Problem set #8
Due: Tues. April 24

Problem 1: Electrical resistance heating

While a heat engine is at best around 50% efficient at turning heat to mechanical work, electrical generators are as much as 99% efficient at turning mechanical work to electrical energy. Does that mean efficiency in generators is a non-issue? 99% efficiency means that 1% of the kinetic energy that goes to drive a generator is lost. In a heat engine or turbine, the lost energy is waste heat that remains in the working fluid and is carried away with the exhaust gas. In a generator, the lost energy is waste heat is produced by resistive heating in the coils of the generator’s wiring. The generator acts like a really big space heater.

A. Modern generators can convert > 500 MW of power. What is their rate of heat production? Give it in W and, for a visual comparison, as the equivalent number of space heaters radiating heat onto the generator. You can assume one space heater is about 1500 W.

B. Is this a problem? Come up with some proposed solutions if so. (Think in part B before Googling anything).

C. Now browse around to see what modern generators actually do, and discuss whether actual generator design is consistent with your strategies in B. You can look at anything, but for speed this interesting website, sort of a Craigslist for power generation systems, has details on individual generating systems.

http://www.powerplantsonline.com/steamturbinegenerator.cfm

Problem 2: Generating AC voltages

The power distributed to your household wiring is all AC (“alternating current”), and is produced by generators operating on similar principles to the rotating loop we discussed in class. AC generating systems produce voltage that rises and falls in a sine wave, as in the picture below. The voltage in turn drives an alternating current that moves back and forth at the same frequency.
The generator consists of a current loop rotating in a magnetic field, or, in modern generators, a magnet rotating between coils of wire. In both cases the magnetic field “captures” by the wire loops varies, producing a voltage.

Faraday’s law expresses this as (in our notation):

\[ V = \frac{d(A\perp \cdot B)}{dt} \]

where \( V \) is the voltage produced in the current loop, \( B \) is the amplitude of the magnetic field, \( A\perp \) is the area of the loop that is perpendicular to the magnetic field, and \( d(\ )/dt \) means a rate of change.

You can produce this voltage by the simplest possible generator, a single loop of wire rotating in the field of a permanent magnet:

If you want to look at a nice animation, see [http://www.walter-fendt.de/ph14e/generator_e.htm](http://www.walter-fendt.de/ph14e/generator_e.htm) and make sure that the “without commutator” option is set to make the simplest AC generator.

There are two options on this problem, one for those not comfortable with calculus/dot products/sines and cosines, the other for those who are comfortable

If not comfortable:

A. **Draw the system at various time points over the course of a single oscillation of period \( T \).** (For example, at \( t=0, \ t=\frac{1}{4} \ T, \ t=\frac{1}{2} \ T, \ t=\frac{3}{4} \ T, \ t=T \).) If you want to make the problem somewhat different (and more like real generators), draw a magnet rotating in a loop instead.

B. **Make a graph of both \( A\perp \cdot B \) and of the resulting voltage.**

Assume that you are now rotating the loop (or magnet) twice as fast, so that you can complete two full revolutions in your original period \( T \). If it helps, draw the system again (and add some more intermediate time points now that you’re going faster).
C. On your graph of $A\perp B$, add the new $A\perp B_{\text{faster rotation}}$.

D. On your graph of voltage, add the new voltage generated by faster rotation. Make sure that you draw its amplitude correctly to scale...Think carefully about what maximum amplitude you’d produce if you rotated twice as fast.

E. From this drawing, come up with an expression for the average of the absolute value of the voltage $|V|_{\text{average}}$. (The “absolute value” means ignore the fact that it goes negative – make the negative parts positive as well). Write your expression in terms of B, A, and the frequency of rotation (1/T). If it helps you think about this, consider only ¼ of the whole cycle.

For those comfortable with calculus/dot products/sines and cosines:

A. Write a formula for $A\perp B$ as a function of time
B. Write the formula for voltage as a function of time
C. Write the formula for for $A\perp B$ as a function of time with twice the rotation rate
D. Write the formula for voltage as a function of time with twice the rotation rate
E. Integrate the absolute value of the voltage to get $|V|_{\text{average}}$ (with your answer in terms of not the period of the oscillation but the frequency).

For everyone:

F. What will happen if you add more loops alongside one another? (As in, coil your wire multiple times instead of just looping it once). Remember the force that pushes the charges through the wire on each loop is proportional to the magnetic field and velocity of the loop, so every turn of wire gets its own “kick”. The purpose of this question to think, and we won’t grade too rigorously, but think about it. If you can, incorporate your answer into your formula from E – that is, write a new formulate for $|V|$ for when you have N loops.

Problem 3: Specifying AC voltages

The voltage in the U.S. electrical system is alternating current, switching back and forth 60 times a second, with a voltage specified as between 110-120 Volts. But what does that specified voltage really mean? Since voltage in an AC system varies from 0 to some maximum amplitude $V_{\text{max}}$ back down through 0 and then to a negative $-V_{\text{max}}$, the specified voltage is some single value that represents that whole pattern.
A. Could the specified voltage be the average voltage? What is the average voltage?

The specified voltage could be the maximum voltage $V_{\text{max}}$, or it could be the average of the absolute value of the voltage $|V|_{\text{average}}$ that you just calculated in problem 2. But it isn’t. Instead it’s the “root-mean-squared” voltage, i.e. the square root of the average of the square of the voltage:

$$V_{\text{rms}} = \sqrt{\langle V^2(t) \rangle}$$

Where the horizontal bar represents a time average.

B. Determine $V_{\text{max}}$, the peak voltage you get on your household 110-120 V electrical system, from your known $V_{\text{rms}}$. For anyone with calculus, integrate to find $V_{\text{rms}}$. For others, use the formula $V_{\text{rms}} = V_{\text{max}} / \sqrt{2}$

The diagram below describes a simple system, with current flowing from an applied voltage $V$ to ground through a resistive load $R$. (That resistive load could be a lightbulb, for example).

![Diagram of a simple electrical system with current flowing from an applied voltage $V$ to ground through a resistive load $R$.]

The power dissipated in the resistive load is the product of current and voltage:

$$P = I \cdot V$$

where current $I$ has units of charge/time and $V$ has units of energy/charge, so that $I \cdot V$ has units of energy/time, or power. The current that flows in a resistive system is proportional to a voltage drop (Ohm’s law: $\Delta V = I \cdot R$, where $\Delta V$ is the voltage drop).

C. Write an expression for the power dissipated in the resistive load in terms of both $I$ and $R$ and also in terms of $V$ and $R$.

D. Why do we use $V_{\text{rms}}$? What’s so special about squaring the voltage?

E. If you hooked the generator of problem 2 up to some resistive load $R$, you’d have a means of turning kinetic energy into electrical energy into heat. Using your insight from the problems above, write an expression for the power output...
F. If the generator is putting out constant current and constant voltage (as you’d hope in the modern electrical system), and the resistance of the load decreases, will the power output of the generator increase or decrease?

G. (Optional, worth thinking through even if you don’t answer) Imagine a system of single generator being turned by a turbine. The turbine can only put out so much power. What happens if you change the resistance of the electrical load on the system so that it “wants” more power than the turbine can put out? What changes? (Because something must change, if the turbine can’t give any more power. This is the same as asking, what happens to our electrical system if everyone turns on their lights at the same time?)

H. (Optional, ditto as above). What happens if you change the resistance of the system so that the load “wants” no power? (This is the same as asking, what happens to our electrical system if everyone turns out the lights at the same time?)

Problem 4: Power draw of household appliances.
Heating due to resistance in wires is usually not a problem in household wiring – the wiring is designed so for typical electricity use it wouldn’t be. If however you tried to pull huge current through your household wiring, you could produce enough heating in the wires to be a fire danger. For this reason all U.S. houses have built-in protection systems to prevent too much current from flowing and melting the wires.

The wiring in your house is arranged in several individual circuits, each of which has thick enough wire to safely carry 15A or sometimes 20A of current, and each circuit is connected to a fuse or circuit breaker that will trip and cut off all current from flowing if you try to carry any more than that. Probably at some time you have plugged in too many appliances and blown a breaker – nearly everyone has had that experience.

A. Calculate how many space heaters or hairdryers you can plug in to a single household circuit before you blow a breaker.

Info: a space heater or hairdryer typically draws ~ 1500 W of power.

Note that to use an electric stove, which draws more power than hairdryer, you have to have a separate, high-current circuit installed for it.

B. For extra credit (VERY optional), test your hypothesis and report the results.

Make sure you know where your breaker box is first! And make sure you have breakers rather than fuses, which are more of a pain to change. Resetting a breaker is just flipping a switch, so there is no harm done by this experiment, but you don’t want to waste time hunting around for the box with a flashlight.
C. The power consumption of the average American is 10,000 W, and you saw from the “spaghetti” chart in class that about 1/3 of that power is transmitted as electricity. **If you were designing a household electrical system, what would you set the maximum current draw per inhabitant of your house/dorm/apartment per person to be to be safe?** This is just a judgment call – there is no single answer – but explain your reasoning.

D. **For extra credit (optional), find the breaker box in your apartment, dorm, or house, photograph it and attach the photo.** How many circuits do you have? What is the total current draw your building can have before blowing the main breaker? What is the maximum current drawn per inhabitant of your building? Is that consistent with C? Is the main breaker equal to the sum of all the individual circuits, or could you trip the main even while not overloading any individual circuit? If you are in a dorm you can probably not get access to the breaker box unsupervised, but you can possibly talk to the facilities/maintenance personnel and persuade them to take you down to look at it. If it’s for a class, and if you show interest in their responsibilities and their job, people are often very willing to help.

**Problem 5: AC motors and generators**

We discussed in class that every motor can be a generator and every generator could be a motor. AC motors – any motor that you plug into wall power – are based on the same principles as the AC generator of problem 2.

First, go back to the shopping page for generators [http://www.powerplantsonline.com/steamturbinegenerator.cfm](http://www.powerplantsonline.com/steamturbinegenerator.cfm) and compare the generators’ mechanical rotation rates and the frequencies of the power they put out. Typically rotation rates are given in rpm (revolutions per minutes) but the electrical current they produce is specified in Hz (oscillations per second).

A. **For some reasonable number of generators (as many as you need to pick out patterns), write down their rotation rates and electrical frequencies, converting everything to Hz for consistency. Is there a pattern?**

B. **Do the same exercise for motors.** Here you have to Google to find some, but there are many online. (You can e.g. Google with the “shopping” tab) for AC motors that run off of ordinary household power. Make sure the motors you choose have no gears and aren’t “AC/DC” motors. The only electrical frequency you can find should be 60 Hz. The voltages should be 110-120V for ordinary wall power or, for three-phase motors, something like “208-230”. **Again, write down the rotation rate and electrical frequency for a selection of AC motors, in Hz. Does the same pattern apply that you found in A?** Can you get an AC generator that turns at any speed you want?
C. If the AC motor and generator rotation rates don’t seem to behave purely like the simple loop of problem 2, then **try to design a modification to the simple loop generator that would produce this behavior. How would you get the rotation rates you see?** (This will not be graded rigorously, but do think hard, especially if you intend to come to lab on Tuesday before class. Optional + for inferring the solution. Googling/background reading is OK and indeed welcome if it comes with thought.).

Note: AC motors come in two types, synchronous and induction, and induction motors have a slight frequency lag which can complicate patterns, but you should still be able to pick something out. The big AC generators will all be synchronous.

Problem 6: DC motors and generators

The primitive motor you built in lab with a 9V battery and paperclips was a DC motor – the current from the battery flowed only in one direction, from one terminal to the next.

A. Go online shopping for DC motors, as you did above. For DC motors there’s no electrical frequency involved, since they’re driven by direct current (as from a battery), but **write down a selection of different rotation rates you can buy. Is there a pattern? Comment on the differences you see (or don’t) in AC vs. DC motors.**

B. Go back to the animation of the AC generator at [http://www.walter-fendt.de/ph14e/generator_e.htm](http://www.walter-fendt.de/ph14e/generator_e.htm) and now click the “with commutator” button to turn your AC generator into a (sort of) DC generator. **Describe what the commutator is doing and why switching from the slip ring (“without commutator”) to the split ring (“with commutator”) changes the generator output.**

C. The “DC” output this generator producing is actually pretty bad. If you want direct current from your generator, what you want is the current and voltage to be constant, or as constant as you could reasonably ask for. This output is swinging up and down from a maximum value all the way to zero. It’s got to be possible to do better than that (and in fact DC generators and motors do do much better than this). **How could you make a “better” generator that would give you smoother output, just by adding more windings of wire? Describe.**

Again, outside reading is OK if it goes with thought beforehand – as in, think first and Google second. If you read outside to figure out the answer do report what sources you looked at. The solution is identical for motors and generators, and we will look at DC motors in the lab next week and you can see how they are designed to produce much better DC.
Problem 7: (Optiona). Motor dissection.

If you have something electrical that has moving parts, it will have a motor in it. If you have something with a motor that is broken or that you don’t mind sacrificing, bring it into class and/or lab. If you are coming to class but not to lab, extract the motor first to show people. (Even better, get the motor apart so that you can see the inside). If you are coming to lab, it’s OK to bring in the whole object and you & others can dismember it during lab to find and inspect the motor.