GEOS 24705 / ENST 24705
Problem set #2
Due: Tues. April 6

Problem 1

What are the limits on the Earth's ability to support people?

In class we made a bunch of estimates of people's food use and photosynthetic efficiency. You estimated that

- The solar flux of radiation reaching the ground is about 200 W/m^2
- The average person requires 100 W power to sustain life
- To feed oneself on good land one would need ½ acre or 2000 m^2
- The efficiency of food production is ~ .03% on good land
  (i.e. only .03% of the sun's energy turns into chemical energy we can eat).
- Total photosynthetic efficiency is ~ 0.3%

Note that I told the class that you were a bit off – you made assumptions about farm size that were suitable for subsistence farming in pre-industrial times, not for hybridized seeds and fertilizer. The average photosynthetic efficiency over the entire Earth (including oceans) is indeed ~0.4%, but good fertile land can do better. Up to now your estimates have been dead on, this is the first time you're off.

But you don't have to take my word for this .. you can check it yourself.

If your estimate for the photosynthetic efficiency of food production was right, could the current population of the Earth be supported?

A. If you divide the Earth up among its current population of $\sim 7 \cdot 10^9$ people, how much land does each person get? Don't Google the surface area of the Earth, at least not at first (you can check your answer afterwards by Googling, but show in your answer that you're working it out yourself first.

   Hint: If you don't know the radius of the Earth, guess it by thinking about distances on the globe that you do know (e.g. the distance across the U.S.) That lets you calculate the Earth's surface area. You can then look at a map (or visualize one) and estimate what fraction of that surface area is land.

B. Assume that the land that you are allotted is representative of the Earth's land surface – it has the same proportion of ice caps, mountains, deserts, scrubland. What fraction of your land is nice, flat, fertile, well-watered, and suitable for farming?

C. From your answer in B, what is the total area of good farmland that you have? (If it makes it easier to visualize, give the number not only in m^2 but also in units of acres, with 1 acre ~ 4000 m^2)

D. What is the flux of solar energy that can be converted to chemical energy in food, in W/m^2, based on your previous estimates of photosynthetic efficiency of food production? (Since you've been given an average piece of the Earth, you also get the average solar insolation, 200 W/m^2).

E. Can you feed yourself on your allotted land, if you eat only plant material? (Your calculation in D is valid only for vegans…).

F. Can you feed yourself if all your calories come from meat? The Worldwatch Institute (a relatively respectable source) estimates that it takes 5 pounds of grain to produce one pound of beef. Assume for now that grain and beef have the same energy density in J/kg: if so, you would need 5 times as much land to raise grain to feed the cow to get your beef dinner as you would need if you just ate the grain instead.

G. What is the "carrying capacity" of the Earth, given your assumptions: the maximum population it can support? Do the calculation for a planet of vegans. What is the current "safety factor" of the Earth's agricultural productivity, i.e. number of people we could support/ number of people living? Does this estimate make you think your estimate of photosynthetic efficiency of food production is off?

H. (O*ptional, extra credit):* Repeat the above calculation for a planet of people eating like Americans (~ 30W/person out of one's daily diet, i.e. ~1/3 of total caloric intake, from animal products, according to the World Resources Institute, 2002).

I. *(Optional, extra credit):* The Earth's population is currently growing at about 1%/year. If that rate continues, in what year will the Earth hit the carrying capacity that you calculated? (If it's not there already).

J. Reality check: the Iowa Corn Producers Association says that the average production (on excellent farmland, with lots of fertilizer) is 183 bushels/acre of shelled corn per year (though in bad years yields can be as low as 50). Use this to estimate the W/m2 of food production for the most productive modern farms.

Guidance: Here you see the full awfulness of English units, and get to practice the kind of unit conversions that plague anyone working in the energy field. It's tedious but necessary if you want to interpret or especially compare reports.

So: bushels is a unit of volume: 1 bushel ~ 35 liters. You need to turn that volume of corn into an energy – and so you need to know an *energy density* for corn, i.e. energy/volume or energy/mass. (That should remind you of the latent heat calculation, where we estimated the energy/volume or energy/mass of evaporating water). The Corn Association says that shelled corn weighs 56 pounds/bushel at something like 15% moisture content by weight, i.e. only 85% of that mass is really the corn itself, so the DRY mass of corn is 0.85*56 pounds/bushel ~48 pounds/bushel, which we can call 50. Use the rule of thumb that carbohydrates have about 4 calories/g (that's food calories, not physicists' calories, be warned); remember your other unit conversions; don't forget that the corn yield is per year; and go for it. You shouldn't need a calculator for any of this – do it by hand.

Your final answer should be in W/m2.

How off was your estimate in D?

Think of D as the pre-modern productivity, and J as the maximum productivity modern farming can squeeze out of the land.

K. *(Optional, extra credit)* Let's assume that everyone on Earth gets to be like Americans – they farm as productively, but also eat like Americans (lots of animal products). How much land does each person need to support themselves? What is the carrying capacity of the Earth?

Problem 2 Energy density.

Energy density is a powerful concept that is deeply important to many applications, especially those involving transportation.

Consider a few possible energy sources, and estimate their energy density, both the mass energy density (in J/kg, i.e. energy per mass) and the volume energy density (in J/m^3 , i.e. energy per volume). The lower the mass energy density, the heavier the fuel you have to carry to get the same range. The lower your volume density, the bigger your fuel tank (or battery pack) needs to be to get that range.

For these problems, in many cases you have the info already, or can readily get it, you just need to convert units. Please don't Google the Wikipedia page on energy density and pull answers off – not educational. It is OK to Google the Wikipedia page on gasoline.

A. Orange (or other fruit) juice. You can get the calories off a label. Remember that American food labels write "calories" for what physicists would term "Calories" or "kilo-calories".

B. Gasoline. OK to get info from web, and convert units as necessary.

C. Cooking oil of some kind. From the food label. How does your answer compare to that in part B? Is it consistent with your knowledge that cars can be run on biodiesel made from waste cooking oil?

D. Batteries. The power put out by a battery is

$$P = I \cdot V$$

Where I is the current in Amps, V is the battery voltage, and P is power in units of J/s. If you know the total current the battery can produce over its life (usually given in Amp · hours) you can readily get the total energy stored in the battery in J.

Unfortunately there seems to be no consumer-protection requirement that household battery vendors list their Amp · hours. Imagine going to the grocery store to buy cereal and seeing rows of identically sized boxes, none of which stated how much cereal was actually inside! We'd never tolerate that, but we accept buying batteries without knowing their energy content.

So, the simplest thing is to use your laptop battery instead, which is always labeled with V and A · hr.

E. *(Optional)* Do an additional, different type of battery. Consumers use a whole spectrum of batteries. Car batteries are typically lead-acid. Household batteries are generally cheap disposable alkaline ones but for renewability or other performance characteristics can be NiCad (nickel-cadmium) or NiMH (nickel metal hydride) or Lithium-ion. Laptops are always the one of the latter three. Batteries can therefore have quite different energy densities.

Here's one of the few websites that explicitly lists the energy content of some kind of batteries (high-quality alkaline household batteries):

http://www.batterycountry.com/ShopSite/alkaline-batteries.html

As in D, you'll have to find some alkaline batteries and weigh them to get an approximate mass. (If your scale is designed for greater masses, try the trick of weighing many batteries at once and then dividing to get the mass of a single battery). To estimate volume, remember the volume of a cylinder is $\pi R^2 L$, where R is the radius of the cylinder and L the length.

Is your answer consistent with the fact that laptop batteries are expensive compared to household batteries?

F. You frequently hear people discussing electric cars. Why do you never read any proposals for electric airplanes?

Problem 3

Next week we will discuss the efficiency of the human body as an engine: a device to turn chemical energy (food) into mechanical work. Get a start on this subject by using information you already know to answer:

What is more energy efficient, bicycling or driving?

A. First, compare the two forms of transportation in terms of miles per gallon of gasoline equivalent. It's OK to stick with mpg rather than Standard International units (km/J or km/m^3 equivalent), because most Americans have a better feel for mpg. (Also OK to use SI). You know mpg for a car already. For bicycling, assume some reasonable number of extra calories/hour burnt while biking at some reasonable speed. You can convert energy content to equivalent volume of gasoline using the values just found in Problem 2.

B. The person-engine plus bicycle looks like a pretty good option compared to cars when you consider mpg. But remember, a bike is very light. A car has to move some 4000 pounds of metal around, so you'd expect it would use more energy to do so. How does a car compare to a bike if the unit of comparison is (miles/gallon)*lb instead of miles/gallon? (Or flip the units to be gallons/(mile*lb). This is the relevant factor if we wanted to haul significant amounts of freight rather than a single driver). How does the person-engine compare to the internal combustion engine now? Would you use rickshaw drivers or Priuses to move freight around?