USING SMALL PARAMETER CHANGES TO ACCESS MANY DIFFERENT & INTERESTING LIMIT CYCLES IN RANDOM NETWORKS

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1 Introduction

Randomly connected networks with synchronous update, asymmetric connections, idealized integrate and fire nodes, and constant parameters, when given different initial conditions, typically converge to only a small number of different and dynamically interesting limit cycle attractors (e.g: Clark, Kürten & Rafelski, 1988). We study one method for accessing a richer repertoire of behavior, and find that with small (but not tiny, nor huge) changes in *all* of the connection strengths or all of the thresholds, we can access a quantitatively different and interesting limit cycle (for more details, see: McGuire, Bohr, Pershing, & Rafelski, 2000). Due to the large number of combinations of parameters in a given network, we can therefore access a large number of different and interesting attractors without drastically changing the network.

1.1 Volatility

Volatility is defined as the ability to access a large number of limit-cycles, where a large number of neurons participate in the dynamics. We define volatility also as an entropy-weighted entropy:

$$\mathcal{V} = -\sum_{t_s=1}^{t_s^{\max}} e(t_s) P(t_s) \ln P(t_s) \ , \tag{1}$$

where t_s denotes the trial number, $P(t_s)$ denotes the probability of the network being in a given limit cycle attractor, and $e(t_s)$ is a similar entropy measure that measures the dynamical participation of all the neurons in each limit cycle, based upon the time-averaged neuronal firing rates, $< s_i >_t$: $e = -\sum_i < s_i >_t \ln < s_i >_t$, where i denotes neuron number.

We simultaneously vary the thresholds of all the McCullough-Pitts neurons from their nominal values by adding a noise term to each neuron, chosen from a spatially uniform gaussian distribution of width ϵ.

For $(\epsilon_1 < \epsilon < \epsilon_2)$ (Fig. 2), volatility is high, corresponding to two distinct transformations to volatility. At $\epsilon = \epsilon_1 \sim 5 \times 10^{-4}$, the amplitude of slow threshold noise causes a transformation from few accessible limit-cycles to a large number of different, accessible limit-cycles; at $\epsilon = \epsilon_2 \sim 5 \times 10^{-1}$, the slow threshold noise is so large that all limit-cycles become fixed points. We accordingly label three different regimes for the RSANN with slow threshold noise:

- Stable Regime: $\epsilon < 5 \times 10^{-4}$
- Volatile Regime: $5 \times 10^{-4} \leq \epsilon < 5 \times 10^{-1}$
- Trivially Random Regime: $\epsilon \geq 5 \times 10^{-1}$.

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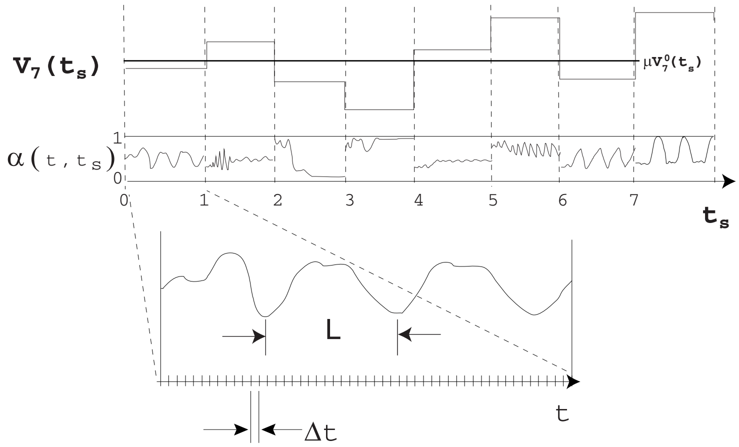

Figure 1: A qualitative sketch showing the threshold for neuron #7 ($V_7(t_s)$) (all thresholds are varied simulataneously) and the spatially-averaged firing rate, $\alpha(t, t_s)$ as a function of trial number (slow time) t_s and in the inset the $\alpha(t, t_s)$ as a function of fast-time t.

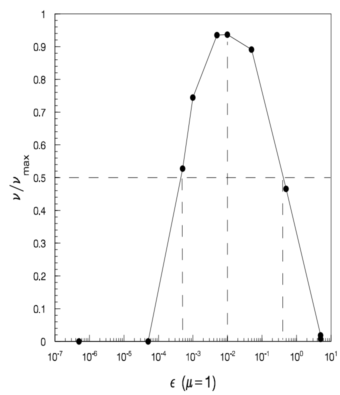

Figure 2: The volatility is large and nearly maximal only within a broad range of slow-time noise amplitudes: $5 \times 10^{-4} < \epsilon < 0.5$ ($N = 40$ neurons, and $t_s^{max} = 100$). The maximal volatility for $t_s^{max} = 100$ slow-time steps is $\mathcal{V}_{max} = \frac{1}{2} \ln 2 \ln 100 = 1.5960$.

Our key result is that a random neural network can be driven easily from one to another stable recurrent mode. While such behavior can be always accomplished by radical modifications of some of the network properties, the interesting result we have here presented is that a plausible, small O(0.1%), random change to the neural threshold parameters suffices to access new dynamical behavior. We are aware that this does not yet create a network that can self-sequence a series of modes. The development of self-control algorithms needed to access mode sequences is a natural and very complex next task, potentially leading to the understanding of random neural network information processing.

References

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