Problem 1: Three-phase power transmission

The standard means by which electricity is transmitted in the U.S. is as 3-phase AC power. Each of the 3 conductors you see coming off of high-voltage transmission towers carries a single phase. We discussed in class how 3-phase power transmission has two advantages: 1) the voltages always sum to zero, so there is no net movement of charge, and 2) the powers carried in each phase sum to a constant, so that you have smooth constant power transmission. The diagram below shows $V$ and $V^2$ for the 3 phases.

A. Show that the voltages sum to zero.

For those who are comfortable with sines & cosines, consider the sum of $V_1=\sin(\omega t)$, $V_2=\sin(\omega t + \phi_2)$, and $V_3=\sin(\omega t + \phi_3)$ and prove that it equals zero. (You need standard trig identities but nothing fancier than that).

For those who are not comfortable, pick four locations on the top figure above, and at each location measure $V_1$, $V_2$, and $V_3$ by hand and add them.

B. Show that sum of the $V^2$s is constant. Same procedure: for those comfortable with sine and cosines, show analytically that $V_1^2 + V_2^2 + V_3^2 = \text{constant}$. For those not comfortable, pick four locations along the figure and measure and add by hand.

C. Can you get the same properties if you use only two phases?
**Problem 2: Electricity distribution – a very local field trip.**

In the questions below, answer what you can from your local electrical system. If you can’t answer a few, don’t worry. These are look-see questions, not time-consuming.

The electrical power in your house is at a relatively low 110-120 V. You know from class discussions the transmission and distribution lines have some resistance and there can be heating in them, and that resistive heating losses would be severe if you tried to transmit power at this low voltage for any significant distance. In a sensible system, power is transmitted at high voltage. That means there must be a transformer quite close to your house. Remember, Edison transmitted DC power at 110 V and couldn’t extend his lines longer than 2 km, so you are sure to find a transformer within 2 km. In modern systems it is much closer than that.

A. Go outside and find the transformer that serves your house /dorm / building.

B. Photograph the high(er) voltage distribution lines coming in to the transformer and attach the image to your problem set solutions (or make a hand sketch if you don’t have a camera). Label the picture with interesting /relevant features of the power lines. (Refer to the Hayes chapter). Note whether your local distribution lines have a fourth “neutral” or “return” wire, or whether they carry the three phases without a neutral, as do transmission lines. (Both are possibilities).

C. Photograph the transformer and lines coming from it and, as in B., label and comment. How many of the primary phase conductors (the 3 “hot wires” of the distribution system) feed the transformer? If only one conductor feeds it, the transformer can only produce single-phase power.

D. From the “low side” of the transformer (the low-voltage power leaving the transformer), how many wires go to each building the transformer serves? Note: in the U.S., most buildings get three wires in: two hot wires that carry a single phase (inverted on one of the lines), plus a neutral wire. Having 3 wires coming in doesn’t mean you’re getting 3-phase power. For three-phase you’d need four wires, 3 for the three phases plus a neutral. From both C & D: is your building getting 3-phase or single-phase power?

E. How many buildings are served by the transformer that serves your house/dorm/apartment?

F. If you can see a max power rating written on the transformer, write it down. This will be usually written in “kVA”, which is essentially kW (as you know if you remember that 1 Volt at 1 Amp = 1 Watt).

G. Photograph the wires that lead from the transformer and enter your house / dorm / apartment building. Are they thicker or thinner than the main distribution lines? Why?
H. **(Optional, extra credit)** Find the substation that serves your neighborhood (this is likely a bike ride or even a drive away). Take pictures of the substation and the high-voltage transmission lines that serve it and discuss the components you see, referring to the Hayes chapter for explanations. The substation brings voltage down from the very high levels of long-distance transmission lines (up to 500 kV) down to ~10s of kV for distribution within a city.

**Problem 3 (Optional): Field trip part 2 – household electricity distribution**

These questions should be easy and fast, but we’re making them optional because people in apartment buildings or dorms may not have access to their breaker box. It’s OK to do A-C but not D-F if the latter group is not possible for you.

Because flowing too much current into your house would be a fire hazard, your house is protected by circuit breakers (or less likely, fuses) that trip when too much current flows.

Find the breaker box (or boxes) in your building. You’ll likely see two very thick wires coming in to the main breaker carrying all the power to the house. The box will contain a single master breaker that sets the maximum current that can flow, and then a number of individual circuits that take some of that current. Most circuits will be 120 V and 15 A. A smaller number of circuits will be twice that, 240 V and 30 A. The 240 V breakers take up two spaces in the breaker box, which is logical since they control twice the current as the smaller 120 V breakers. If you have an electric stove, you likely also have a special circuit for it that carries 40 A or even 50 A.

Some breaker panels are enclosed by a metal panel so that you’ll see only the switches; others are totally open so that you can see all the wiring as well. If your panel is open, you should see two “bus bars”, which the breakers connect to; these connect to the two hot wires and are at high voltage. Under no circumstances should you touch the bus bars! They will be happy to discharge current through you to the ground. Observe that the 240V breakers span both bus bars, while the 120 V circuits connect to only one.

If the electrician who installed the box was responsible, either the breakers will be labeled or you’ll have a circuit diagram on the inside of the breaker box door.

- **A. Identify the main circuit breaker that would cut off all power to the house / dorm / apartment building.** If it has an amperage written on it, write that down, and calculate the max power draw of your house. If you could see a max power on the transformer, compare to that, & state whether the comparison makes sense.

- **B. Write down the number of individual circuits of each type, add up their currents and compare to the main breaker limit (if you could read one).**

- **C. Take photos and attach them (with labels if that aids in understanding)**

Identify the conduits – metal casings that look like pipes – that carry wiring throughout the building, and notice that the conduits split off and travel to different places. Probably you will see fewer separate conduits than you have circuits, because at this point each
conduit will be carrying the wires for several circuits.

Look at a monthly electric bill (if you get one) to do the questions below.

D. Estimate (if possible) how close to the monthly power draw of your house / apartment is to its max capacity. You will have to convert the electric-bill power usage (given in kW*hr/month) to standard units of W.

E. Give the per capita electric power use of your residence (i.e. divide by the number of people, if multiple people live in your house/apartment/dorm).

F. How does your electricity usage compare to the U.S. average? (If you did problem 2.1 in PS13, you’ve already estimated U.S. per capita electricity usage).

Problem 4 (Optional): Atmospheric heat engine – Carnot limit

We estimated in class that the efficiency of extraction of wind energy from the atmosphere

$$\varepsilon_{\text{extraction}} = \frac{W_{\text{extractable}}}{Q_{\text{sun}}}$$

was about 1%.

(We got that from ~2 W/m² areal power density for a wind farm / 200 W/m² incoming average solar flux)

The atmosphere therefore appears to be an inefficient heat engine...bad at turning solar energy into kinetic energy in the winds. We can’t say that for sure from the wind farm calculation, though, because we don’t know how much kinetic energy there is in the atmosphere at higher altitudes than the 100 m towers we considered.

You can however estimate an upper limit for the atmosphere’s efficiency as a heat engine by considering its Carnot limit. You don’t know the thermodynamic cycle of the atmosphere, but you know it can’t be better than Carnot.

Estimate a reasonable $T_{\text{hot}}$ and $T_{\text{cold}}$ for the atmosphere and therefore the limiting Carnot efficiency.

Atmospheric circulation is driven by temperature gradients both vertically and horizontally; considering either will give you a reasonable approximate number.

Problem 5: Solar thermal

In extraction of both wind and hydro power, we are using the atmosphere as a heat engine powered by solar radiation. And we estimated that the atmosphere is a very inefficient heat engine. It might occur to you at this point that it could make sense to find a way to use the solar radiation more efficiently.

Solar thermal plants capture incoming solar radiation, direct it onto a tube of water to heat it up and make steam (this is not particularly high-tech) and then runs a steam
turbine, which spins a generator. Think of the Crawford plant, only instead of a coal combustion chamber providing the heat, you have mirrors shining concentrated sunlight onto the boiler.

A. Assume you're in a good sunny location with incoming solar at 350 W/m² rather than the 200 W/m² of the world average. (Note: this is an average over both day and night, so you don't need to correct for the fact that the sun doesn't shine at night). Assume that the collected heat is used to drive a steam turbine that runs similarly to turbines in coal-fired plants. From the picture below (assuming this is typical), make an estimate of what % of the area is actually used to collect sunlight. **What is the W/m² you can generate from a solar thermal plant?**

![Image of solar thermal plant](image)

B. Does solar thermal look promising for providing enough power for the world, at least based on areal power density (W/m²)? (i.e. does it beat our 10 W/m² goal?)

C. How does it compare to the area power density (W/m²) of solar photovoltaic?

D. How much land would be required to fill current U.S. electricity needs with solar thermal? Decide you are Energy Dictator and can appropriate as much land as you need to set up your energy system of choice. The U.S. “sunbelt”, the highest-insolation part of the country, is a continuous area comprising Arizona, New Mexico, W. Texas, SE California, and S. Nevada. Get a map of the U.S. Block out on the map the land you will seize to fill U.S. electricity needs.

E. The average install cost of solar thermal in a good right now is about $3/W_{\text{peak}}$, (where the peak insolation is assumed to be 1000 W/m²). **How does this cost compare to that of solar PV?** (Note: solar PV prices are also given in W_{\text{peak}}).
Residential installations of systems that make electricity from the sun are overwhelmingly solar PV instead of solar thermal – is this a cost decision, and if not, what governs the decision?

F. (Optional) The Barstow, CA solar thermal plant pictured here was built in 1984 and is still operational. That means that the lifetime for solar PV is at least 30 years, and in fact is probably close to that of coal-fired plants (~ 50 years). Using that lifetime, what is the cost of electricity from solar thermal? (Give the answer in cents/kWh). The cost of coal-fired electricity is currently about 3 cents/kWh. How much more would the U.S. have to pay each year to meet its electricity needs from solar thermal instead of coal? What fraction of GDP is this?