Problem 1: Engines and Carnot efficiency

A heat engine operates by extracting mechanical work (W) while moving heat down a temperature gradient, from something hot (at a temperature $T_h$) to something cold (at $T_c$).

Carnot first derived the limiting efficiency (work out / heat in) for a heat engine:
$$\varepsilon = 1 - \frac{T_c}{T_h}$$

A By Carnot's theory, what was the maximum efficiency that Newcomen’s or Watt’s early atmospheric heat engines could have achieved?

B How close did Watt actually get to the Carnot limit?

C By the end of the steam era (early 1900s), engine efficiency approached 25%, but in practice, steam engines never actually achieved their Carnot limits. Assume that each engine’s efficiency (at 25%) was only 1/2 its own Carnot limit. What temperature would the early 1900s engines then have to run at to achieve 25% efficiency?

D If you want to achieve an engine efficiency of 75%, what temperature would you have to run the engine at if were otherwise ideal?

E Steel loses 90% of its strength above 800 C ($\sim$ 1000 K). Can you make a 75% efficient engine out of steel?

F Why is there a lot of research into ceramics in engine design at present?

Problem 2. Engine cycles

For the two “engines” below, draw the piston strokes and match them to corresponding segments on the P-V diagram, as we did in class. Indicate on the diagram: a) when heat is being put into the engine, b) when heat is leaving the engine, c) when the piston is doing work, and d) when work is being done ON the piston. Explain what you can about the relative size of the heat fluxes and work on the different segments.

A. The totally uninsulated, isothermal-only engine. (That is, you forgot about Carnot’s rule that you can only derive net work out from an engine if you let heat flow down a temperature gradient, and built a dumb engine).

B. A Carnot-cycle engine.
Problem 3: Heat engines and heat pumps

A heat engine is a device that moves heat down a temperature gradient (from hot to cold) and extracts some of that heat flow as mechanical work. A heat pump is basically a heat engine run in reverse: it takes mechanical work and uses it to push heat up a temperature gradient (from cold to hot). A refrigerator is a heat pump, using mechanical work to move heat from the interior of the refrigerator to the warmer exterior.

We stated in class that the best heat engine would be a purely reversible one – that is, it would suffer no frictional or other losses. In this case the values of all the heat flows are identical no matter which direction the engine is run in.

Carnot’s thought experiment showed that the Carnot limit was the maximum efficiency possible for ALL reversible heat engines.

A. What would happen if you could hook up a hypothetical better-than-Carnot-efficiency heat engine with a Carnot engine reversed as a heat pump? (So that the engine drives the heat pump). Draw the diagram and explain what would happen. Is this possible?

B. What would happen if you hooked up a worse-than-Carnot heat engine with a Carnot heat pump? Draw the diagram and explain what happens. Is this possible?
Problem 4: Evaluating heat pumps

If you want to describe how good a heat pump is, the efficiency isn't the right intuitive metric. The efficiency tells you the mechanical work you get out given a certain amount of heat transfer. With a heat pump, you want to do the LEAST mechanical work and transfer the MOST heat.

A Make up a metric (call it “COP”, for “coefficient of performance”) that describes how good a heat pump is (i.e. how much heat you get transferred per work done) and write down its definition.

B Assume that your heat pump is ideal (that is, that it is a Carnot engine run in reverse, the best heat pump you can have). Now rewrite your definition in terms of $T_{\text{hot}}$ and $T_{\text{cold}}$ alone.

C Does it take more work to pump heat across a large thermal gradient or a small one? Is that consistent with your intuition?

D What is the physical meaning if COP > 1? Is it possible to have COP < 1?

Consider the difference between heating your house with a heat pump vs a space heater (or furnace) – see diagram next page. With a furnace or space heater, all the energy you put in is converted to heat. (All your furnace fuel burns and converts its chemical energy to heat; all of the electrical power you put into your space heater is consumed in resistance heating in the heating elements).

Consider your room in a chilly Chicago winter (make reasonable assumptions). Assume that in your heat pump system, the input electrical power gets converted to mechanical work (for the heat pump compressor) without any losses.

E How much more power would you need to heat your room with a space heater than with an ideal heat pump?

So... why don't more people use heat pumps? Well, an actual real heat pump won't reach ideal efficiency, and in practice COP is lower than the Carnot case by a factor of 2-4. And most importantly, furnaces (big tubs to burn stuff in) and space heaters (big resistors, basically just giant toasters) are very simple and therefore cheap. That upfront cost difference can lure people to buy the product with the lowest immediate cost, or can actually make the lifetime costs actually lower. Energy is cheap at present, and may not be the driver in a decision.
Problem 5: Energy transformation technology
In class we discussed the 17th-18th century energy dilemma – that economies limited by lack of energy, in part because the only transformations that were possible were motion -> motion and chemical energy -> heat. In modern life we have developed technologies to perform many more energy transformations. These
technologies are represented on the “energy conversion grid” (along with some natural processes) downloadable with this problem set.

Use this grid, trace out the chain of energy transformations involved in some aspects of your everyday life, and book-keep the energy losses that occur during that chain. The table on the following page gives conversion efficiencies for many common transformations. (You have all the info here you need, except that there are 7% losses in U.S. electricity transmission lines – i.e. on average, electricity transmission and distribution is about 93% efficient). Assume for now that all electricity is produced by burning some fossil fuel and spinning a steam turbine.

For problems A-F below, you should turn in 1) a printout of the technology grid numbering (or otherwise marking) the progression of transformations, with 2) a calculation of the net efficiency of the entire process written on it.

Since the primary source of all the energy we’re considering here is solar radiation, you properly have to include the efficiency of photosynthesis ($\varepsilon_{\text{photo}}$) for any conversions involving biomaterial (i.e. food, biofuel, or fossil fuels). But, if you’re comparing two processes that both start with biofuels, they’d both be multiplied by this factor, and it’s more intuitive to leave it out. Until you get to I, leave $\varepsilon_{\text{photo}}$ out of the calculation (just write “x $\varepsilon_{\text{photo}}$” rather than multiplying through and getting a tiny number that gives you no intuition).

A. Cooking with a gas stove
B. Cooking with an electric stove
C. Cooking with a microwave (assume efficiency of the microwave itself is $\sim$65%, a typical number. The rest of the power doesn’t get into the food but is lost in e.g. turning the fan.)
D. Lighting (assuming you power your house with biofuels). Basically this is a calculation of how many photons you need outside to produce one inside. It’s also a measure of how silly (or sensible) a “vertical farm” would be.
E. Heating your house with a furnace
F. Heating your house with a heat pump with COP = 3.5. Here you can assume that the rated COP includes the losses of the electric motor that drives the compressor used in the heat pump (i.e. don’t include the small-electric- motor transformation; it’s already wrapped into the rated COP).
G. From your answers above, is a heat pump necessarily better than a furnace? What is the minimum COP you need to beat out a furnace?
H. (Optional) Come up with another activity involving energy transformations and chart it out and calculate its efficiency
I. (Optional) Think of some other process or technology that has been left off the grid, and add it.
Table 7  Efficiencies of Common Energy Conversions  
(percent)

<table>
<thead>
<tr>
<th>Conversions</th>
<th>Energies</th>
<th>Efficiencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large electricity generators</td>
<td>M → e</td>
<td>98–99</td>
</tr>
<tr>
<td>Large power-plant boilers</td>
<td>c → t</td>
<td>90–98</td>
</tr>
<tr>
<td>Large electric motors</td>
<td>e → m</td>
<td>90–97</td>
</tr>
<tr>
<td>Best home natural-gas furnaces</td>
<td>c → t</td>
<td>90–96</td>
</tr>
<tr>
<td>Dry-cell batteries</td>
<td>c → e</td>
<td>85–95</td>
</tr>
<tr>
<td>Human lactation</td>
<td>c → c</td>
<td>85–95</td>
</tr>
<tr>
<td>Overshot waterwheels</td>
<td>m → m</td>
<td>60–85</td>
</tr>
<tr>
<td>Small electric motors</td>
<td>e → m</td>
<td>60–75</td>
</tr>
<tr>
<td>Large steam turbines</td>
<td>t → m</td>
<td>40–45</td>
</tr>
<tr>
<td>Improved wood stoves</td>
<td>c → t</td>
<td>25–45</td>
</tr>
<tr>
<td>Large gas turbines</td>
<td>c → m</td>
<td>35–40</td>
</tr>
<tr>
<td>Diesel engines</td>
<td>c → m</td>
<td>30–35</td>
</tr>
<tr>
<td>Mammalian postnatal growth</td>
<td>c → c</td>
<td>30–35</td>
</tr>
<tr>
<td>Best photovoltaic cells</td>
<td>r → e</td>
<td>20–30</td>
</tr>
<tr>
<td>Best large steam engines</td>
<td>c → m</td>
<td>20–25</td>
</tr>
<tr>
<td>Internal combustion engines</td>
<td>c → m</td>
<td>15–25</td>
</tr>
<tr>
<td>High-pressure sodium lamps</td>
<td>e → r</td>
<td>15–20</td>
</tr>
<tr>
<td>Mammalian muscles</td>
<td>c → m</td>
<td>15–20</td>
</tr>
<tr>
<td>Traditional stoves</td>
<td>c → t</td>
<td>10–15</td>
</tr>
<tr>
<td>Fluorescent lights</td>
<td>e → r</td>
<td>10–12</td>
</tr>
<tr>
<td>Steam locomotives</td>
<td>c → m</td>
<td>3–6</td>
</tr>
<tr>
<td>Peak crop photosynthesis</td>
<td>r → c</td>
<td>4–5</td>
</tr>
<tr>
<td>Incandescent light bulbs</td>
<td>e → r</td>
<td>2–5</td>
</tr>
<tr>
<td>Paraffin candles</td>
<td>c → r</td>
<td>1–2</td>
</tr>
<tr>
<td>Most productive ecosystems</td>
<td>r → c</td>
<td>1–2</td>
</tr>
<tr>
<td>Global photosynthetic mean</td>
<td>r → c</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Energy labels:  c — chemical,  e — electrical,  m — mechanical (kinetic),  r — radiant (electromagnetic, solar),  t — thermal

From V. Smil
Physics & Geosci. Students recommended to do or at least to attempt #6 & #7

**Problem 6 (Optional): Describing the Carnot cycle.**

Following the discussion below, **derive P as a function of V for each of the segments in the Carnot cycle.** The relationship for the adiabatic legs is known in atmospheric science as “Poisson’s relationship” or “Poisson’s equation”.

Background: the gas in the cylinder is governed by both its equation of state, the ideal gas law

\[ PV = nRT \]

(where \( n \) is the number of moles and \( R \) the universal gas constant) and by conservation of energy. The heat input into (or removed from) the system must be accounted for by changes in the internal energy of the system or by work done on or by the system

\[ dQ = dU + dW \]

(The differential notation, e.g. “\( dW \)” is standard in thermodynamics; if it bothers you, you can rewrite this as a differential equation in time). Work is given by \( p*dV \), as we’ve used in class. A change in internal energy is manifested as a temperature change.

\[ dU = cv*dT, \]

where \( cv \) is the specific heat at constant volume (J/K). (This is in fact the definition of temperature).

Use some simplifying definitions in your derivation: the specific heat at constant pressure is related to that at constant volume by \( cp = cv + nR \) (in our units), and their ratio is defined as a term \( \gamma = cp/cv \). **Explain why \( cp > cv \).**

**Problem 7 (Optional): Deriving Carnot’s limit from the P-V diagram**

In class we used the T-S diagram to show that this engine had the ideal Carnot efficiency of \( \varepsilon = 1 - T_{\text{cold}}/T_{\text{hot}} \). With a little effort at integration you can do the same with the P-V diagram. **integrate P*dV around the cycle and divide by the heat input to derive the same Carnot efficiency.**

**Some hints for #5 and #6 are on the next page if you need them. When you finish, state whether you read the hints or not.**
Problem 7 hints

To get rid of the dT term, use the ideal gas law and take d(PV)

Problem 8 hints

Hint: The math for this is relatively simple – this is not an exercise in integration - but the problem can still seem confusing; the challenge is in thinking your way through to make it uncomplicated. Problem 6 was designed to help you do that. Appreciate how easy Carnot made this for you by his choices!

Hint: consider the adiabatic and isothermal legs separately for simplicity, i.e. first consider the two adiabatic stages, and then the two isothermal ones. Use your intuition to predict first the answer you should get from the adiabatic stages.

Hint: You do need to use some information about the adiabat in your final efficiency calculation. Remember you can always rewrite your Poisson’s relationship in terms of any two state variables (i.e. a relationship between P and V can become one between T and V)