GEOS 24705 / ENST 24705 / ENSC 21100
Problem set #7
Due: Th. Apr. 21

Problem 1: History of efficiency of engines

The early engine-men, or "engineers" as they were soon called, were very interested, as you've seen, in tweaking their engines for maximum "duty" or, as we would say it, efficiency. In PS 6, you looked at the historical growth of steam engine efficiency. Gains were steep at first but then efficiency seemed to be leveling out. By 1825, more than a hundred years after Newcomen's engine, Carnot was engaged in developing the theory of the maximum limiting efficiency of heat engines. We'll continue that derivation on Thursday (and below), but before we do, you can get some perspective by looking at efficiencies in modern engines.

Below are two figures. First, the figure from Smil that shows the history of steam engine efficiency. The data stop in the 1950's, because at that point the steam engine was largely abandoned in favor of the internal combustion engine. In this problem you'll estimate the efficiency for two parts of the modern energy system: the internal combustion engines that power our transportation, and the steam turbines that make most of our electricity. (A steam turbine is still technically a heat engine, but it's not an engine of the type we've discussed, with pistons and cylinders.)

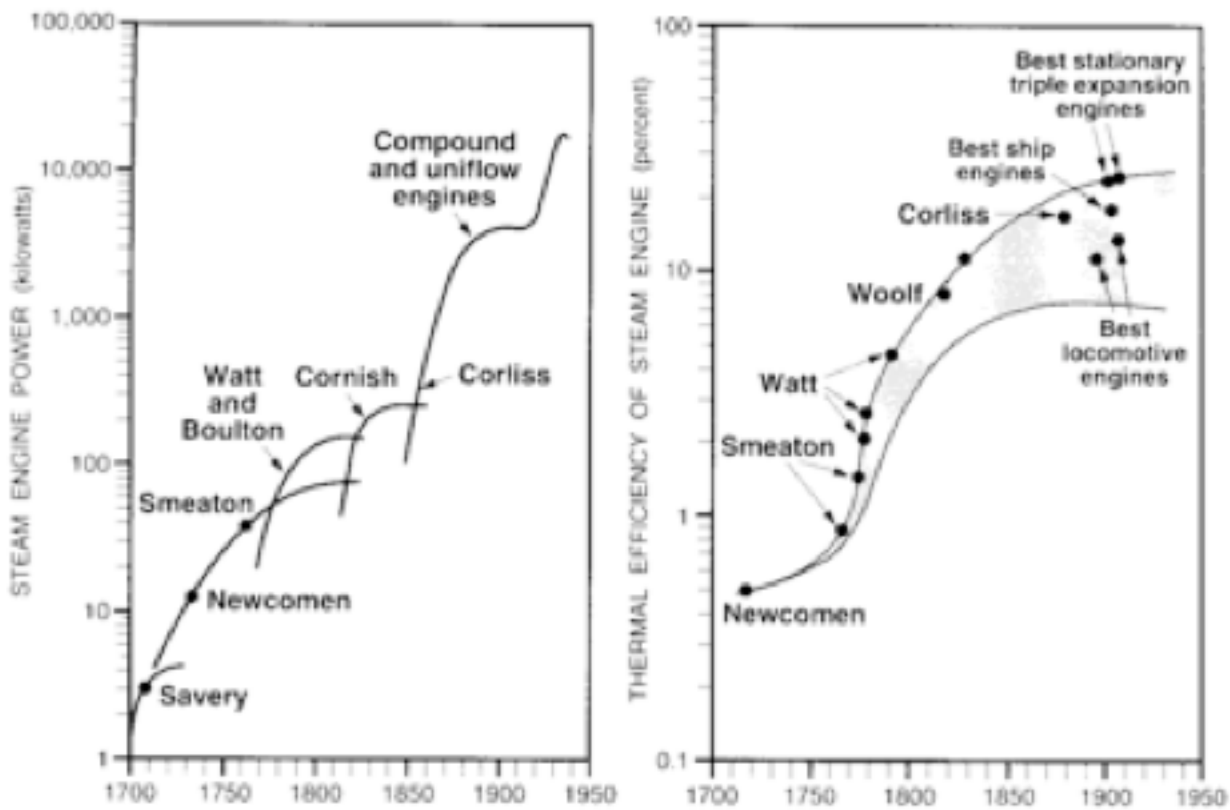
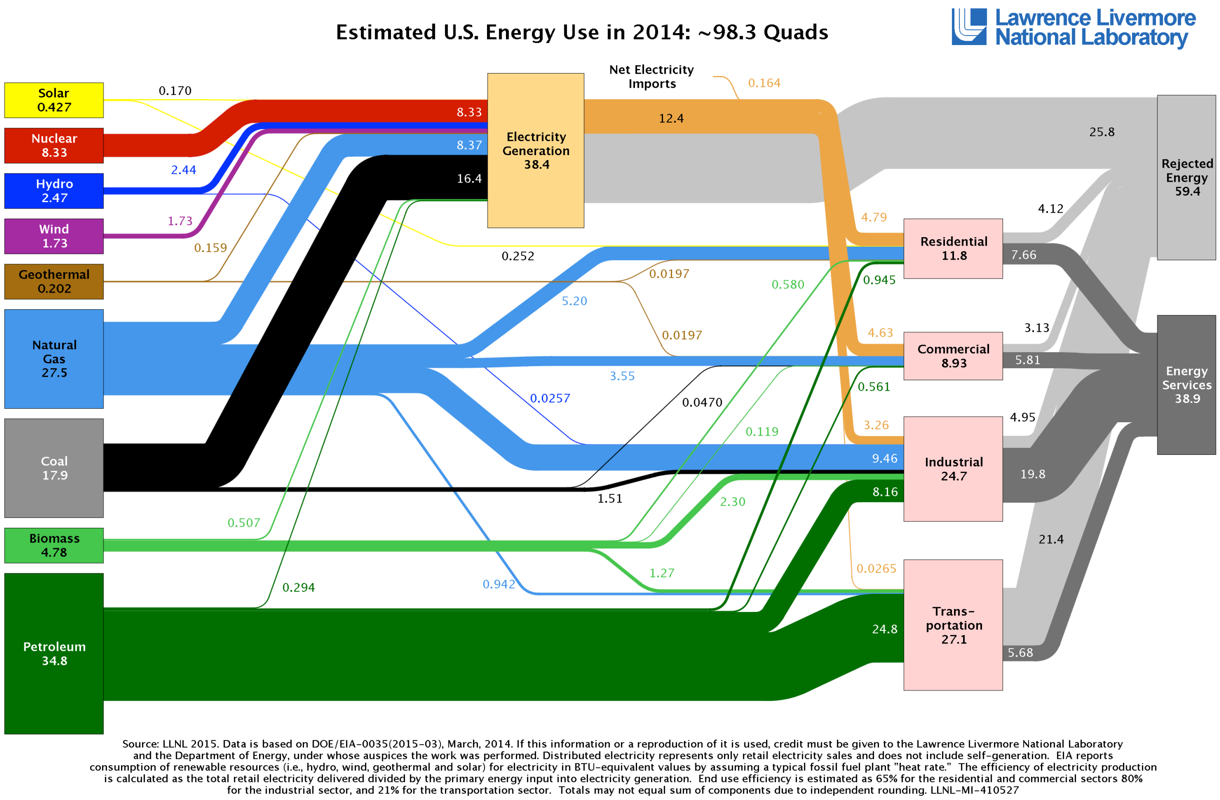


FIGURE 5.3 The rising power and improving efficiency of the best steam engines, 1700–1930. *Sources:* Plotted from data in Dickinson (1939) and von Tunzelmann (1978).

From V. Smil

The second figure is a "Sankey diagram" showing energy flow in the U.S. human energy system. The Department of Energy laboratory at Lawrence Livermore puts one of these out each year; I'm giving you 2014. The left side of the Sankey diagram shows "primary energy sources", using mostly the same convention for "primary" as we did in our problem using numbers from Braudel[1]. The thicknesses of the lines are proportional to the size of the energy flow. The right side of the diagram shows final uses of energy after the various transformations it undergoes in the energy system. The dark grey is energy that we get out in forms that we want. The light grey is energy that gets converted into heat that we don't want and can't use. It is basically energy wastage.



You can learn a lot by looking at a Sankey diagram. For now you need not worry about the actual units. (A "Quad" is a unit of energy; sensibly this figure uses units that produce values in the 1-100 range.)

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[1] Defining what is meant by a "primary"energy flow involves somewhat arbitrary choices. You saw this already in the problem where you calculated European "primary" energy uses with numbers from Braudel. Both our problem and the Sankey diagram consider as "primary" energy the chemical energy from fossil fuels and biomass, which gets liberated as heat with 100% efficiency. The Sankey diagram analogously considers "primary" the heat liberated by nuclear reactions in nuclear reactors (which again is converted from nuclear energy with 100% efficiency). In the Braudel problem we also included the biomass used to feed human and animal labor; that's left out of the Sankey diagram probably because human and animal work is negligible now relative to the rest of our energy system. In both cases it's awkward deciding how to book-keep energy derived from wind and hydro (and solar). In the Braudel problem we counted as primary the mechanical work done by windmills rather than the kinetic energy of the wind itself. Similarly, the Sankey diagram counts as the primary energy flow the electricity made by renewables, not the kinetic energy of wind, the gravitational potential energy of falling water, or the radiation energy of the sun falling on the solar panel.

Questions

A. **What is the overall efficiency of the U.S. energy system?** That is, what fraction of our primary energy flows can be extracted ultimately in forms that we want?

B. **What fraction of U.S. energy use comes from fossil fuels?**

C. **What fraction of U.S. primary energy flows go through a heat engine?** Here just count transportation, which is all engines, the steam turbines used to make electricity discussed above, and natural gas turbines (all use of natural gas to make electricity is also a heat engine).

D. **What is the mean efficiency of the engines in the transportation sector in 2014?**

E. **What is the mean efficiency of turbines used to make electricity in 2014?**

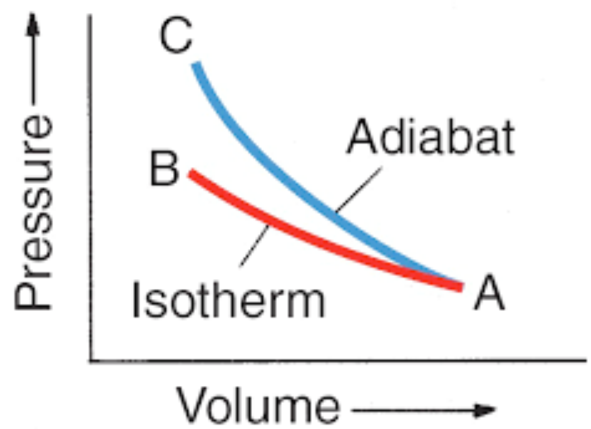
F. **Add your datapoints in D and E to the Smil diagram.** Discuss. Was the switch to internal combustion engines driven by a desire for better efficiencies?

G. ***(Optional).*** **Use the data in this figure to calculate the per-capita U.S. power use.** Give your answer in Watts/person. First write down what you expect that number to be, based on class discussion. The calculate. One quad is 10^{15} British thermal units (BTUs). One BTU is 1055 Joules. You can look up the 2014 U.S. population.

Problem 2: Limits to efficiency of heat engines - Carnot's theory

In class on Tuesday we started going through Carnot's arguments that let him propose a limiting efficiency for heat engines. We discussed that any heat engine is a mechanism that transfers heat from a hot reservoir to a cool one, but extracts some of that heat flow as mechanical work. We also talked about Carnot's assumption that for the most efficient engine, you should add heat only at the hottest temperature and remove it only at the coldest temperature. That requirement immediately let us derive the four stages of the "Carnot cycle" without any math at all. The Carnot cycle must consist of a stage at a constant temperature T_{hot} during which heat is added and a stage at a constant temperature T_{cold} where heat is removed, and those two stages must be connected by two additional stages during which NO heat is added or removed. Any process that does not involve adding or removing heat we call "adiabatic", so the Carnot cycle must consist of two "**isotherms**" and two "**adiabats**".

We then thought about the **shape** of isotherms and adiabats on a P-V diagram. From the ideal gas law PV = nRT, we realized that the isotherm (with T constant) is a curved line with $P \alpha 1/V$. We don't know the equation for an adiabat, but we could get insight by a mental experiment. Imagine yourself standing over an insulated cylinder (something like a thermos with a piston in it), and squeezing that piston downwards. People's insight concurred: compressing the gas inside the cylinder would make it hotter. (Similarly, if you punch a hole in a bottle of compressed gas and let it escape rapidly, too rapidly to exchange heat with its environment, the escaping, expanding gas will get colder.) In physics jargon we'd say "adiabatic compression" raises the gas temperature and "adiabatic expansion" lowers the gas temperature. The class then considered side-by-side experiments where we compress two cylinders (both going from $V_1 \rightarrow V_2$), but one case is insulated and adiabatic while the other is done isothermally ($T_2 = T_1$). In the adiabatic case, we just decided that compression makes the gas hotter, so $T_2 > T_1$. Since the ideal gas law holds at all times (that is, PV = nRT), and both cylinders are squeezed to the same volume V_2, and the adiabatic case has a higher final temperature T_2, it must necessarily also have a higher final pressure P_{2a} than in the isothermal case. We have thus concluded something about the shape of an adiabat on a P-V diagram! We know that it must be *steeper* than the isotherm, i.e. to produce a greater P change per V change. (I'll give deriving the exact adiabat shape as an optional problem.)



Let's now think about **heat flow** and **work** during adiabatic and isothermal process. First compare the adiabatic and isothermal cases in the figure above. Everyone should be able to look at the picture above and think, hmm, if the gas at C is hotter and higher pressure than the gas at B, it must somehow have more energy. But the adiabatic compression, by definition, didn't involve any transfer of energy from the outside environment. The gas in the adiabatic cylinder could not have gained energy as it was compressed from A to C. There's only one way to reconcile these facts: the gas in the isothermal case had to LOSE energy to its environment as it was compressed from A to B. The energy loss allowed it to avoid the temperature rise of the adiabatic case.

But now think about the weirdness of the adiabatic case. The gas being compressed adiabatically got hotter, even without heat being added! How is that possible? Because the temperature of the gas is a measure of its "internal energy", and the laws of conservation of energy allow energy to be converted back and forth between work and internal energy. The usual notation for describing energy conservation in an ideal gas is:

$$\underset{\text{(heat)}}{dQ} = \underset{\text{(work)}}{p\,dV} + \underset{\text{(internal energy)}}{C_v\,dT}$$

Here the "d" notation means a "differential", a small incremental change. It's standard notation in thermodynamics but be aware that some mathematicians and physicists hate it. Q heat, p is pressure, V is volume, and C_v is the specific heat[2] of the gas itself, the energy required to change the temperature of the gas a given amount (in units of J/K)[3]. Basically this expression says that if you add

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[2] The subscript "v" is there for a reason. It means this is the energy required to change the temperature of the gas if there were NO change in volume. That is, it's the energy that would go ONLY to changing internal energy and NOT to doing work.

[3] We talked about specific heats for liquid water earlier in class, but the units were different. I said that the specific heat of water, that we called just "c", was actually the definition of the calorie: one calorie is the amount of energy required to raise 1 gram of liquid water by 1 degree Celsius (or Kelvin). That means the units of specific heat are J/g K, i.e. c is a specific heat *per mass*. That's actually the normal definition for gases as well – we'd usually write

$$dq = p\,dv + c_v\,dT$$

where q is heat per mass, c_v is specific heat per mass, and v is the volume that would be occupied by a given mass, i.e the inverse of mass per volume, which we know as density. So $v = 1/\rho$.

some energy (heat) to a gas, that energy can either go to raise the gas temperature (the C_v dT term) or it can go to doing work (the p dV term). In the adiabatic compression, you had no heat flow (dQ = 0) but you personally were doing work on the gas as you pushed the piston down. That squeezing effort you made produced a negative volume change dV. To keep dQ=0, that negative term has to be balanced by a positive dT term – that is, by a rise in temperature. The temperature rise in an adiabatic compression is a necessary consequence of the conservation of energy. If you do work on a gas by squeezing it, that energy has to go somewhere, and it goes into the internal energy of the gas. Realize the subtlety: "adiabatic" means no heat flow to or from the environment – but the gas can do ***work*** on its environment, or have work done to it.

The same conservation of energy explains the cooling when you burst a cylinder of compressed air. As the high-pressure gas expands into a larger volume, it does work *on* its environment (the differential pressure p is positive and dV is positive). The energy lost to that work must be compensated for by a drop in temperature (a negative dT term). This all sounds logical, but might make you feel a bit uncomfortable. In this example the expanding gas is pushing on its environment, and it's taking the energy for that work directly from its own thermal energy. But, didn't I just argue strongly in class that it wasn't possible to borrow heat to do work? Is this a violation of the 2^{nd} law of thermodynamics?

The key thing to realize is that to build an actual engine, you have to make a repeating cycle. It's not a violation of the 2^{nd} law that you borrow some thermal energy from a compressed gas to produce work as it un-compresses. It would be a violation of the 2^{nd} law if you got work from thermal energy and left the system otherwise exactly as you found it. But in this case you would have to "put work back" in to re-compress the air and re-heat it. Both of the "dumb engines" that we considered in class were systems of this type. You could get work out during part of the cycle, but you then had to put the same amount of work back. Both systems were useless as engines. (Instead, they were systems that let you store energy in compressed air.) In the adiabatic case, no heat flowed at all – you just swapped energy between the work term and the internal energy term. In the isothermal case, energy flowed out of the cylinder into the environment on the downstroke, but an equal amount flowed back in on the upstroke.

Carnot knew, from looking at real engines, that people could in fact get net work out of a repeating-cycle engine as long as there was a net heat flow. If he integrated the conservation of energy equation all the way around a cycle

$$\oint dQ = \oint p\, dV + \oint C_v\, dT$$

then the third tem MUST be zero, because the system returns to its original temperature, so there's no net change in internal energy. But Carnot could get net work produced: the term $W = \oint p\, dV$ need not be zero. That net work would equal $\oint dQ$, the net heat flow. That is, the net work is the part of the original heat flow into the engine that DIDN'T flow back out again. In our terminology in class,

$$W = \oint dQ = Q_{in} - Q_{out}.$$

The question that Carnot pondered was, how MUCH work can you get out of the whole cycle for a given heat flow in? What is the limiting efficiency: $e = W / Q_{in}$? In this problem you'll write out the parts of Carnot's logic that we discussed in class, and you'll go a little bit forward in the derivation.

Questions

A. Draw the four stages of the Carnot cycle. That is, for each stage, draw the cylinder and piston and explain what is happening during this stage of the cycle. Then, on a single P-V diagram, draw each of the four stages and clearly label it.

B. On the P-V diagram, indicate for each stage if/when heat is being added to the gas and if/when heat is flowing out of it.

C. Draw each of the four segments of the P-V diagram separately, and for each one, indicate if work is being done on the gas, by the gas on the environment, or neither. Since work is the integral $\int p\, dV$, it's just the area under each segment on the P-V diagram, so shade in that area, and think about (and mark) when that work is positive and when negative. Discuss. Can you convince yourself that there is more net work done by the engine than on it?

D. (Optional) In the footnote I said that C_v was the specific heat, the energy required to raise the gas temperature one degree IF the volume of the gas does not change. Think of this as the heat you'd add to gas in a fixed, static box of gas while watching a temperature readout. But what if you do the experiment not with a fixed box but with a cylinder with a piston in it, whose volume can change? **Would you need to add more or less or the same heat to raise the temperature of the gas by one degree if volume is not constant?** Discuss. For those who remember some physics, explain what this question has to do with the quantity termed C_p, the specific heat at constant pressure.

E. (Optional) **Compare the specific heats of liquid water and of air.** Air is considered a good insulator, while water is a terrible insulator. You can run naked around Hyde Park on a cool (50 F or 10 C) Fall day and you'll be just fine, as long as the police don't apprehend you, but jump naked into Lake Michigan when the water is 50 F and you'll die of hypothermia very quickly. Does that meant the specific heat of water is much much greater than that of air, i.e. that it takes more energy from your body to warm up the water around you, so that you lose heat and die? Look up the specific heat of air (use c_v) and discuss.

F. ***(Optional)*** **Derive the P-V relationship for the adiabatic segments of the cycle.** (For an isothermal leg, the ideal gas law is all that you need, and the "rule" is just P*V = constant). If T is not constant then the ideal gas law isn't enough to tell you how P, V, and T all evolve together. You need another equation. But, you have a second equation here: the conservation of energy equation $dQ = p\, dV + C_v\, dT$. That's an equation about changes (dV and dT); how do you use it in conjunction with the ideal gas law? Try differentiating the ideal gas law (in cheap thermo notation where you aren't taking a derivative with respect to anything but are just writing a differential). Then substitute in to get a single equation in just P and V, then integrate.