Water and wind turbines

Problem 1: big-dam hydro with Francis turbines

Consider Hoover Dam, one of the signature pieces of American engineering. Basic stats on the dam, reservoir (Lake Mead) and power plant are:

- rated power: 2080 MW
- dam height: 221 m
- dam width: 380 m
- Lake Mead volume: 35 km$^3$ (when full)
- Lake Mead surface area: 640 km$^2$
- Lake Mead max length: 180 km
- Number of turbines: 17
- Inlet diameter for each turbine: assume ~ 4 m

Remember the Bernoulli equation for water:

$$\frac{1}{2}v^2 + gh + \frac{p}{\rho} = ct. \text{ (incompressible fluids)}$$

A. When the power plant is running at full power, what is the flow out of the dam? Give the flow $Q_m$ in kg/s but also convert to a volume flow rate $Q_v$ in m$^3$/s.

B. What is the “turnover time” of water in Lake Mead, if the plant runs at full power continually? (i.e. how long does water stay in the reservoir before it is flushed out?) How many times is the lake effectively re-filled in a year?

C. How fast is water moving when it enters the Francis turbines?

D. Is your answer in C. consistent with your idea of how a good reaction turbine would be designed? Would you prefer a high or low velocity? Explain.

E. How fast would water be moving if you disconnected the generators from the grid and let the water flow freely out? (Assume it would then flow out at maximum velocity, since there is no impediment to flow that keeps pressure high in the turbine).

F. Would the water continue to exit at that velocity as the reservoir drains? Explain.

G. Optional: If it did continue to flow at that velocity, how long would it take for Lake Mead to be drained if the generators are disconnected?

H. Optional: Given assumptions of E, write down the equation for the drop in water height in Lake Mead over time. Solve the resultant differential equation. How long would it take Lake Mead to empty by 90%?
Note: you have to assume some shape for the reservoir behind the dam. One option is to assume that the reservoir is rectangular and flat-bottomed, i.e. \( V = A \cdot h \). Another option (which sounds better, but also proves a bit unrealistic) is to assume the reservoir is a half-cone of radius \( h \) and length that shrinks as the reservoir dries up.

I. Optional: We have previously calculated the W/m^2 for hydro, considering the area to be the entire catchment area on which rain falls (.03 W/m^2 if you have a single dam per river but caught every raindrop on earth in it, 0.3 W/m^2 if you terraced every river completely with dams). People in class have pointed out that you can use land for multiple energy-generation strategies, and the actual area taken up by rivers and reservoirs is quite small. Perhaps for completeness we should consider land requirements of hydro that involve no alternative uses, i.e. the land consumed in reservoirs. So, what are the W/m^2 for Hoover Dam/ Lake Mead hydro with this definition?

Note that this number doesn’t tell you how much hydro you can extract worldwide, but it does give you an idea of you how much land you have to drown to produce a given amount of hydropower.

Problem 2 – do one of the following (either 2A or 2B). Problem 2A should be quicker for those who don’t have a lot of physics background. It should be a half hour of reading. All Physics majors should do at least the first question in 2.2 (2.2 A), even if you’re choosing to do 2.1, because this is the first and only time in this class we get to think about reference frames. So, that’s non-optional for the physicists. Obviously doing both problems gives extra credit.

Problem 2.1: Hydro in developing countries

Ethiopia, one of the poorest countries on Earth, with a correspondingly low energy use per capita, has essentially no fossil fuel-fired electricity generation. The vast major of its power comes from hydro, and that percentage will rise, because the country is in the process of damming its Omo River with a series of dams, collectively known as the Gilgel Gibe project. (Estimates are that when Gilgel Gibe is done, 96% of Ethiopia’s electricity will be hydro).

Are the dams a good idea? Funding for the dams comes from outside sources. There are obvious points of concern, including that the construction contract was given to a well-connected Italian firm as a no-bid contract. It is rumored that much of the power is already contracted to be sold to neighboring (and better developed) Kenya and Sudan. That might be a good thing, if it brings in needed cash, or a bad thing, if revenues go directly into officials’ pockets.

Do some reading, answer a few questions, and come to a decision (if you can) on the worth of the dam. Is the project worth it?

Readings: glance through the Voith-Siemens brochure on the project that is posted on the class website, mostly for drawings of the giant Pelton wheels. Skim (extremely lightly) through the negative report from a group called “Counterbalance” linked below. Most importantly, read through an article in the Ethiopian journal Tadias, skim the three articles it links to, and make sure to read the many comments by Ethiopians. You need not read any of this in detail, and
especially don’t spend too much time on the long Counterbalance report. This is just to get a feel for the arguments.

**Tadias article, links, and comments**

http://www.tadias.com/05/14/2009/ethiopia-big-dam-bigger-problems-debating-gilgel-gibe/

“**Counterbalance**” report: http://www.counterbalance-eib.org/component/option,com_datsogallery/Itemid,86/func,detail/id,44/

A. From information in the Nazret.com article linked to from Tadias, what is the per capita electricity consumption of the average Ethiopian? The article gives it in kWh/yr; convert to W.

B. Compare this to the U.S. per capita electricity consumption. (You can assume that 1/3 our primary power consumption goes to electricity, as we saw on the “spaghetti” chart of U.S. energy use, and that electricity production is ca 40% efficient).

C. What is the capital cost of Gilgel Gibe II and III, in $/W, from the Counterbalance report (p. 7)? Is that reasonable? (See figure in hydro lecture slides if you want a comparison)

D. Do you have an opinion on the project? Express it if so. If you’re of mixed mind, say that. Just a few sentences explaining your position or confusion.

**Problem 2.2: Pelton wheels for high-head hydro**

Pelton wheels are used only in low-flow, high-head situations. Why? The low flow criterion occurs because they Pelton wheels take up so much space for a given flow. In the Francis turbine, water fills the entire turbine volume, but the air-filled Pelton wheel enclosure must be many times larger than the volume occupied by the flow.

The high head criterion occurs because it’s difficult to build a wheel to produce AC power at 60 Hz unless you have high head; at low head any reasonably sized wheel won’t spin fast enough. This problem asks you to show this.
The Pelton wheel is driven by a jet or jets of water that strikes the vanes (often at the top or bottom of the wheel). The jet bounces off the vane, imparting the momentum of its water molecules and pushing the vane forward. But because the wheel is already rotating, the vane isn’t stationary when the jet hits it, but is moving at some velocity \( v \).

The optimal power extraction from a Pelton wheel occurs when the linear velocity \((v)\) of the wheel at the vanes has some specific relationship to the velocity of water coming out of the nozzle \((v_{\text{nozzle}})\).

**A.** For optimal power extraction from the Pelton wheel, what is \( v \) in terms of \( v_{\text{nozzle}} \)?

*Two additional pages posted give (Hint 1) the relationship you’re trying to prove and (Hint 2) suggestions on proving it. Use either or both of these if you want, but state what hints you use. If you can’t prove this but want to use the relationship and still do the rest of the problem, that’s fine too, just say so.*

**B.** Design a Pelton wheel generating system to produce 60 Hz electricity from a 10 m high dam.

That means computing \( v_{\text{nozzle}} \), determining the optimal \( v \), determining the dimensions of the Pelton wheel to produce that \( v \) (for the rotation rate you’ve chosen), and choosing the type of generator (and hence rotation rate).

First, try using the simplest possible AC generator, a 2-pole synchronous generator, so that the Pelton wheel will rotate at 60 Hz. How big a wheel will you need?

**C.** You might decide that this is impractical for serious power generation – you’ll never be able to put serious flow into a wheel that size. Now try to go with as many poles on your magnet as is normally handled, about 24. Now how big is the Pelton wheel you’d design? Is this practical?

**D.** What if the head were 1000 m instead of 10 m? What size would your Pelton wheel be now? Is this practical?

**E.** Some Pelton wheels can have as many as 6 jets. Does the size of the wheel depend on the number of jets you add? What is the effect of adding more jets?

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**Problem 3: Can wind power the world?**

In class we discussed our target for finding a power source that could run the whole world in the future and let everyone live like Americans. The criterion we picked was that energy flux had to be 10 W/m\(^2\) or better.

Can wind get us there? In this problem you’ll design a wind farm and calculate the energy flux it produces.
Notes: remember the power carried as kinetic energy by any moving fluid:

\[ P = \frac{1}{2} A \rho v^3 \]

where \( A \) is the cross-sectional area considered, \( \rho \) is the density of the fluid, and \( v \) is the velocity. Remember that Betz’s law limits the power extracted even by the ideal free-stream wind of hydro turbine to 0.59 of the power carried. In practice even the best wind turbines don’t quite reach this limit, but top out at about 0.5. And the wind doesn’t blow all the time – the “capacity factor” for wind turbines is usually around 0.3, for an overall efficiency of ~ 0.15.

We also discussed briefly the spacing of windmills. Because each windmill disrupts the velocity field of the wind, they can’t be placed right next to each other. The rule of thumb is to place them at least 3 rotor diameters apart along the direction facing the wind, i.e. to leave room for two other wind turbines in between any two that you build. And the velocity field downstream of a wind turbine takes a long time to recover. The engineering rule of thumb is to leave 10 rotor diameters behind a turbine before you put up another one.

Finally, you need to pick a reasonable wind speed to use as your “\( v \)” in computing power. If wind is to be scaled up, we’ll run out of the best and windiest sites. A reasonable number for \( v \) for wind on a large scale is 7.5 m/s.

A. How much power/area can you extract from a wind farm? Draw a diagram of a wind farm layout and wind turbine spacing, and derive power/area in W/m². Does your answer depend on the size of the turbines?

B. For a 500 MW wind farm (the size of the Crawford generating plant), how much area would you need?

Problem 4: Pumped hydro storage

A dam serves two purposes: 1) it stores a large volume of water that can be released through the turbines as desired, and 2) it builds up depth in the stored water and thereby increases energy density in the water. But those purposes can be separated, and you can get the benefit of high head without building a dam. If you build a reservoir on top of a mountain, a pipe leading down the mountain from your reservoir will build up the pressure of the column of water in the pipe itself, i.e. its pressure head will be due to the height of the mountain. You can’t store as much water on a say 1000 m mountain than you could behind a 1000 m dam, but a mountaintop reservoir is a way to get high head cheaply if you don’t need big storage.

Of course a huge drawback of mountaintop storage is that, well, streams don’t flow uphill. If you want to get water to the top of the mountain, you have to pump it there. That means you can’t even get back all the energy that you put in to it, much less gain anything. Mountaintop reservoirs are therefore useless for power generation. The are very useful however for storage of energy. Think of a mountaintop reservoir as a big battery. And energy storage is a need that comes with alternative power sources like wind and solar that are intermittent. The wind
doesn’t always blow (or sometimes blows too hard), and the sun doesn’t always shine.

In this problem you’ll design a pumped-hydro system to back up the wind farm you designed in the previous problem. Assume your wind farm is 500 MW and that you need storage capable of replacing all the wind turbines for a week if the wind quit blowing. Assume you have a 1000 m mountain to build on, and that it’s practical to dig your reservoir 50 m deep. The total efficiency of pumped hydro storage (which requires the conversions electricity -> gravitational potential via a pump -> pressure head-> spin a turbine and make electricity again) isn’t 100%, but in good modern systems can reach 80%.

Remember that, unlike in a dam system, in a mountaintop reservoir you don’t lose much energy density as the water level in the reservoir drops, since most of the pressure head is caused by the height of the mountain, not by the depth of water in the reservoir. (In fact, for the purposes of this problem, you can ignore changes in height as you drain the reservoir).

A. What volume of water will you need in your reservoir?

B. What is the area of the reservoir?

C. Compare to your answer in 3B. Is adding pumped hydro to your 500 MW wind farm a significant extra requirement in area?

D. What are the effective W/m² of wind power, if you now require every wind farm to provide backup energy storage in the form pumped hydro, i.e. to require additional area for energy storage?

E. (Optional). Pumped hydro is also used by energy “wildcatters” who make money by buying power at night when demand is low and prices are cheap, then selling it back to the grid during the day when demand is high and prices are high.
Write down an expression that describes when pumped hydro is a profitable strategy, involving the cost ($/J) of buying power at night ($C_{\text{night}}$), the cost ($/J) of selling it during the day ($C_{\text{day}}$) and the efficiency of the pump/turbine system ($\varepsilon$).

**Problem 5 (Optional): Why wind turbines need gearboxes**

Most wind turbines in the world are directly connected to the grid. Unlike steam or natural gas turbines, though, wind turbines aren’t directly connected to their generators with direct drive but use gearboxes so that the turbine blades can spin slower than their generators. Is this really necessary?

A. Assume that you have a generator (let’s say an ordinary 4-pole generator). How fast must that generator be spinning to make 60 Hz power?

B. If you have an ordinary commercial wind turbine with ~40 m long blades, what is the “tip speed”, the velocity at which the tips of the blades are moving?

C. Is this mechanically feasible? For comparison, the speed of sound in air at room temperature and pressure is 340 m/s.

**Problem 6 (Optional): Bernoulli equation for air**

A. Derive the Bernoulli equation for compressible fluids:

\[ \frac{1}{2} v^2 + g \cdot h + \frac{p}{\rho} \cdot \gamma/(\gamma-1) = ct. \]

With incompressible fluids, you were allowed to neglect the internal energy term ($c_vT$) because changing the fluid pressure did not affect its density or temperature. Now, you don’t have that luxury.

Hint: Remember that the ideal gas law (often seen as $PV=nRT$) can be rewritten as

\[ P = \rho \cdot R_m \cdot T \]

since $n/V$ is the number density. We divide the universal gas constant by the molecular mass ($R \rightarrow R_m$) and multiply $n/V$ by the same factor, and recognize that $n \cdot M/V = \rho$. We used the form with the specific gas constant $R_m$ when integrating around the Carnot cycle.