GEOS 24705 / ENST 24705
Problem set #12
Due: Tues. May 10

Water and wind turbines

Problem 1: big-dam hydro with Francis turbines

Consider Hoover Dam, one of the signature pieces of American engineering. Basic stats on the dam, reservoir (Lake Mead) and power plant are:

- max power: 2080 MW
- dam height: 221 m (max hydraulic head 180 m)
- dam width: 380 m
- Lake Mead volume: 35 km^3 (when full)
- Lake Mead surface area: 640 km^2
- Lake Mead max length: 180 km
- Number of turbines: 17
- Inlet diameter for each turbine: assume ~ 4 m
- Flow rate of Colorado River into Lake Mead: $610\ \mathrm{m3/s}$

Remember the Bernoulli equation for water:

$$\tfrac{1}{2} v^2 + g \cdot h + p/\rho = \text{ct. (incompressible fluids)}$$

A. What is the energy density of water behind Hoover Dam, when the reservoir is full?

B. When the power plant is running at full power, what is the flow of water out of the dam? Give the flow Q_m in kg/s but also convert to a volume flow rate Q_v in m^3/s.

C. What is the "turnover time" of water in Lake Mead, if the plant runs at full power continually? (i.e. how long does water stay in the reservoir before it is flushed out?) How many times is the lake effectively re-filled in a year?

D. If you had drained Lake Mead, how long would it take to refill?

E. From B and C can you run the hydro station at Lake Mead at full power constantly? State the maximum capacity factor for using Lake Mead hydro. *(In practice capacity factors are less because you increase evaporative losses when you make a big lake surface – outflow from a reservoir is less than inflow).*

F. How fast is water moving when it enters the Francis turbines, if the turbines are operating at their rated power?

G. Is your velocity in F. consistent with your idea of how a good reaction turbine would be designed? Would you prefer a high or low velocity? Explain.

H. How fast would water be moving if you disconnected the generators from the grid and let the water flow freely out? (Assume it would then flow out at maximum velocity, since there is no impediment to flow that keeps pressure high in the turbine).

I. How long would it take Lake Mead to drain out, if water continued to exit at that velocity?

J. Would the water actually continue to exit at that velocity as the reservoir drained? Explain.

K. *Optional: using whatever assumptions you made in part H, write down the equation for the drop in water height in Lake Mead over time, and solve the resultant differential equation. How long would it take Lake Mead to empty by 90%?*

   *Note: you have to assume some shape for the reservoir behind the dam. One option is to assume that the reservoir is rectangular and flat-bottomed, i.e. V=A*h. Another option (which sounds better, but also proves a bit unrealistic) is to assume the reservoir is a half-cone of radius h and length L that shrinks as the reservoir dries up.*

L. *Optional: We have previously calculated the W/m2 for hydro, considering the area to be the entire catchment area on which rain falls. What if we're not concerned with total energy available, but want to get a sense of the amount of land you need to consume to produce a given amount of hydropower? So.. what are the W/m2 for Hoover Dam/ Lake Mead hydro with this definition?*

   This number doesn't tell you how much hydro you can extract worldwide, but it does give you an idea of you how much land you have to drown to produce a given amount of hydropower.

Problem 2 – do one of the following (either 2.1 or 2.2). Problem 2.1 should be quicker for those who don't have a lot of physics background. It should be a half hour of reading. Doing both problems gives extra credit.

Problem 2.1: Hydro in developing countries

Ethiopia, one of the poorest countries on Earth, with a correspondingly low energy use per capita, has essentially no fossil fuel-fired electricity generation. The vast major of its power comes from hydro, and that percentage will rise, because the country is in the process of damming its Omo River with a series of dams, collectively known as the Gilgel Gibe project. (Estimates are that when Gilgel Gibe is done, 96% of Ethiopia's electricity will be hydro).

Are the dams a good idea? Funding for the dams comes from outside sources. There are obvious points of concern, including that the construction contract was given to a well-connected Italian firm as a no-bid contract. It is rumored that much of the power is already contracted to be sold to neighboring (and better developed) Kenya and Sudan. That might be a good thing, if it brings in needed cash, or a bad thing, if revenues go directly into officials' pockets.

Do some reading, answer a few questions, and come to a decision (if you can) on the worth of the dam. Is the project worth it?

Readings: glance through the Voith-Siemens brochure on the project that is posted on the class website, mostly for drawings of the giant Pelton wheels. Skim (*extremely* lightly) through the negative report from a group called "Counterbalance" linked below. Read a 2009 compilation of short newspaper articles in the Ethiopian journal Tadias, and most importantly read the many comments, most of which are by Ethiopians. You need not read any of this in detail, and especially don't spend too much time on the long Counterbalance report. This is just to get a feel for the arguments.

Tadias article, links, and comments

http://www.tadias.com/05/14/2009/ethiopia-big-dam-bigger-problems-debating-gilgel-gibe/

"Counterbalance" report: http://www.counterbalance-eib.org/?p=334

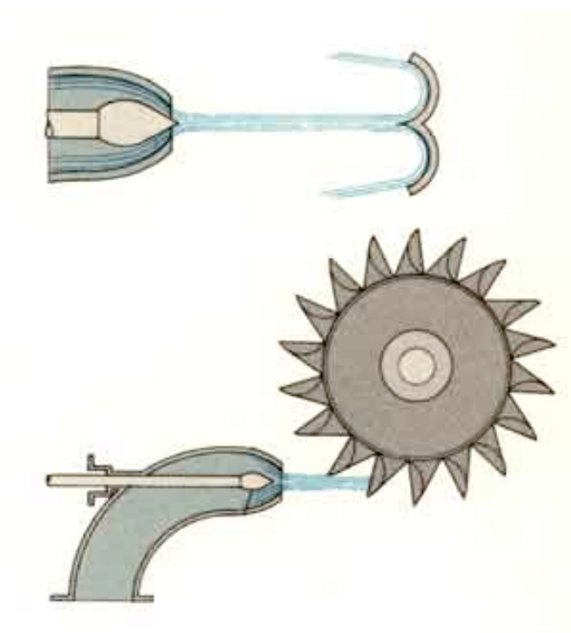
A. From information in the Nazret.com article linked to from Tadias, what is the per capita electricity consumption of the average Ethiopian? The article gives it in kWh/yr; convert to W.

B. Compare this to the U.S. per capita electricity consumption. (You can assume that 1/3 our primary power consumption goes to electricity, as we saw on the "spaghetti" chart of U.S. energy use, and that electricity production is ca 40% efficient).

C. What is the capital cost of Gilgel Gibe II and III, in $/W, from the Counterbalance report (p. 7)? Is that reasonable? (See figure in hydro lecture slides if you want a comparison)

D. Do you have an opinion on the project? Express it if so. If you're of mixed mind, say that. Just a few sentences explaining your position or confusion.

Problem 2.2: Pelton wheels for high-head hydro

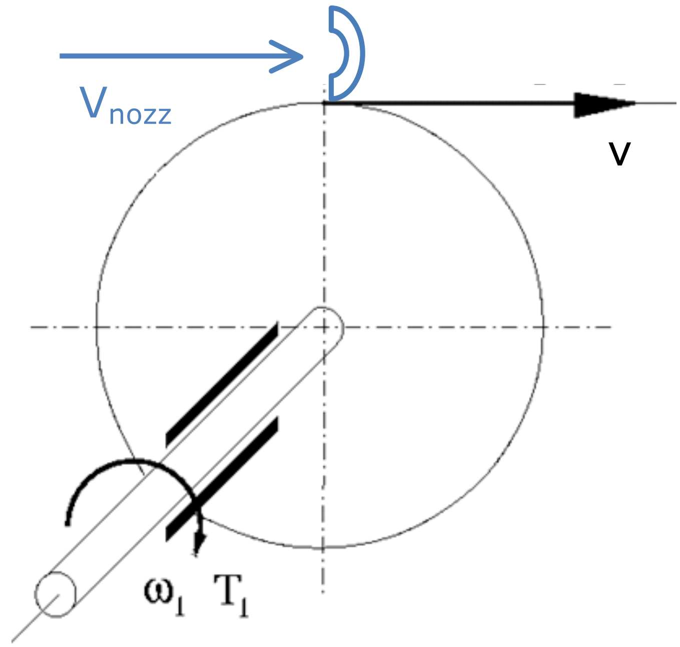


Pelton wheels are used only in low-flow, high-head situations. Why?

The low flow criterion occurs because they Pelton wheels take up so much space for a given flow. In the Francis turbine, water fills the entire turbine volume, but the air-filled Pelton wheel enclosure must be many times larger than the volume occupied by the flow.

One of the reasons for the high head criterion is that power is proportional to flow * head, and if you want to get reasonable power and flow is low, you'd better have high head. But another is that it's difficult to build a wheel to produce AC power at 60 Hz unless you have high head. You'll show this in this problem.

The Pelton wheel is driven by a jet or jets of water that strikes the vanes (often at the top or bottom of the wheel). The jet bounces off the vane, imparting the momentum of its water molecules and pushing the vane forward. But because the wheel is already rotating, the vane isn't stationary when the jet hits it, but is moving at some velocity v. The optimal power extraction from a Pelton wheel occurs when the linear velocity (v) of the wheel at the vanes has some specific relationship to the velocity of water coming out of the nozzle (v_{nozzle})



A. For optimal power extraction from the Pelton wheel, what is v in terms of v_{nozzle} ?

   Two additional pages posted give (Hint 1) the relationship you're trying to prove and (Hint 2) suggestions on proving it. Use either or both of these if you want, but state what hints you use. If you can't prove this but want to use the relationship and still do the rest of the problem, leave blank and move on.

B. Design a Pelton wheel generating system to produce 60 Hz electricity from a 10 m high dam.

   "Design" means compute v_{nozzle}, determine the optimal v, determine the dimensions of the Pelton wheel to produce that v (for the rotation rate you've chosen), and choose the type of generator (and hence rotation rate).

   First, try using the simplest possible AC generator, a 2-pole (i.e. one magnet) synchronous generator, so that the Pelton wheel will rotate at 60 Hz. How big a wheel will you need?

C. You might decide that this is impractical for serious power generation – you'll never be able to put serious flow into a wheel that size. Now try to go with as many poles on your magnet as is normally handled, about 24. Now how big is the Pelton wheel you'd design? Is this practical?

D. What if the head were 1000 m instead of 10 m? What size would your Pelton wheel be now? Is this practical?

E. Some Pelton wheels can have as many as 6 jets. Does the size of the wheel depend on the number of jets? What is the effect of adding more jets?

Problem 3: Rotation rates of wind turbines

Wind is such a diffuse energy resource that turbines have to be big, but their size is a bit problematic – it means that wind turbines can't rotate too fast or they'll shake themselves apart.

Data: the speed of sound in air at room temperature and pressure is 340 m/s. The linear velocity of the different parts of the turbine blade will differ, slower near the hub and maximum at the tip of the blade. It is not survivable for the turbine if the tip speed of the blades breaks the sound barrier. (Remember that in one revolution of the turbine, the tip of the blade travels around a circle of distance $2\pi R$)

Can you design a modern wind turbine to make 60 Hz power directly?

A. First, make reasonable assumptions (based on class slides) about the size of your turbine, and state them

B. What rotation rate would the turbine need to have to make 60 Hz power directly with a simple 2-pole AC generator?

C. What is the tip speed? Is this survivable?

D. Now increase the number of poles on your generator to 24, allowing you to spin 12 times more slowly. Can you make 60 Hz power directly now?

E. Since your turbine does in fact have to make 60Hz power eventually, how might it manage that? State at least one possibility.

F. Based on your experience (or look at videos of wind turbines), estimate/guess a rotation rate for normal turbines. Also estimate/guess a normal wind speed at which the turbine operates.

G. We said in class that the maximum extraction of energy that a wind turbine could achieve a maximum efficiency of extraction of kinetic energy from wind of 59%. In practice however wind turbines can only approach that ideal if their "tip speed ratio" or tsr - the ratio of the speed of the tip of the blade over the speed of the wind – is near an optimal number. What is the tsr of your "normal" wind turbine?

H. Google on "optimal tip speed ratio" and see if your experience has led you to design a reasonable turbine. *(Google only after doing the rest of the problem, and please don't change your answers in response to Googling – you won't be penalized for reasonable assumptions even if they give you a somewhat non-optimal tsr).*

I. How much slower are your blade tips traveling than the speed of sound?

Problem 4: Can wind power the world?

In class we discussed our target for finding a power source that could run the whole world in the future and let everyone live like Americans. The criterion we picked was that area energy flux had to be 10 W/m^2 or better.

Can wind get us there? In this problem you'll design a wind farm and calculate the energy flux it can extract from the wind.

Remember the power carried as kinetic energy by any moving fluid:

$$P = \frac{1}{2} A \rho v^3$$

where A is the cross-sectional area considered, ρ is the density of the fluid, and v is the velocity. Remember that Betz's law limits the power extracted even by the ideal free-stream wind of hydro turbine to 0.59 of the power carried. In practice even the best wind turbines don't quite reach this limit, but top out at about 0.5. And the wind doesn't blow all the time – the "capacity factor" for wind turbines is usually around 0.3, for an overall efficiency of extraction of wind kinetic energy of ~ 0.15

We discussed briefly the spacing of windmills. Because each windmill disrupts the velocity field of the wind, they can't be placed right next to each other. The velocity field downstream of a wind turbine also takes a long time to recover. The rule of thumb is to place them at least 3 rotor diameters apart along the direction facing the wind, i.e. to leave room for two other wind turbines in between any two that you build, and to leave 10 rotor diameters behind each turbine.

You need to pick a reasonable wind speed to use as your "v" in computing power. If wind is to be scaled up, we'll run out of the best and windiest sites. A reasonable number for v for wind on a large scale is 7.5 m/s.

A. How much power/area can you extract from a wind farm? Draw a diagram of a wind farm layout and wind turbine spacing, and derive power/area in W/m^2. Does your answer depend on the size of the turbines?

B. Can wind alone power the world

C. For a 500 MW wind farm (the size of the Crawford generating plant), how much area would you need?

Problem 5: Video tours of wind turbines *(extra credit for finding new videos for suitable for future class use and commenting on them)*

A. Usually the point of turbine design is to extract the most power from a fluid. But sometimes you don't want all the power a fluid contains. If you try to extract power out of too fast a wind for your turbine, you might overstress mechanical components or melt the generator. Explain either failure mode.

B. How would you engineer a turbine so that you could adjust it to take up some power, but less than the maximum? *(Hint: sailors have the same problem – when the wind is too strong they don't want all that power).*

C. If the wind is too strong even for these strategies, turbine operators set a brake so that the turbine shouldn't move at all. But sometimes the brake fails. Watch the videos below and comment on something interesting.

http://www.youtube.com/watch?v=CqEccgR0q-o&feature=related
http://www.youtube.com/watch?v=-YJuFvjtM0s&feature=related

http://www.youtube.com/watch?v=jCyQD83NLDc
http://www.youtube.com/watch?v=C_oFPF6Anwo

D. Turbine installation and operation are now very standardized. Watch the following video and comment on something interesting.

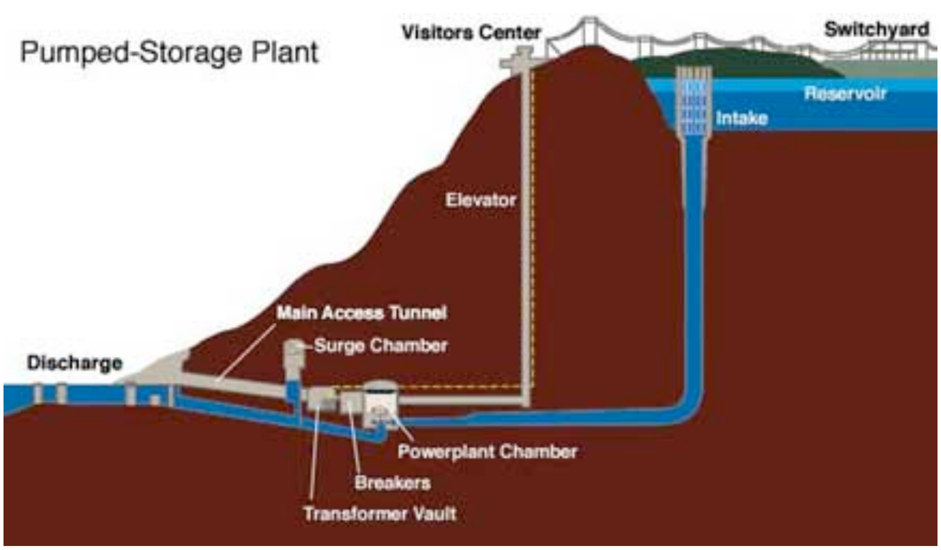
    http://www.youtube.com/watch?v=Jf-Q1wyowWc

OPTIONAL PROBLEMS

Problem 6: (Optional) Pumped hydro storage

A dam serves two purposes: 1) it stores a large volume of water that can be released through the turbines as desired, and 2) it builds up depth in the stored water and thereby increases energy density in the water. Those purposes can be separated, and you can get the benefit of high head without building a dam. If you build a reservoir on top of a mountain, a pipe leading down the mountain from your reservoir will build up the pressure of the column of water in the pipe itself, i.e. its pressure head will be the height of the mountain. You can't store as much water on a say 1000 m mountain than you could behind a 1000 m dam, but a mountaintop reservoir is a way to get high head cheaply if you don't need big storage.

Of course a huge drawback of mountaintop storage is that, well, streams don't flow uphill. If you want to get water to the top of the mountain, you have to pump it there. That means you can't even get back all the energy that you put in to it, much less gain anything. Mountaintop reservoirs are therefore useless for power *generation*. They are very useful however for *storage* of energy. Think of a mountaintop reservoir as a big battery. And energy storage is a need that comes with intermittent alternative power sources like wind and solar. The wind doesn't always blow (or sometimes blows too hard), and the sun doesn't always shine.



Raccoon Mountain pumped hydro energy storage, Tennessee. 1600 MW.

In this problem you'll design a pumped-hydro system to back up the wind farm you designed in Problem 4. Assume your wind farm is 500 MW and that you need storage capable of replacing all the wind turbines for a week if the wind quit blowing. Assume you have a 1000 m mountain to build on, and that it's practical to dig your reservoir 50 m deep. The total efficiency of pumped hydro storage (which requires the conversions electricity -> gravitational potential via a pump -> pressure head-> spin a turbine and make electricity again) isn't 100%, but in good modern systems can reach 80%.

Remember that, unlike in a dam system, in a mountaintop reservoir you don't lose much energy density as the water level in the reservoir drops, since most of the pressure head is caused by the height of the mountain, not by the depth of water in the reservoir. (In fact, for the purposes of this problem, you can ignore changes in height as you drain the reservoir).

A. What volume of water will you need? What is the area of the reservoir?

B. Compare to your answer in 3B. Is adding pumped hydro to your 500 MW wind farm a significant extra requirement in area? That is, would the effective W/m^2 of wind power be significantly altered if you now require every wind farm to provide backup energy storage via pumped hydro?

C. Pumped hydro is also used by energy "wildcatters" who make money by buying power at night when demand is low and prices are cheap, then selling it back to the grid during the day when demand is high and prices are high.

   Write down an expression that describes when pumped hydro is a profitable strategy, involving the price ($/J) of buying power at night (C_{night}), the price ($/J) for selling it during the day (C_{day}) and the efficiency of the pump/turbine system (ε).

Problem 7 (Optional): Bernoulli equation for air

A. Derive the Bernoulli equation for compressible fluids:

$$\tfrac{1}{2} v^2 + g \cdot h + p/\rho \cdot \gamma/(\gamma - 1) = \text{ct.}$$

   Hint: with incompressible fluids, you were allowed to neglect the internal energy term ($c_v T$, where c_v is the specific heat at constant temperature) because changing the fluid pressure did not affect its density or temperature. With gases, you don't have that luxury.

   Background: The constant γ is the ratio between the specific heats at constant temperature and constant pressure (c_v/c_p). The difference between the specific heat at constant pressure and that at constant value is the specific gas constant by $c_p = c_v + R_s$. (R_s , the specific gas constant, is the same as the universal gas constant R of the ideal gas law PV=nRT, which has units of J/mol K, except that R_s is divided by the mass per mole of gas, so that its units are J/kg K).

   For additional extra credit, explain why you know that $\gamma < 1$ (i.e. that $c_p > c_v$). Material from this class is sufficient to explain this – you don't need additional physics.