysis. However, we also expect that locally much of the flow will be diverted along conductive paths – like fractures and pipes – whose orientations do not match the theoretical potential gradient. In this case, water still flows in the direction of decreasing ϕ_h, but water pressures are perturbed in the vicinity of the conductive paths.

One type of conductive path, a moulin, forms where water flows into a crevasse. The inclinations of moulins and crevasses therefore ought to be the same when they first form. Crevasses meet the surface at a right angle, but if they penetrate to deep layers undergoing shear, they tilt; newly formed crevasses should penetrate basal layers at a down-slope angle of roughly 45° and then rotate toward the vertical due to ice deformation. The orientation of moulins may change over time as their walls melt differentially, a likely process because water runs down one side of a moulin when it is not completely filled. Holmlund (1988) found that even at a depth of 60 m moulins in Storglaciären did not trend perpendicular to the equipotential surfaces. Some moulins in the Greenland Ice Sheet remain vertical to at least 90 m depth (Peter 1996).

6.3.1.2 Flow Along the Glacier Bed

The following discussion applies whether the overlying ice is temperate or not. The gradient $-\nabla\phi_h$ again determines the direction of flow, but now $z = \mathrm{B}$, the elevation of the bed. From

Eqs. 6.8 and 6.10,

$$-\nabla\phi_h = -\rho_i\, g\, [\nabla \mathrm{S} + 0.09\, \nabla \mathrm{B}]\,, \tag{6.12}$$

where the coefficient 0.09 is an approximate value for $\rho_w/\rho_i - 1$. The surface slope thus has a dominant role steering flow along the bed.

Basal waters also flow in the across-glacier direction. In the accumulation zone of a valley glacier, the surface is usually higher at the sides than at the center and so water flows toward the centerline. In the ablation zone, the convex surface profile impels water toward the sides. However, the steep slopes of the underlying bedrock valley walls probably permit water to drain toward the center. Thus, in either zone, drainage may concentrate along the centerline of the bed. The dependence of $\nabla\phi_h$ on bed slope implies that subglacial water tends to follow valley floors and cross-divides at the lowest point, even while moving in a general down-glacier direction given by the surface slope. *Eskers*, winding ridges of sand and gravel deposited by subglacial streams, are usually found in such places (Shreve 1985).

According to this analysis, water running along the bed must accumulate upstream of places where the bed slopes at more than 11 times the surface slope and in the opposite direction; this situation reverses the hydraulic gradient on the bed. The build-up of pressure might then force water out of passageways and into a sheet or into englacial conduits. Perhaps water passes an overdeepened section in this way. Water can also accumulate anywhere on the glacier bed if the capacity of the water system to conduct flow decreases downstream. In this case, however, the water does not stagnate.

In Eq. 6.12, the factor 0.09 indicates that the bed slope must be larger than the surface slope by a factor of 11 to have an equal influence steering the water flow. Thus water should often flow uphill at the bed of a glacier. Indeed the paths of eskers and eroded channels on formerly glaciated terrain demonstrate uphill flow. The value 0.09 only applies, however, with the assumption that water pressure equals the ice overburden; if, in effect, the ice is floating. Beneath valley glaciers with daily cycles of surface melt, this is probably a poor approximation much of the time. Lower pressures often occur in the conduits conveying the meltwater. Define the "flotation fraction" f_w as the ratio of water pressure to overburden pressure, P_w/P_i. Then Eq. 6.12 is

$$-\nabla\phi_h = -\rho_i\, g \left[f_w\, \nabla \mathrm{S} + \left[\frac{\rho_w}{\rho_i} - f_w \right] \nabla \mathrm{B} \right]. \tag{6.13}$$

If $f_w = 0.8$, the bed slope needs to be only 2.7 times greater than surface slope to have an equal influence steering the water. With $f_w = 0.56$, the flow direction responds equally to bed and surface slopes.

Equations 6.12 and 6.13 provide a useful indication of the overall direction of water flow at the bed, but not the detailed pattern. Water tends to flow in channels determined by the local bedrock topography, rather than melt out new ones in the ice. In addition, the theory applies only to conduits full of water. The pressure in partially filled channels is atmospheric, and the assumption that water pressure varies in proportion to ice pressure, already a crude approximation, is then invalid. On the other hand, conduits beneath ice sheets and long glaciers are likely to remain water-filled.

Björnsson (1982) used Eq. 6.12 to define the water drainage basins underneath three outlet glaciers of Vatnajökull, a major ice cap in Iceland. The magnitude and direction of surface slope (∇S) were measured from conventional maps, and ∇B from bedrock maps based on radar soundings. The vectors of water flow could then be plotted and the water drainage divides mapped. For comparison, the ice-drainage basins were delimited from surface contours (Section 8.2.1). For one of the glaciers, the water-drainage basin coincided with the ice-drainage basin. For the other two glaciers, the streams emerging at the termini drained water from only about half of their corresponding ice-drainage basins; the bed topography partially diverted basal waters to adjacent outlet glaciers.

6.3.2 Drainage in Conduits

Drainage in conduits was originally analyzed by Röthlisberger (1972) and Shreve (1972). Subsequent work of Nye (1976), Spring and Hutter (1981, 1982), and Clarke (1982, 2003) extended the model to describe channels with varying flows, to include some physical refinements and to analyze glacier outburst floods.

The analysis refers to flow in conduits large compared with intergranular veins and tubes. Although the conduits are initially regarded as being within the glacier, most of the analysis also applies to channels in ice at the bed.

Two opposing effects determine the size of a conduit:

1. Water flowing in a conduit enlarges it by melting ice from the walls. Viscous dissipation in the water and friction of water against the walls produce the necessary heat. In addition, water from the surface or subglacial sources may be warmer than 0 °C.

2. If the pressure of the overlying ice exceeds the water pressure, ice flows in and reduces the diameter of the conduit. The diameter decreases at a rate proportional to the third power of the difference between the ice and water pressures (Nye 1953). The power 3 is the exponent on stress in the creep relation for ice, discussed in Chapter 3.

By these processes, the capacity of the system adjusts, though not instantaneously, to changes in the strength of water sources such as surface melt.

The objective of the following analysis is to derive an expression for the water pressure in a conduit, assumed to be full, in terms of the discharge Q_w, the volume of water flowing in the conduit in unit time at a particular location. The water pressure is no longer assumed to be equal to the ice-overburden pressure.

6.3.2.1 Fundamental Mechanics

This analysis was largely developed by Röthlisberger (1972). For convenience, in the following discussions we use the symbol G instead of $|\nabla\phi_h|$ for the magnitude of the hydraulic gradient.

From Eq. 6.8, the force per unit volume driving water flow in a conduit is

$$G = \frac{dP_{\mathrm{w}}}{ds} + \rho_w\, g\, \sin\theta\,. \tag{6.14}$$

Here s denotes distance along the conduit, measured up-glacier from the exit (opposite to the water flow), and θ denotes the inclination of the conduit to the horizontal, positive when descending downstream. As before, P_{w} signifies water pressure.

Consider a straight, circular channel with radius R_c and cross-sectional area $\mathcal{A}_c$ (Figure 6.11). The analysis must account for three processes:

1. The tunnel contracts because the ice flows. Nye (1953) showed that a cylindrical hole, of circular cross-section, contracts under an effective pressure N at rate

$$\frac{1}{R_c}\frac{\partial R_c}{\partial t} = A\left[\frac{N}{n}\right]^n \quad \text{with} \quad N = P_i - P_{\mathrm{w}}. \tag{6.15}$$

A and n are the creep parameters for ice (Eq. 3.35), and P_i is the ice overburden pressure, usually greater than P_{w}.

2. The work done by gravity and by the water pressure gradient produces heat in the water. The amount produced in a length ds of the conduit in unit time is $Q_w G$. Instantaneous heat transfer from water to ice is assumed. However, the temperature of the ice, which is always at melting point, changes along the conduit because the melting point depends on pressure. Thus some of the heat is needed to warm the water to keep it at the same temperature as the ice. In a distance ds following the water flow, the pressure in the surrounding ice drops by an amount dP_i, and the ice temperature increases by an amount $\mathcal{B}\, dP_i$. Here $\mathcal{B}$ denotes the change of melting point of ice for unit change of hydrostatic pressure (Section 9.4.1). To warm the water by this amount requires a heat $\rho_w c_w Q_w \mathcal{B}\, dP_i/ds$, for a specific heat capacity of water c_w. The walls of the conduit melt at rate $\dot{M}$ (mass per unit length of wall in unit time). With a latent heat of fusion of L_f, the heat used for melting is simply $\dot{M} L_f$.

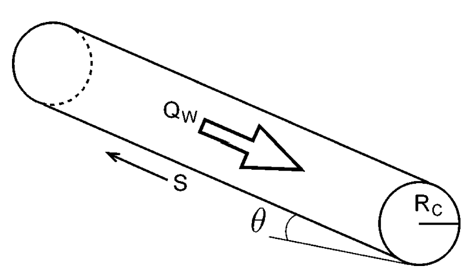

Figure 6.11: Variables for analysis of water flow in a tunnel surrounded by ice.

Heat balance thus requires that

$$\dot{M}\,L_f = Q_w\,G - \rho_w\,c_w\,Q_w\,\mathcal{B}\,\frac{dP_i}{ds}. \tag{6.16}$$

Surface meltwater is assumed to enter the glacier at 0 °C; otherwise it carries heat into the ice and another term must be added.

3. The size of the tunnel, an unknown quantity, can be related to Q_w and G using empirical formulae from hydraulics. The most commonly used expression, the Manning formula, gives the mean velocity of turbulent flow in a pipe as:

$$v_w = \frac{Q_w}{\mathcal{A}_c} = \frac{1}{[\rho_w\,g]^{1/2}\,n_{\mathrm{m}}}\,R_h^{2/3}\,G^{1/2}. \tag{6.17}$$

Here R_h is the hydraulic radius (cross-sectional area $\mathcal{A}_c$ divided by the perimeter) and n_{m} denotes the *Manning roughness coefficient*, a parameter with values in the range 10^{-2} to 10^{-1} s m$^{-1/3}$. For a circular pipe $R_h = R_c/2$ and the equation can be written

$$G = [4\pi]^{2/3}\,\rho_w\,g\,n_{\mathrm{m}}^2\,Q_w^2\,\mathcal{A}_c^{-8/3}. \tag{6.18}$$

6.3.2.2 Steady-state Tunnels

At steady state, ice melted from the walls balances the inflow of ice given by Eq. 6.15. Thus

$$\dot{M} = 2\pi\,\rho_i\,R_c\,\frac{\partial R_c}{\partial t} = 2\,\rho_i\,\mathcal{A}_c\,A\left[\frac{P_i - P_{\mathrm{w}}}{n}\right]^n. \tag{6.19}$$

To combine the equations, a relation is needed between the gradients of water and ice pressures, dP_{w}/ds and dP_i/ds. Given that water pressure is usually less than ice pressure, the value of dP_{w}/ds averaged over the glacier bed should also be less than the average of dP_i/ds; both pressures fall to atmospheric at the terminus. Therefore we write

$$\frac{dP_{\mathrm{w}}}{ds} = f_w\,\frac{dP_i}{ds}, \tag{6.20}$$

where f_w, the flotation fraction, nominally ranges from zero to one. If the water pressure equals the ice overburden, $f_w = 1$, the case considered originally by Röthlisberger. With this definition, and using values $\rho_w = 10^3\,\mathrm{kg\,m^{-3}}$, $c_w = 4.22\,\mathrm{kJ\,kg^{-1}\,K^{-1}}$, and $\mathcal{B} = 7.42\times10^{-8}\,\mathrm{K\,Pa^{-1}}$, Eq. 6.16 reduces to the relation

$$\dot{M}\,L_f = Q_w\left[\rho_w\,g\,\sin\theta + [1-\gamma]\,\frac{dP_{\mathrm{w}}}{ds}\right], \tag{6.21}$$

where $\gamma = 0.313/f_w$ (dimensionless). The value of $\mathcal{B}$ for pure water has been used; for air-saturated water, the value should be about 30% larger. In addition, we assume for now that $f_w = 1$.

Finally, elimination of the unknowns $\dot{M}$ and $\mathcal{A}_c$ from Eqs. 6.18, 6.19, and 6.21 gives a differential equation relating water pressure P_{w} to flux Q_w:

$$G^{11/8} - \gamma\, G^{3/8}\, \frac{dP_{\mathrm{w}}}{ds} = \rho_i\, [\rho_w g]^{3/8}\, K_1\, L_f\, n_{\mathrm{m}}^{3/4}\, A\, [P_i - P_{\mathrm{w}}]^3\, Q_w^{-1/4}, \tag{6.22}$$

where $K_1 = [2/27]\,[4\pi]^{1/4} = 0.139$ and a value $n = 3$ has been used. This is the fundamental equation of steady-state tunnel theory. Note that G depends on dP_{w}/ds and $\sin\theta$ according to Eq. 6.14.

This equation has to be integrated numerically. An alternative approach is to simplify it by considering end members (Fowler 1987a). At the base of a glacier with ice thickness H, $P_i = \rho_i g H$. Moreover, $P_{\mathrm{w}} \leq P_i$, except in local patches. At a distance X from the terminus, the order of magnitude of dP_{w}/ds is thus $\rho_i g H / X$. With typical values for a mountain glacier's ablation zone of $H = 200$ m, $X = 5$ km, then $dP_{\mathrm{w}}/ds \approx 400$ Pa m^{-1}. In one end member, the bed slopes steeply down toward the terminus. For example, with slope $\theta = 10°$, then $\rho_w g \sin\theta \approx 1700$ Pa m^{-1}. This considerably exceeds the pressure gradient, and so to a rough approximation $G \approx \rho_w g \sin\theta$. This is equivalent to assuming the water pressure stays constant along an isolated tunnel and only changes where tributaries join and contribute water. Substituting this approximation for G in Eq. 6.22 and rearranging gives the relation

$$P_i - P_{\mathrm{w}} = K_2\, \frac{G^{11/24}\, Q_w^{1/12}}{n_{\mathrm{m}}^{1/4}\, A^{1/3}} \tag{6.23}$$

$$\text{with} \qquad K_2 = \left[\left[\rho_i L_f K_1\right]^{1/3} [\rho_w g]^{1/8} \right]^{-1}. \tag{6.24}$$

The water pressure in the conduit is less than the pressure in the nearby ice; Eq. 6.23 gives the magnitude of the under-pressure, $P_i - P_{\mathrm{w}}$, which is also the effective pressure (Section 6.1.2). In the other end member, appropriate for the lower regions of large glaciers, the bed elevation varies little. In this case $G = dP_{\mathrm{w}}/ds$. Repeating the substitution gives an equation identical to Eq. 6.23 but with a different value for K_2; the relation of the effective pressure to G, Q_w, A, and n_{m} does not change.

According to the theory, effective pressure increases weakly with discharge and increases as roughly the square root of the driving force. Figure 6.12 shows how some of the characteristics of the tunnel flow vary as a function of discharge. The most important conclusion is that, in a tunnel system, an increase in flux *reduces* the steady-state water pressure, although the dependence is not a sensitive one. As an example, consider the values of water pressure measured by Mathews (1964) in South Leduc Glacier, Canada. Here, the mean value of $P_i - P_{\mathrm{w}}$ was higher in summer than winter by a factor of 1.32. This implies a ratio of summer to winter flux of about 30. Such a large difference seems reasonable because the summer flux results from surface ablation, but the winter flux mainly from basal melting. Indeed, typical fluxes from Gornergletscher (Switzerland) are 10 m^3 s^{-1} in summer but only 0.1 m^3 s^{-1} in winter (Röthlisberger 1972).

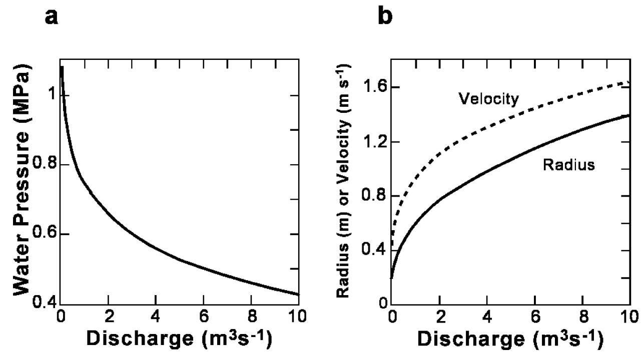

Figure 6.12: Predicted variation of water pressure, velocity, and radius for a tunnel in steady state, beneath 250 m of ice, with various discharges and one set of parameters ($\theta = 2.5°$, $A = 5 \times 10^{-24}$, $n_m = 0.1$).

The inverse relation between flux and pressure implies that in two tunnels side by side the water in the larger one has the lower pressure; it will tend to draw water away from the smaller. Thus water tends to collect into a few main arteries. Although these equations have been derived for a tunnel in the ice, they can also be applied to one at the glacier bed.

Despite these conceptual insights, the theory has limited value for calculating water pressures, because of uncertainties in the values of A and n_{m}, and because some of the assumptions fail. Calculations with best-guess values for the parameters often give negative values for P_{w}. To make it positive, the ice needs to be very soft; an A of five to ten times the generally accepted value needs to be used (Röthlisberger 1972). This is not necessarily wrong, because a water content of even 1% softens ice significantly, and basal ice layers are sometimes unusually soft (Chapter 3). On the other hand, the calculated negative pressures might reflect complexities not considered in the theory so far.

6.3.2.3 Complexities

Partial Filling of Tunnels Because the supply of meltwater fluctuates rapidly, it is possible that for much of the time water only partly fills tunnels, and the pressure is atmospheric (Lliboutry 1983a). Kohler (1995) investigated this possibility. He used tracers injected into moulins to measure average flow velocities v_w in the drainage system of Storglaciären, Sweden. He simultaneously measured water flux Q_w from the glacier. The data showed that v_w increased as a linear function of Q_w, an expected result if the cross-sectional area of the tunnels does not change with time (because $Q_w = v_w \mathcal{A}_c$). If the tunnels were only partly filled with water, the relationship would be nonlinear. Kohler therefore concluded that water completely filled much of the tunnel system.

Channel Shape The cross-section of a tunnel at the glacier bed is probably not semicircular. The bed should retard the inward flow of the ice immediately above it. If the tunnel is not full, as is likely near the terminus, water melts ice from the sides but not from the roof. For both reasons, the tunnel is likely to be broad and low, as tunnels observed at glacier termini often are. Hooke et al. (1990) used numerical finite-element models to examine the closure of low, broad channels by ice flow. Hock and Hooke (1993) then used modelled channel geometries to interpret measurements of discharge and water transit times at Storglaciären, where they had injected tracers into moulins to measure transit times. They concluded that the subglacial drainage most likely follows an arborescent network of broad, flat channels. Cutler (1998) extended this analysis further by focusing on changes over the melt season; he found evidence that subglacial channels change from a semicircular form early in the melt season to a flatter shape as the season advances. Evidence also suggested that the channels partly fill with air during extended cold periods and at night.

Channel Roughness Low, broad channels at the bed resist the water flow more effectively than do circular channels in ice (Hooke et al. 1990; Cutler 1998). The rock material of the bed can be very rough, and the hydraulic radius of a flat channel is smaller than for a circular channel of the same cross-sectional area. Fracture systems that feed tunnels, like those observed by Fountain et al. (2005), are narrow and discontinuous and so also hydraulically rough.

The empirical relationship between flow velocity and hydraulic gradient, Eq. 6.17, depends on the parameterization for roughness. There are two common options for this (Spring and Hutter 1981; Clarke et al. 1984b), but no good observational or theoretical basis for choosing between them. Clarke (2003) found that either one works in simulations of glacier floods.

Temporal and Spatial Variability The steady-state approximation may be reasonable for conduits draining large glaciers, but extreme variations of melt and water flux occur daily in most alpine glaciers (Figure 6.6). In early morning, rapidly draining conduits are only partly filled with water. Conduits receiving the largest fluxes from the surface fill rapidly; the water backs up, increasing the pressure and its gradient. Thus water pressure and flux increase and decrease together in short period variations, not inversely as expected for steady state (Clarke 2003).

Spatial variability matters too. Constrictions of flow cause the pressure upstream to increase; it can even exceed the overburden pressure. When this happens, the ice might fracture, allowing water to leak away, or the conduit might bifurcate. Sometimes the water reaches the surface and emerges as artesian upwellings (for examples, see Stenborg 1968, and Copland et al. 2003b).

Water pressure can differ greatly between neighboring conduits. Fudge et al. (2008) measured water levels in a cluster of 16 boreholes drilled in a 60 m by 60 m region of Bench Glacier, Alaska. Mean water levels differed by as much as 100 m, equivalent to 1 MPa of pressure, even though all the boreholes showed diurnal fluctuations indicating connection to the conduit system (Figure 6.13 illustrates a period in mid-July). Moreover, as the figure shows, groups of boreholes behaved the same way, suggesting that each group was responding to conditions in a unique conduit. The most likely explanation for these observations, given the locations of the boreholes, was a system of pipes that all trend down-glacier but remain isolated from each other – and from

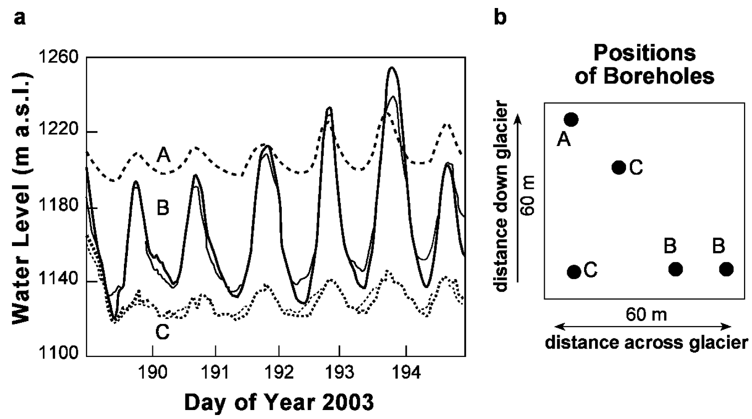

Figure 6.13: (a) Day-to-day variation of water level (a proxy for subglacial water pressure) in three sets of boreholes in Bench Glacier, Alaska. (b) A map showing the relative positions of boreholes belonging to each group. Adapted from Fudge et al. (2008).

large low-pressure conduits – for perhaps hundreds of meters. At other glaciers, dye-tracing experiments have suggested networks of parallel conduits that span much of the ablation zone (Sharp et al. 1993).

Weertman (1972) discussed a mechanism by which the water in a conduit might remain isolated from nearby conduits and from water distributed on the bed nearby. Because water pressure in a conduit is less than the ice-overburden pressure, some of the weight of the overlying glacier must be taken up by increased pressure between ice and bed on either side of the conduit. Such zones of enhanced pressures act like dams, preventing water from leaking out of the conduit or draining into it. On the other hand, a rough or porous substrate should lead to considerable spatial variability of the pressures at submeter scales, so the dams must leak. With a permeable bed, water pressure variations in a conduit should therefore propagate into surrounding regions (Hubbard et al. 1995).

Accretion by Super-cooling According to Eq. 6.21, under some conditions water freezes to the conduit walls instead of driving melt, a process referred to as *accretion by super-cooling* (Section 4.5.1). Specifically, accretion occurs if the right-hand side of Eq. 6.21 is negative, or $\rho_w g \sin\theta < -[1-\gamma]\, dP_\mathrm{w}/ds$. (Recall that $\theta > 0$ for a conduit that descends down-glacier; $dP_\mathrm{w}/ds > 0$ if pressure increases up-glacier; the factor $\gamma = 0.313/f_w$; and f_w indicates the ratio of the water-pressure gradient to the ice-pressure gradient.) If the water pressure gradient is much smaller than that of ice-overburden pressure (small f_w), accretion should occur in many situations. With $f_w < 0.313$, for example, more heat is used to warm water to melting point than is generated by the work of the pressure gradient, and accretion occurs anywhere the conduit rises down-glacier ($\theta < 0$).

Even with water pressures equal to the ice overburden ($f_w = 1$), accretion occurs if the conduit rises down-glacier at a sufficiently steep slope. Consider a conduit on the glacier bed (so $\theta = \beta$, the bed slope) in a region where the bed rises down-glacier ($\beta < 0$). Such a region is said to have an "adverse slope." For accretion,

$$\rho_w \, g \, |\beta| > [1-\gamma] \, \frac{dP_{\mathrm{w}}}{ds}. \tag{6.25}$$

But $P_{\mathrm{w}} = f_w \rho_i g H$, so Eq. 6.25 means

$$\rho_w \, g \, |\beta| > [1-\gamma] \, f_w \, \rho_i \, g \, [\alpha + |\beta|] \, , \tag{6.26}$$

where α denotes the surface slope, positive down-glacier. Rearranging then shows that ice accretes to the conduit walls only if the bed rises down-glacier with a slope

$$|\beta| > \left[\frac{\rho_w}{[f_w - 0.313] \, \rho_i} - 1 \right]^{-1} \alpha. \tag{6.27}$$

The coefficient multiplying α is 1.7 for $f_w = 1$ and 0.81 for $f_w = 0.8$.

Where accretion occurs, constriction of the conduit will inhibit water flow. Conditions favorable for rapid accretion might prevent tunnels from forming at all on certain regions of the bed.

Heat Advection and Reservoir Temperature So far the temperature of the water has been assumed to equal the temperature of the adjacent ice. In fact, exchange of heat between water and ice is not instantaneous, and water partly retains its original temperature if a strong flow moves it quickly along a conduit. Nye (1976) and Clarke (1982) added terms to the heat balance (Eq. 6.16) to account for transport of heat energy by the flowing water ("heat advection") and exchange of heat between water and ice of different temperatures.

For a length ds of a channel, advection supplies or removes energy at rate

$$E_{ad} = -\rho_w \, c_w \, \frac{\partial}{\partial s} \, [Q_w T_w] \cdot ds. \tag{6.28}$$

Water draining out the fronts of glaciers carries a small amount of frictionally produced heat that would otherwise be available to melt the channel walls. Hock and Hooke (1993) found, for example, that although water entering Storglaciären as melt should be at about 0.0 °C, it warms to about 0.07 °C as it passes through the glacier. This represents a loss of only about 1% of the heat generated by the net drop of hydraulic potential between the glacier surface and front.

More significantly, with a source water warmer than 0 °C heat advection adds energy to the drainage network in the glacier. Water in lakes on and beside glaciers can warm to a few degrees above zero. Flow of such waters into the glacier supplies a significant quantity of sensible heat. Likewise, subglacial lakes on volcanic terrain supply heat. Measurements in one subglacial lake in Iceland found water temperatures of nearly 5 °C (Jóhannesson et al. 2007). Clarke (1982) extended Nye's (1976) analysis of glacier floods to account for the effects of "warm" lake waters feeding conduits. The model simulates well a variety of modern and historical floods (Clarke 1982; Clarke et al. 1984b).

6.3.3 Drainage in Linked Cavities

As ice slides over a rough bed, cavities can form on the downstream sides of bumps. Examination of recently deglaciated bedrock reveals a pattern of chemical precipitates formed in subglacial water. Such patterns indicate that the cavities fill with water and link together through a network of narrow Nye channels in the bed (Walder and Hallet 1979). Water may also flow between cavities by way of conduits in the ice, or in a discontinuous water sheet or permeable sediment horizon at the interface.

Lliboutry (1969, p. 953) was the first to postulate the existence of such a linked-cavity system. Because the high normal stress in the ice on the upstream side of a bump should rapidly close conduits, the drainage system will largely avoid these regions. The result is a complex network of channels in which water flows in both across-glacier and down-glacier directions (Figure 6.14a). The following analysis of steady flow in such a system largely follows that of Walder (1986).

The main differences from tunnel flow are:

1. Cavities stay open not only because their walls melt but also because the ice slides.
2. Because cavity size depends on bedrock topography, which varies along the path, a model with uniform channel cross-section is inappropriate.

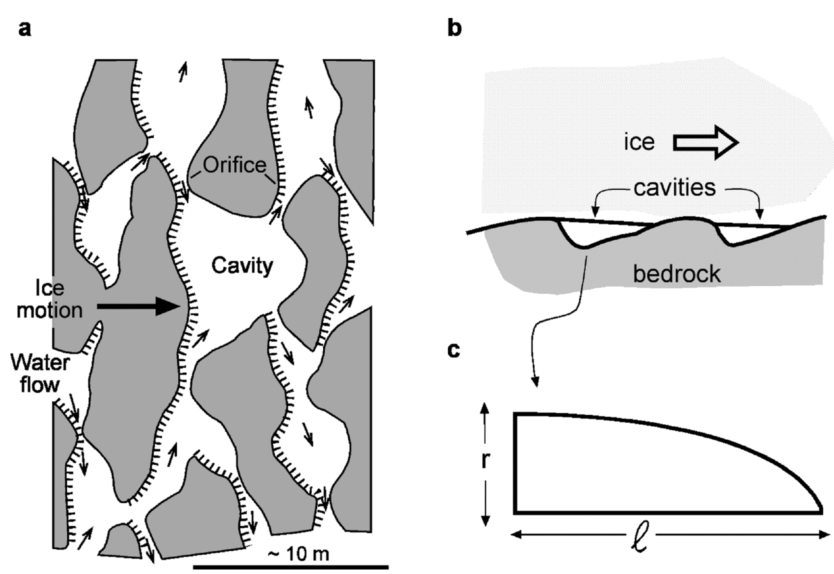

Figure 6.14: Schematic diagrams of linked cavities: (a) plan view, (b) cross-section. Panel (c) shows the idealization used in model formulation. Adapted from Fountain and Walder (1998) and Kamb (1987) and used with permission from the American Geophysical Union, *Journal of Geophysical Research*.

The following assumptions are made:

1. The bed is impermeable.
2. For simplicity, suppose that the bed is horizontal. Thus only the pressure gradient drives water flow: $G = dP_w/ds$.
3. Initially, assume a constant pressure gradient along the path.

Figure 6.14b shows the model of an idealized, water-filled ellipsoidal cavity formed in the lee of a vertical bedrock step of height r. The cavity's length ℓ is small compared with its transverse dimension, b. The glacier flows parallel to ℓ, while water flow can, in general, take any direction. In the cavity, the pressure gradient and water flow are regarded as oriented in the transverse direction (with length increment ds).

By analogy with a circular cylinder (hence Eq. 6.15) the closure rate, measured vertically, is approximately

$$w_c = A\,r\left[\frac{P_i - P_w}{n}\right]^n. \tag{6.29}$$

(Using ℓ instead of r in this relation ultimately gives the same qualitative conclusions.) This closure has to compensate for opening by both melting of the roof and sliding of ice over the top of the bump. Closure slows down as the water pressure rises. If the pressure approaches the overburden value, the closure rate will be too small for a stable cavity system to exist.

Let $\dot{M}$ denote the mass of ice melted from the roof in unit time per unit length in the direction of flow (kg s^{-1} m^{-1}). Melt thus removes a thickness per unit time from the cavity roof of $w_m = \dot{M}/[\rho_i\,P_r]$, where P_r measures the perimeter of the roof in a section perpendicular to the water flow direction. For an ellipsoidal cavity that is long and low, $P_r = k_1\ell$, where $k_1 \approx 1.1$ (Walder 1986).

As in conduits, the energy for melt originates as viscous dissipation of potential energy. Because water moves in the transverse direction, the ice pressure and temperature can be regarded as constant and so the factor γ in Eq. 6.16 is set to zero. Thus Eq. 6.16 indicates that

$$w_m = \frac{1}{\rho_i\,L_f\,k_1\,\ell}\,Q_w\,G. \tag{6.30}$$

The cavity roof moves downward with net velocity $w_c - w_m$. An ice particle in the roof will descend back to the bed in a time $r/(w_c - w_m)$. For a sliding velocity u_b, this time also equals ℓ/u_b. Hence the cavity length is

$$\ell = \frac{r\,u_b}{w_c - w_m}. \tag{6.31}$$

Fast sliding and rapid melt both increase the cavity size, while rapid closure by ice flow diminishes it. Assuming turbulent flow in the cavity system, the Manning formula (Eq. 6.17) again relates the water flow rate (v_w) and discharge to the pressure gradient. For cross-sectional area $\mathcal{A}_c$,

$$v_w = \frac{Q_w}{\mathcal{A}_c} = \frac{1}{[\rho_w\, g]^{1/2}\, n_\mathrm{m}}\, R_h^{2/3}\, G^{1/2}. \tag{6.32}$$

For the cavity shape shown in Figure 6.14b, $\mathcal{A}_c \approx \pi r \ell/4$ and $R_h \approx \mathcal{A}_c/2P_r = \pi r/8\,k_1$. Hence

$$Q_w = \frac{K_2}{[\rho_w g]^{1/2}}\,\frac{r^{5/3}\,\ell}{n_\mathrm{m}}\,G^{1/2} \qquad \text{with} \qquad K_2 \approx \frac{\pi^{5/3}}{16\,k_1{}^{2/3}} \approx 0.4\,. \tag{6.33}$$

Substitution for ℓ using Eqs. 6.29, 6.30, and 6.31, and rearranging gives an expression for the effective pressure, analogous to Eq. 6.23 for tunnels:

$$[P_i - P_\mathrm{w}]^n = \frac{K_3}{n_\mathrm{m}\,A}\left[r^{5/3}\,\frac{u_\mathrm{b}\,G^{1/2}}{Q_w} + K_4\,r^{2/3}\,G^{3/2}\right] \tag{6.34}$$

$$\text{with} \qquad K_3 = \frac{n^n\,K_2}{[\rho_w g]^{1/2}} \qquad \text{and} \qquad K_4 = \frac{1}{k_1\,\rho_i\,L_f}\,. \tag{6.35}$$

As with tunnels, the effective pressure $P_i - P_\mathrm{w}$ increases with the pressure gradient driving the water flow. In contrast to tunnels, however, an increased discharge is transmitted with an increased water pressure, which opens the cavity further. For two cavities side-by-side, the one carrying the higher discharge will have the higher water pressure and consequently will not tend to capture water from the other. The stable configuration of a linked cavity system, in contrast to that of a tunnel system, is therefore a large number of relatively small channels distributed over a large part of the glacier bed.

Equation 6.34 can be written, alternatively, as an expression for discharge:

$$Q_w = \frac{K_3\,r^{5/3}\,u_\mathrm{b}\,G^{1/2}}{n_\mathrm{m}\,A\,[P_i - P_\mathrm{w}]^n - K_3\,K_4\,r^{2/3}\,G^{3/2}}\,. \tag{6.36}$$

The discharge that a cavity system conveys generally increases with the pressure gradient and with the sliding velocity; fast sliding opens cavities more. Note, however, that if water pressure rises to near the ice pressure, the equation breaks down; Q_w rises to infinity. This is the situation, previously mentioned, in which a stable cavity system cannot exist; the cavities expand continuously because creep closure is too slow compared with melt. For $G = 10\,\mathrm{Pa\,m^{-1}}$, $r = 0.1\,\mathrm{m}$, $n = 3$, and a range of plausible values for A and n_m, the critical value of $P_i - P_\mathrm{w}$ is about 0.1–0.4 MPa. This compares with the estimate of Iken (1981, Eq. 2), based on a simple balance-of-forces argument and an idealized bed geometry, that unstable ice sliding may arise when the water pressure comes within 0.2 to 1 MPa of the overburden pressure. Iken's instability requires that the bed has a mean down-glacier slope; the quoted range of pressures applies for bed slopes between 5° and 25° and a basal shear stress of 0.1 MPa.

Because cavities are wide features that easily conduct water and because the discharge passing through a single cavity is expected to be small, the pressure gradient in cavities must usually be very small. Thus melting of the roof should typically play an insignificant role in keeping a cavity open; the closure from ice flow is balanced by opening due to sliding. Walder (1986) verified this for plausible ranges of values of G and Q_w. In general, cavities stay open largely because of sliding. The narrow passageways ("orifices") that connect them, however, are sustained by melt, because reducing the dimensions of a cavity increases the pressure gradient and the melting rate.

The analysis so far has concentrated on the cavities, although the orifices control flow through the system. The same equations should apply, although the heat used for warming water can no longer be neglected (the factor γ in Eq. 6.21 should be included in the analogous relation for melt in cavities; $1-\gamma$ replaces 1 as the numerator in Eq. 6.30). In a steady state, the flux must be the same in both a cavity and the orifice that drains it. Because Eq. 6.33 shows that, for constant Q_w, G increases in proportion to $\ell^{-2}\,r^{-10/3}$, pressure gradients in the orifices can easily be 100 or even 1000 times those in the cavities. The mean pressure gradient in the network is

$$\overline{G} = \frac{G_c\,s_c + G_o\,s_o}{s_c + s_o}, \tag{6.37}$$

where s denotes length measured along the direction of water flow, and suffixes c and o refer to cavities and orifices. If $s_c/s_o \approx 10$ and $G_c/G_o \approx 0.01$, $\overline{G} \approx 0.1G_o$. Because in a linked-cavity system water flows obliquely to the direction of ice flow, the path length may be 10 times that of a conduit in the same glacier. Thus $\overline{G} \approx 0.1G'$ where G' is the hydraulic gradient in a tunnel. It follows that gradients in the orifices are probably comparable with those in a tunnel, whereas gradients in the cavities are very much less. Again, the equality of the fluxes and the difference in cross-sectional areas implies that water moves perhaps 100 times faster in an orifice than in a cavity. Thus the orifices control the flow but the water spends most of its time in the cavities.

In reality, the distinction between cavities and orifices must be much less sharp than assumed in this model because bedrock bumps have a wide range of sizes. Kamb (1987) argued that cavities of height ~1 m and length ~10 m and orifices of height 0.1 m or less are the important ones.

The preceding analysis treated the sliding velocity as an independent variable. Observations show, however, that sliding can speed up when water pressures rise. A complete model should incorporate an inverse relationship between u_b and $P_i - P_w$. Kamb (1987) proposed a model accounting for this complexity. He assumed that u_b varies inversely as $[P_i - P_w]^3$, a relation that is itself highly uncertain and inconsistent with some observations (Section 7.2.6). Kamb found that, as in the preceding analysis, flux and water pressure increase together. Processes in an orifice were considered in detail. With a small sliding velocity, a small increase in flux or water pressure might lead to rapid enlargement of the orifice. In this way, a linked-cavity system might change into a tunnel system. Conversely, a tunnel system can switch to a system of linked cavities. Because Kamb found that the pressure in a linked-cavity system is much higher than in a tunnel system carrying the same flux, such a switch might explain surges (see Chapter 12). The beds of surging glaciers are probably deformable, however, so the relevance of the proposed mechanism is unclear.

Weertman (1969a, 1972, 1986), although agreeing that surface meltwater reaching the bed in tunnels will remain in them, has argued that water formed by melting of basal ice flows in a thin sheet. A sheet of water between two flat, parallel surfaces conveys a flux proportional to powers of the thickness of the sheet and the hydraulic gradient driving flow. Values for the exponents depend on whether the flow is laminar or turbulent. Weertman originally assumed a uniform sheet thickness but later considered a sheet that pinches out on the upstream side of a bump and enlarges on the downstream side. The distinction between this and a linked-cavity system is fuzzy.

Does water flow along the bed in a sheet or in channels? Walder (1982) argued that sheet flow is unstable. If part of the sheet were to thicken slightly, more water would flow along that route, concentrating heat generation. More ice would melt from the roof and the thickness would increase further. By this feedback, a sheet would organize itself into a pattern of channels. However, rapid sliding on a rough surface or downward creep of ice (Creyts and Schoof 2009), might disrupt this process. So might a deformable bed, the next situation we consider.

6.3.4 Subglacial Drainage on a Soft Bed

If the bed consists of a layer of deformable and permeable sediments such as glacial till (a "soft" bed), some subglacial water can escape through the pores. Groundwater emerges in seeps and springs in the forelands of many glaciers. At Trapridge Glacier (Yukon), for example, fluorescent tracers injected at the glacier surface appeared years later in winter ice deposits that formed in front of the terminus (Stone and Clarke 1996). Calculations with plausible values of sediment permeability and thickness suggest that porous flow could evacuate much of the meltwater produced locally by geothermal and frictional heat (Alley et al. 1986c). However, if basal meltwater originates from a large catchment area upstream, porous flow is inadequate to remove it. Nor can flow through the pore spaces remove surface meltwater that penetrates to the bed. Furthermore, if the sediment beneath the glacier deforms, it carries some water along with it. Such "advection" accommodates the water sources only with exceptionally high sediment deformation rates and a thick deforming layer (Alley et al. 1986c). Much of the water must therefore flow along the ice-sediment interface.

Does the water at the ice-sediment interface flow as an irregular sheet or in channels carved in the ice or the sediment? Again, feedback between melt, thickening of a sheet, and concentration of flow and heating might cause a sheet to separate into some sort of channel system (Walder 1982). In addition, the flow along the thick parts of a sheet would exert a greater shear stress on the sediment than would thin parts; the thick regions would erode the sediment preferentially. These mechanisms should produce a system of channels. On the other hand, creep and displacement of the sediment itself might prevent such channels from forming, whereas rapid sliding around large rocks protruding into the ice would form cavities.

A different possibility is suggested by data from numerous borehole experiments at Trapridge Glacier. Water is rapidly redistributed along the bed, yet subglacial channels have never been

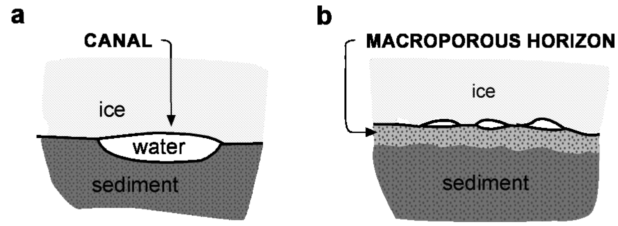

Figure 6.15: Schematic depiction of water system elements on beds of sediment.

identified at this site. Apparently, the water flows instead through a *macroporous horizon*, a thin, permeable, quasi-continuous sheet of water and sediment (Figure 6.15). This may be a combination of intergranular pore spaces in the uppermost layer of sediment, and thin films, cavities or larger gaps ("blisters") at the interface with the ice. Flowers and Clarke (2002a,b) concluded, by comparing observed water pressures to model calculations, that the water discharge (per unit width) along the bed obeys a relation analogous to Darcian flow of groundwater:

$$q_w = -\frac{k_h\, d_w}{\rho_w\, g}\,\frac{\partial \phi_h}{\partial s}\,, \tag{6.38}$$

for a hydraulic potential ϕ_h, effective hydraulic conductivity k_h, and thickness of horizon d_w. The conductivity itself varies, depending on how permeability increases as the sediment deforms and how gaps between the ice and substrate open under high water pressures.

The till beneath Trapridge Glacier is a coarse-grained sediment, typical of mountain glaciers. A macroporous horizon is less plausible for fine-grained sediments, like those beneath parts of the Pleistocene ice sheets and the modern West Antarctic ice streams. Here, and elsewhere, water might flow in channels. Because sediment is a complex material with poorly constrained deformation properties, theoretical analyses of channels in sediment are speculative. A channel cut into sediment should tend to close by creep of ice from the top and creep of sediment from the sides and bottom. Water counteracts closure by melting the ice roof and by flushing away fine-grained particles.

Walder and Fowler (1994) attempted a detailed analysis. For steady flow, these authors concluded that water could flow either in R-channels cut upward into the ice, as with a "hard" bed, or in a network of broad shallow channels, called *canals*, cut downward into the the sediment (Figure 6.15). For a canal, the approximate relationship between water flux and pressure was found to be

$$Q_w = B_c \sin^2\alpha\; d_c^3\; [P_i - P_{\mathrm{w}}]^{-n} \tag{6.39}$$

Here α is the ice-surface slope, d_c the depth of the canal, and n the exponent in the ice creep relation. B_c depends on the thermal and mechanical properties of ice and sediment as well as the roughness of the canal's floor. The slope α is a surrogate for the hydraulic gradient in the canal.

Equation 6.39 shows that the flux in a canal increases with water pressure. Thus the larger canals do not tend to draw water away from the smaller ones; individual canals remain separate. In this respect, canals behave like linked cavities on a hard bed rather than like R-channels.

The analysis showed that low surface slopes – typical of ice sheets or ice streams on deformable beds – favor canals over R-channels. Walder and Fowler discussed how the distribution of eskers in North America supports their picture of subglacial drainage. Eskers are abundant over the Canadian Shield, where the ice sheet rested mainly on bedrock, but rare in areas mantled with thick deposits of till, such as the prairies. Eskers form in large R-channels which, according to the analysis, form under an ice sheet only if its bed is hard.

For slopes typical of valley glaciers, on the other hand, both types of features might exist. Equation 6.39 indicates that, for a given flux, water pressures in a canal system are relatively high. If both canals and R-channels exist, the channels draw water away from the canals. Thus, according to the analysis, R-channels are the preferred system in valley glaciers, even on a soft bed.

Ng (2000) proposed an alternative model for steady-state canals. He used a more detailed description of how sediment erosion and deposition depend on the water flow. Conservation of sediment mass along the channel provided an additional constraint. As in the Walder and Fowler canals, the water flux was found to increase with the pressure. Ng found, in addition, an inverse relation between pressure and the sediment flux conveyed by the canal. Sediment enters the canals because it creeps inward and erodes. The difference between ice-overburden pressure and canal water pressure drives the creep of sediment (Alley 1992b). Therefore, the lower the pressure, the greater the supply of sediment.

The available data provide little constraint on theories of canals, or other sorts of channels on soft beds of fine sediment. A meters-thick layer of water-saturated sediment underlies the Siple Coast ice streams in West Antarctica (Section 8.9.2.4). Basal water pressures measured in boreholes through three of the ice streams were within 0.2 MPa of the approximately 9 MPa ice-overburden pressure – and, at many sites, within 0.05 MPa (Kamb 2001). Such high pressures are consistent with flow in films or canals, as postulated by theory; if R-channels exist, they are widely spaced. Kamb (2001) and Engelhardt and Kamb (1997) reported on numerous experiments conducted in these boreholes. The major finding was lack of consistency from site to site. At some places, the flux of water that could be pumped out of the subglacial system or exchanged between nearby boreholes indicated the presence of a gap along the ice-sediment interface, with an average thickness of a few millimeters. At other locations, such a gap could not exist because pressures differed between nearby boreholes by as much as 0.2 MPa. Such differences would be neutralized by flow along the interface if such a gap were present. In these regions. the authors concluded, a system of canals must exist to transport the meltwater from upstream. The most likely explanation for the diverse observations is a highly complex and inhomogeneous water system, with both films and channels.

Recent observations of this region in West Antarctica reveal that large volumes of ponded subglacial water move from one place to another, on timescales of a few years. Specifically, satellite altimetry measurements show that the ice sheet surface swells and subsides throughout regions of several kilometers' dimension. The regions are located on Whillans and Mercer Ice Streams and near the grounding zone of the Ross Ice Shelf (Gray et al. 2005; Fricker et al. 2007). The observed vertical displacements amounted to several meters over the years 2003 to 2006. The most plausible explanation is partial filling and draining of subglacial lakes. According to Fricker et al.'s estimates, one subglacial lake near the Engelhardt Ice Ridge (formerly Ridge B/C) drained an estimated 2 km^3 of water to the Ross Sea over a period of about three years. The basin subsequently started to refill. Roughly 1.6 km^3 of water was redistributed between other subglacial basins over the same period. Such volumes are comparable to the total amount of water supposedly produced over a few years by subglacial melt in the entire catchment of Whillans Ice Stream (0.53 km^3 yr^{-1}). Thus, it appears that most subglacial water in this region moves through a network of subglacial lakes before draining to the Ross Sea.

By what mechanism does water flow in these events? The fluxes appear to follow gradients of hydraulic potential, a useful fact to know but not a helpful one for distinguishing between possible drainage system types. A well-developed tunnel system would accommodate faster transfers of water than those observed. Till canals, a macroporous horizon, or a free water sheet are all, in principle, consistent with the observations.

6.3.5 Summary of Water Systems at the Glacier Bed

Four systems whose characteristics have been analyzed in the preceding sections (with varying degrees of success) carry water along the beds of glaciers.

1. A tunnel system. The pressure and its gradient vary inversely with the flux over periods long enough for steady-state theory to apply. Thus, of two adjacent tunnels, the larger one has the lower pressure and so draws water from the smaller one. The system therefore develops into one, or a few, large tunnels – at least near the front of the glacier. Over short periods, like diurnal cycles, the opposite behavior applies: water accumulates in the upper parts of the tunnel system when inputs are strong, leading to simultaneous increases of pressure, pressure gradient, and flux. At either timescale, pressures may differ considerably in nearby tunnels. Although they have a tendency to join eventually, two tunnels can run parallel to each other for a distance much greater than the spacing between them.

   The tunnel system carries a large flux at low pressure in approximately the same direction as the ice flow. Dye tracing experiments in the ablation zone show that water travels from surface to terminus in a few hours at velocities of the order of 1 $m\,s^{-1}$, which is comparable with that in an open channel (Lang et al. 1979). Because it evacuates water quickly, a tunnel system weakens or nullifies the influence of variable water sources on the flow of a glacier (Chapter 7).

2. A linked-cavity system or, more precisely, a distributed system of passageways whose diameter fluctuates widely along the path. The wide parts are cavities on the downstream

sides of the larger bedrock bumps; the narrow parts occur downstream of small bumps or as channels cut in the bed. The steady-state water pressure, unlike that in a tunnel, increases with the flux and so larger channels should not grow at the expense of the smaller ones. The system is therefore a complicated meandering network of many interconnected cavities. The existence of such networks is confirmed by observations of chemical precipitates on bedrock exposed by retreating glaciers. Transit over a given distance takes much longer than in a tunnel because the path length is longer and the narrow orifices throttle the flow. Measurements of flow in linked cavities can be obtained by injecting tracers into boreholes; for example, Kamb et al. (1985) measured a mean velocity of only 0.02 m s^{-1} in a dye-tracing experiment at Variegated Glacier (Alaska) during a surge (see Section 12.3.2). Because sliding tends to keep cavities open, the sliding velocity influences their size. At the same time, the sliding velocity depends on the water pressure and volume in the cavities (see Chapter 7), so complex feedbacks link together the processes of sliding and water drainage.

3. A network of broad, shallow "canals" in the surface of a soft bed. The water pressure nearly matches the ice-overburden pressure. Canal drainage is expected only beneath ice sheets and ice streams with low surface slopes. The sediment might need to be fine-grained.
4. A permeable macroporous horizon of saturated granular material at the ice-bed interface. Water flows through the pore space, generally at high pressure, and perhaps also along films at the interface. Hydraulic conductivity of the horizon depends on sediment properties and water pressure. This is the soft-bed equivalent of a linked cavity system. Water in the macroporous horizon is distributed widely on the bed and has a large influence on flow of the glacier by basal sliding and sediment deformation (Chapter 7).

Drainage in a valley glacier in summer, except during a surge, is probably by way of distributed linked cavities and macroporous horizons between widely spaced branches of a tunnel system. Because a tunnel can carry a large flux at low pressure, the distributed systems normally receive little of the surface melt. The amount of melt varies diurnally, seasonally, and according to the weather, however, and so the system is never in a steady state.

The size of passageways within the ice and at the bed can adjust, though not instantaneously, to the volume of water. The adjustment time probably ranges from a few days to one or two weeks depending on position in the glacier and the amplitude and rate of change of flux. The system cannot adjust to diurnal variations, but for seasonal variations, the system adjusts to the average conditions. Pressures then vary inversely with the flux, and hence water pressure is higher in winter than in summer. Case studies showing how the drainage system evolves over the summer season are discussed by Hock and Hooke (1993) for Storglaciären, and Nienow et al. (1998) for Haut Glacier d'Arolla.

During winter the passageways shrink and many probably close completely, trapping water within the glacier or at its bed. When melting begins in spring, the water pressure initially increases with the flux and starts to enlarge the tunnels and cavities. Whether water initially drains through the distributed systems may depend on conditions in that particular year. In most cases, however, the tunnel system, with its characteristic low pressure, eventually takes over the bulk of the drainage. The short travel times measured by dye tracing confirm this. Again, water reaching the bed in tunnels connected directly to moulins should remain in tunnels except after sudden increases in flux, when the resulting high pressure may drive some water into the cavities and pore spaces. In late summer, the reduced water supply may be insufficient to fill the tunnels. In this case the pressure in them will drop to atmospheric, and, except beneath shallow ice, they will begin to close rapidly. The distributed systems are kept open by sliding, by high water pressures, and, where water passes through orifices, by melt.

6.3.6 System Behavior

To understand how a glacier's plumbing system works, and not just the local behavior of individual elements within it, the entire pathway taken by water through the glacier needs to be analyzed as a complete system. Fluxes, pressures, and pressure gradients all depend, in general, on conditions upstream and downstream as well as on the interplay of melt, ice flow, and sediment movement at a site.

The simplest case is a single pipe running through a glacier (Röthlisberger and Lang 1987). The pressure at the downstream exit is atmospheric or, if the glacier terminates in a lake or the sea, the static water pressure at the exit. At a steady state, the water flux at any point equals the total of upstream sources – melt of conduit walls and inputs from moulins. Upstream integration of the pressure gradient given by the conduit formulae gives the pressure variation along the pipe. Obtaining a solution requires iteration, because melt and hence flux depend on conditions in the conduit. The complexity increases greatly for time-dependent problems and systems with multiple interacting elements.

Clarke (1996) proposed a method to corral the complexity of time-dependent problems into a mathematically tractable form (though with no explicit accounting for spatial variations). He made an analogy between a hydraulic path through a glacier and an electrical circuit, with hydraulic potential taking the place of voltage and with conduits and distributed pathways represented by resistors of various types. With this approach, water pressures obey a system of linked ordinary differential equations in time. The approach allows for diverse elements to be added to the circuit, representing discharge sources, open and closed cavities for water storage, upward and downward steps in the elevation of the passageways, switches between different types of parallel flow paths, and more. The analysis showed how simple forcings – such as regular periodic inputs of water – lead to diverse and complex pressure and flux variations beneath a glacier, because of the interaction of different types of elements. Complex behaviors observed in numerous boreholes reaching the bed of Trapridge Glacier motivated the analysis.

In general, the flux of water (per unit width across-glacier) transmitted by the basal water system increases with both the size of passageways and the size of the hydraulic potential gradient. This leads to a phenomenological view of the water system's behavior as follows. (For an application of this strategy using numerical techniques, see Alley 1996.) Assume the water flux is $q_w = -c_1 H_w \, \partial\phi_h/\partial x$, with H_w the volume of water per unit area, x pointing down-flow, and c_1 taken as constant. Substitution into the relation for conservation of water volume (Eq. 6.2) gives, in one spatial dimension,

$$\frac{\partial H_w}{\partial t} = \dot{S}_w + c_1 \frac{\partial H_w}{\partial x} \frac{\partial \phi_h}{\partial x} + c_1 \, H_w \frac{\partial^2 \phi_h}{\partial x^2}. \tag{6.40}$$

But, on the bed, $\phi_h = P_\mathrm{w} + \rho_w g \beta$, so that $\partial\phi_h/\partial x = \partial P_\mathrm{w}/\partial x - \rho_w g \beta$, where β denotes the bed slope, positive for a bed descending down-glacier. Combining expressions gives a new relation with both H_w and P_w. But these two variables are also interdependent. Consider, specifically, the case of rapid fluctuations – with periods of minutes to perhaps a few days. In a distributed system, increasing the water pressure relative to the ice-overburden pressure P_i opens wider the cavities and pore spaces; thus, H_w and P_w increase together. To illustrate the implications, take the simplest possible relation: $H_w = c_2 P_\mathrm{w}/P_i$. Substituting and rearranging gives the relation

$$\frac{\partial P_\mathrm{w}}{\partial t} = \left[c_2^{-1} \, P_i\right] \dot{S}_w \; + \; \overbrace{c_1 \, P_\mathrm{w} \frac{\partial^2 P_\mathrm{w}}{\partial x^2}}^{A} \; + \; \underbrace{c_1 \left[\frac{\partial P_\mathrm{w}}{\partial x} - \rho_w \, g \, \beta\right] \frac{\partial P_\mathrm{w}}{\partial x}}_{B} - \underbrace{\left[c_1 \, \rho_w \, g \, \frac{\partial \beta}{\partial x}\right] P_\mathrm{w}}_{C}. \tag{6.41}$$

Thus variations of water pressure at the bed are driven by sources of water ($\dot{S}_w$), and propagate diffusively (term A). This means that zones of high water pressure tend to flatten out and spread and that fluctuations of pressure at one location propagate along the bed. Propagation can also occur as simple waves (term B if $\rho_w g \beta > \partial P_\mathrm{w}/\partial x$) or nonlinear waves (the other case for term B). In addition, water tends to collect in bedrock hollows (term C), until the water pressure builds up to permit drainage. This discussion illustrates how the water system can respond in diverse ways to varying sources and pressure boundary conditions – even if the simplest relations between quantities are assumed. Descriptions of diffusive and wave behaviors in other contexts can be found in Chapters 9 and 11.

6.3.6.1 Integrated Modelling

Because a glacier's water system has several different components, each with a complex spatial distribution, a thorough analysis requires numerical modelling with a geographically realistic framework. Flowers and Clarke (2002a, 2002b) made an ambitious attempt to model the entire hydraulic system of a glacier. The model simulates variations over time and in both horizontal dimensions, on a domain covering the entire glacier. The supraglacial, englacial, basal, and groundwater systems were each represented by a conservation relation of the form $\partial H_w/\partial t = \dot{S} - \nabla \cdot \vec{q}_w$ (Eq. 6.2), where H_w and $\vec{q}_w$ take values unique for each system, at every coordinate. Source terms $\dot{S}$ included melt and rainfall on the surface, melt at the bed, and porosity changes in the groundwater system. In addition, all four equations included source terms that represent gains and losses due to exchanges with the other systems.

For example, surface meltwater, calculated from meteorologic data using the positive degree-day method, flows downhill on the model glacier to regions with crevasses or moulins, which carry it to the bed. A Darcian relation like Eq. 6.38 describes its subsequent movement along the bed, from which it either feeds runoff at the terminus directly or goes into an underlying aquifer. At the point of exchange from englacial to basal systems, and from basal to aquifer systems, the rate of exchange depends on a difference in hydraulic potential between them. For surface to englacial exchange, all water that flows to crevassed or moulined regions goes into the glacier, unless englacial reservoirs are full of water already. Transfers from surface to englacial (and englacial to basal) systems are not instantaneous but occur at a rate inversely proportional to time constants, which are adjustable parameters.

The method was applied first to hypothetical glaciers of simple form. Calculations showed that ice surface and bed topographies largely steer the flows of water, as we should conclude from simpler theoretical considerations. The presence of the glacier, which constitutes a source of pressurized water at the surface of the aquifer, strongly influenced the direction of groundwater flow; water was driven away from the glacier (unless the glacier sat in a deep valley) and emerged in the foreland. Diurnal cycles propagated through all the systems at a rate that depended on the time constants in the source-term exchanges. The authors (2002b) subsequently applied the model to Trapridge Glacier, Yukon, for which parameters could be calibrated by comparison to observed basal water pressures. The calibrated model then afforded a detailed look at how the summertime hydraulic pattern developed and terminated.

A related effort to calculate seasonal runoff variations from integrated modelling was made by Arnold et al. (1998) and applied to Haut Glacier d'Arolla, Switzerland. This model treated the subglacial water system using relations for conduits, rather than a Darcian relation as used by Flowers and Clarke; conduits are an important part of the water system at Haut Glacier d'Arolla. Arnold et al. connected surface melt to meteorological conditions using energy balance relations of the sort discussed in Chapter 5.

6.4 Glacial Hydrological Phenomena

6.4.1 Jökulhlaups

A *jökulhlaup* is the sudden and rapid draining of a glacier-dammed lake or of water impounded within a glacier. Such events, which cause extensive flooding and pose a great hazard to people downstream, have been reported from Iceland (where the name *jökulhlaup* originates), Norway, Switzerland, Pakistan, Greenland, Alaska, Canada, South America, and New Zealand. Glacier-dammed lakes form in various situations, but most commonly when a glacier blocks the stream draining a side-valley. A particularly dangerous case, because of the large volume of water impounded, is when a surging or other advancing glacier blocks the drainage from a major valley. A related phenomenon occurs in Peru, where lakes dammed by terminal moraines have formed in the wake of retreating glaciers; collapsing ice sometimes falls into a lake and generates waves large enough to overflow the moraine. The largest frequently occurring jökulhlaups are those released from ice-bound lakes in volcanic areas of Iceland. In particular, large lakes form beneath and beside the Vatnajökull and Myrdalsjökull ice caps. Water to fill the lakes originates as melt at the base of the ice caps, driven by very high heat fluxes from beneath. The largest floods in Iceland occur during volcanic eruptions (Björnsson 1992, 2002). Outside Iceland, well-studied examples of periodic floods include the aptly named Hazard Lake in Yukon Territory (Clarke et al. 1984b; Clarke 2003), Hidden Creek Lake on Alaska's Kennicott Glacier (Anderson et al. 2006; Walder et al. 2006), and Lago Argentino, Patagonia, a lake regularly dammed by advances of Glaciar Perito Moreno (Skvarca and Naruse 2006; Stuefer et al. 2007).

Jökulhlaups may occur once per year or only every several years. The lake starts to drain when it reaches a certain level and the flood may stop before the lake is empty. Occasionally, water from the lake flows across the glacier surface and cuts a channel in it. In the great majority of cases, however, the lake drains under the ice before the water level has reached the ice surface. The floods are sometimes extremely large. The peak flow of the 1996 outburst of Grimsvötn in Iceland was about $5 \times 10^4\,\mathrm{m}^3\,\mathrm{s}^{-1}$, or about one-quarter the flow of the Amazon; $3.4\,\mathrm{km}^3$ of water drained in just 40 hours (Snorrason et al. 2002). A flood of comparable size occurred here previously, in 1934 (Thorarinsson 1953). Yet larger floods have occurred in prehistoric times. Flows greater than $10^7\,\mathrm{m}^3\,\mathrm{s}^{-1}$ were released from glacial Lake Missoula during the last ice age (Pardee 1942; Bretz 1969; O'Connor and Baker 1992). These floods formed the "channelled scablands," a $5000\,\mathrm{km}^2$ area of eastern Washington State that is marked with gigantic channels scoured into the bedrock. Sediment deposits reveal that more than forty such floods occurred during the last ice age alone (Waitt 1980; Atwater 1984).

Fundamentally, glacier floods occur because of feedback between melt and the ability of drainage paths to convey water. Figure 6.16 summarizes the key process (Nye 1976; Walder and Costa 1996). Discharge of water, whether through subglacial tunnels or supraglacial streams, enlarges or deepens the channels because frictional heat production causes melt. In turn, a larger tunnel or more deeply incised stream conveys a larger discharge for a given water level in the lake. Thus the melting and discharge increase together. This positive feedback stops only after significant depletion of the volume or pressure of the source water. (The feedback may fail to develop, however, if the lake is small and cold.)

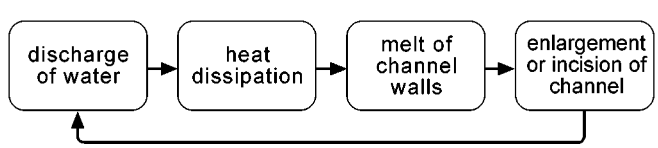

Figure 6.16: The positive feedback mechanism responsible for catastrophic drainage in glacial floods. The feedback can fail to develop, however, if the volume of the reservoir is too small or the water too cold.

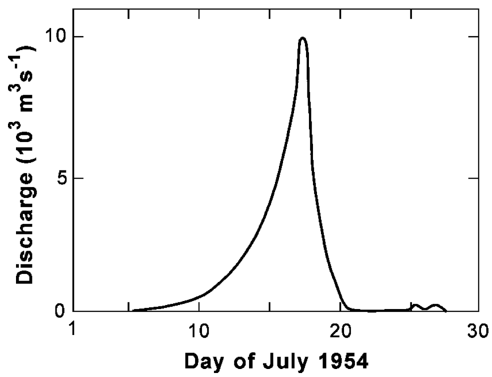

Figure 6.17: Hydrograph of the 1954 jökulhlaup from Grimsvötn, Iceland. Redrawn from Rist (1955).

Grimsvötn, a subglacial lake in the middle of the Vatnajökull Ice Cap, provides one of the best studied cases (Thorarinsson 1953; Björnsson 1974; Nye 1976). A volcanic center supplies heat flow in excess of $50\,\mathrm{W\,m^{-2}}$, which melts ice from the base to form a lake. Melt rates typically reach $5\,\mathrm{m\,yr^{-1}}$. Basal melt also produces a depression in the upper surface, focusing meltwater from the surrounding region into the lake. A floating "ice shelf," about 220 m thick, covers most of the lake. The lake drains catastrophically to the coastal plain through a subglacial passage 50 km long. Drainage stops when the water level has fallen by about 100 m, although the lake is much deeper than this. The water regains the critical level after five or ten years. Figure 6.17 shows the characteristic features of many of the floods: drainage at an increasing rate for about ten days followed by rapid cut-off that takes about two days. In the former phase, drainage increases because melt enlarges the passageways (Liestøl 1956). Collapse or closure of the passageways or the existence of a rock threshold can explain why drainage often stops before the lake empties. Grimsvötn starts to drain when the water is still about 20 m below the depth necessary to float the ice dam (Nye 1976); the observations suggest that drainage occurs when effective pressure in the dam falls below some threshold. As the lake fills, water pressure increases, the drainage system enlarges, and the potential gradient increases; these interdependent processes determine the onset of the jökulhlaup.

Not all floods begin like the one shown in the figure, however. In the exceptionally large jökulhlaup of 1996, the discharge increased rapidly at a nearly constant rate and reached its peak value in only 16 hours. Melt of passageways cannot explain the rise; instead, water probably advanced from the lake as a broad turbulent sheet with pressures high enough to deform the overlying ice and open further the gaps between ice and bed (Jóhannesson 2002).

During a jökulhlaup, water drains from a large reservoir that effectively fixes the water pressure at the entrance to the passageways conducting the water. This differs from the steady-state tunnel case analyzed in Section 6.3.2; that analysis assumed a constant discharge and then found the pressure needed for a balance between tunnel closure and melting. In the present case, when the reservoir starts to drain, the initial discharge is determined by the pressure gradient, which scales with the difference between pressures in the reservoir and at the exit of the tunnel. If, with this discharge, melt does not balance the inflow of the tunnel walls, the tunnel contracts. This may prevent the flood from developing. If, on the other hand, the initial discharge exceeds that required for balance, the tunnel enlarges and the discharge increases, causing yet further enlargement, as sketched in Figure 6.16. This fundamental instability explains the shape and dimensions of the rising limbs of the typical flood (Nye 1976). As the reservoir empties, water pressure decreases at a rate dependent on the shape of the reservoir, and the tunnel starts to close. The closure, and hence the termination of the flood, occurs rapidly – primarily because closure rate depends on the third power of the effective pressure in the tunnel (Eq. 6.15).

To simulate the progress of the Grimsvotn jökulhlaup, Nye (1976) used the equations for nonsteady flow in a conduit: equations for heat balance (Eq. 6.16) and turbulent flow (Eq. 6.18) and two relations for changes of conduit size. First, the conduit expands if melt exceeds inflow of ice, with a rate given by the difference of terms in Eq. 6.19:

$$\rho_i \frac{\partial \mathcal{A}_c}{\partial t} = \dot{M} - 2\,\rho_i\,\mathcal{A}_c\,A\left[\frac{P_i - P_{\mathrm{W}}}{n}\right]^n. \tag{6.42}$$

In addition, conservation of water mass within a unit length of conduit requires that

$$\frac{\partial \mathcal{A}_c}{\partial t} = \frac{\dot{M}}{\rho_w} - \frac{\partial Q_w}{\partial s}. \tag{6.43}$$

This states that, if the conduit is full, the change of cross-sectional area must equal the volume of water melted from the unit length of wall minus the difference between the volumes entering the upstream and leaving the downstream end of the unit section.

Nye assumed that the water was close to 0 °C when it left the lake. This is not plausible for most glacier-dammed lakes. Even subglacial lake waters can be “warm” in volcanic areas (Jóhannesson et al. 2007); the lake water feeding the 1996 Grimsvötn jökulhlaup was about 4 °C. Clarke (1982) added one more equation to take account of such positive water temperatures and found that release of thermal energy by the water was the major factor in enlarging the tunnel. His model obtained a good reproduction of the discharge-time relation for Hazard Lake, Yukon. On the other hand, several observations of the 1996 Grimsvötn flood showed that the flood waters lost their heat to the ice much more quickly than predicted by theory (Jóhannesson 2002); for example, the water was supercooled when it appeared at the terminus. In this flood, high water pressures rather than melt drove the opening of passageways. High pressures in jökulhlaups sometimes force water into the overlying ice along fractures and conduits, which emerge at the surface (Roberts et al. 2000).

Clarke (1982) used his theory to develop formulae for the maximum discharge, based on some simplifying assumptions. However, an equally good prediction of peak discharge can be obtained from an empirical relation that Clague and Mathews (1973) fitted to jökulhlaup data from ten lakes:

$$Q_{\mathrm{MAX}} = 75\left[V_o/10^6\right]^{0.67}. \tag{6.44}$$

Here Q_{MAX} is peak discharge ($m^3 s^{-1}$) and V_o the volume (m^3) of the lake just before it starts to drain. This formula, which takes no account of such quantities as tunnel shape and slope, hydraulic roughness, and ice-flow parameters, "confounds understanding but seems to give reasonable results" (Clarke 1986), a humbling thought for theoreticians. The most recent estimates for the multiplier and exponent in this relationship are 46 and 0.66 for floods draining through tunnels (Walder and Costa 1996).

A relationship of the same form applies to catastrophic drainage of lakes impounded by glaciers blocking side valleys (Walder and Costa 1996). Such floods commonly escape through a breach between the ice dam and the bedrock valley wall. The floods tend to be much larger than for subglacial ones, for a given lake volume; the best-fit multiplier and exponent in Eq. 6.44 are 1100 and 0.44, respectively. The similarity of these values to those describing floods from earthen dams indicates that, regardless of the drainage process that starts the flood, the hydraulics of flow through a rapidly forming subaerial breach controls the subsequent discharge.

Jökulhlaups transport sediment. In Iceland, where the volcanic substrate is fragmentary or highly fractured rock, the floods move great quantities of sediment which are deposited in broad outwash plains called *sandurs* at the periphery of the ice caps. A May 1721 eruption of the volcano Katla beneath the Myrdalsjökull Ice Cap produced a jökulhlaup that buried the nearby sandur in about 90 m of debris and extended the coastline southward by some 5 km. Sediment moving in subglacial conduits and erosion of conduit floors are probably significant processes affecting the size and shape of subglacial passageways and hence their ability to convey the flood. These processes were not included in Nye's theory. Fowler and Ng (1996) incorporated erosion of sediment along the conduit sides into a model for jökulhlaups, assuming a sediment bed. In their view, competition between widening of the conduit by erosion and narrowing by sediment creep determines the conduit shape, which is broad and flat. Erosion was assumed proportional to the rate of energy expenditure by the flowing water.

A further complication for models is that sometimes the flood emerges in several outlet channels and sometimes as a broad irregular sheet several kilometers wide. Clarke (2003) suggested that multiple channels develop because overpressurization of the main conduit at constrictions causes ice fracture or lateral drainage. But in the cases like the 1996 Grismvötn event – in which floods propagate as a sheet of overpressurized water – flow in the sheet and interaction between the sheet and developing pipes are both important processes (Flowers et al. 2004), as is flow upward into the glacier (Roberts et al. 2000).

6.4.2 Antarctic Subglacial Lakes

In Chapter 9 we explain how the thick ice and low accumulation rates in Antarctica allow the base of the ice sheet to warm to melting point despite the frigid surface climate. Many lakes exist beneath the Antarctic Ice Sheet. The lakes were originally identified from bright horizontal reflecting surfaces seen in radio-echo soundings (Oswald and Robin 1973). At least

150 subglacial lakes have now been identified in Antarctica (Siegert 2005; Fricker et al. 2007). Most occupy bedrock basins. Their abundance in Antarctica can perhaps be explained by three factors. First, surface slopes of the ice sheet are small, implying a small typical hydraulic gradient; this permits basins to trap water. Second, subaerial erosion has not acted on the bedrock for millions of years. Thus, numerous bedrock basins have developed by tectonic processes and subglacial erosion, but have not been breached or filled with sediment. Third, the absence of surface meltwater streams implies that no water reaches the bed as a concentrated source capable of sculpting a network of large and abundant tunnels.

Lake Vostok, beneath 3750 m of ice, is the most thoroughly investigated subglacial water body in Antarctica but has not yet been entered or directly sampled (as of 2008). With an area of 15,690 km^2 and an estimated volume of 5400 km^3, it is the world's seventh largest lake. Equally remarkable is that the seminal Russian and French ice-coring studies were conducted at Vostok Station for many years before anyone knew of the lake beneath. Only in the late 1990s did the coring penetrate deeply enough to recover bottom ice, a 210 m-thick layer clearly identifiable as lake water frozen onto the base of the ice sheet (Jouzel et al. 1999; Siegert et al. 2001). The transition from the ordinary glacier ice to the lake ice is abrupt and defined by a switch to large and undeformed crystal textures, with distinct isotopic and chemical compositions. This lake, and others like it in East Antarctica, attract much interest because they presumably have been sealed from direct contact with the atmosphere for millions of years. The meltwater passing through the lakes, however, is derived from ice "only" a few hundred thousand years old.

The ice floats freely on large lakes. Large lakes can be identified because the overlying ice sheet surface is nearly flat in the direction of ice flow. (We discuss ice flow in the vicinity of these lakes in Section 8.9.5.) Water sometimes moves between lakes as slow subglacial "floods," events identified by deflation and swelling of the surface. Such events have been identified in soft-bed regions of West Antarctica (Section 6.3.4) and in East Antarctica in a region of unknown bed type (Wingham et al. 2006). In the latter case, about 1.8 km^3 of water moved within 16 months from one subglacial lake to a group of lakes more than 290 km distant. The drainage path followed a subglacial trench. The positive feedback between melt and drainage (Figure 6.16) makes drainage events inherently unstable. Consequently, it is reasonable to expect that lakes drain episodically rather than continuously (Wingham et al. 2006; Evatt et al. 2006). A lake might steadily fill up as it accepts water from subglacial catchments. This increases the water pressure as the ice surface swells. It also increases the surface slope on the downstream side, increasing the hydraulic gradient. As with the lakes beneath Icelandic ice caps, rapid drainage begins when the hydraulic gradient directs water out of the lake and, simultaneously, effective pressures are low enough to open passageways.

Large transfers of water between subglacial lakes might influence ice sheet dynamics; such events could change the resistance of the bed to sliding motion over large regions, on timescales of years. In addition, lakes underly the upper ends of some of Antarctica's major ice streams (Bell et al. 2007). Water draining from the lakes lubricates the bed downstream, allowing rapid ice flow. Whether the presence of the lakes actually influences rapid ice flow or not remains unknown. Over many years, the amount of water drained along the bed is determined by the basal melt in the catchment; the presence of lakes is largely irrelevant. Perhaps lakes influence ice flow downstream only if they drain episodically.

Although the ice sheet surface over Lake Vostok is flat in the direction of ice flow, in the transverse direction the surface elevation decreases by about 40 m over the 200 km span of the lake (Siegert et al. 2001). By the requirements of floating equilibrium, the ice thickness and basal elevation must also vary. At the same time, hydrostatic equilibrium in the lake beneath requires uniform pressure in the lake on surfaces of constant elevation. Thus if H_1 denotes ice thickness above one side of the lake, and H_2 (smaller than H_1) the thickness above the other side, then $\rho_i H_1 = \rho_i H_2 + \rho_w \Delta\mathrm{B}$, with $\Delta\mathrm{B}$ the elevation difference of the ice base between the two locations. Call the surface elevation difference $\Delta\mathrm{S}$. Then $H_2 = H_1 - \Delta\mathrm{S} - \Delta\mathrm{B}$. The elevation along the roof of the lake therefore rises from one side of the lake to the other by $\Delta\mathrm{B} = \Delta\mathrm{S}\, \rho_i/[\rho_i - \rho_w] = -11\, \Delta\mathrm{S}$. In other words, the base of the ice slopes eleven times more than the surface and in the opposite direction; it follows an equipotential as determined in Section 6.3.1.1. But water can nonetheless flow along this surface because of density contrasts. Increased pressure lowers the melting point, so higher temperatures should occur under the thin-ice side. Water produced by melt on the thick-ice side of the lake supercools as it flows along the roof to the thin-ice side, and so freezes on. This explains the 200 m of accreted lake ice at the Vostok core site. Water can circulate in the lake because its density depends on both salinity and temperature. Within the lake, waters should be slightly warmer at the bottom than at the roof because of geothermal heat. The warmer waters should rise buoyantly. Depending on the salinity of the lake, the fresh water produced by melt of the overlying ice might also be buoyant.

Further Reading

Fountain and Walder (1998) reviewed water flow in temperate glaciers, and the formation of features like moulins. Jóhannesson et al. (2007) presented results of direct measurements in a volcanic subglacial lake in Iceland. Hubbard and Nienow (1997) and Tranter (2005) reviewed measurements and interpretations of sediments and solutes in proglacial streams and subglacial waters – topics of central interest to studies of erosion. Willis (2005) wrote a summary of the hydrology of watersheds containing glaciers.