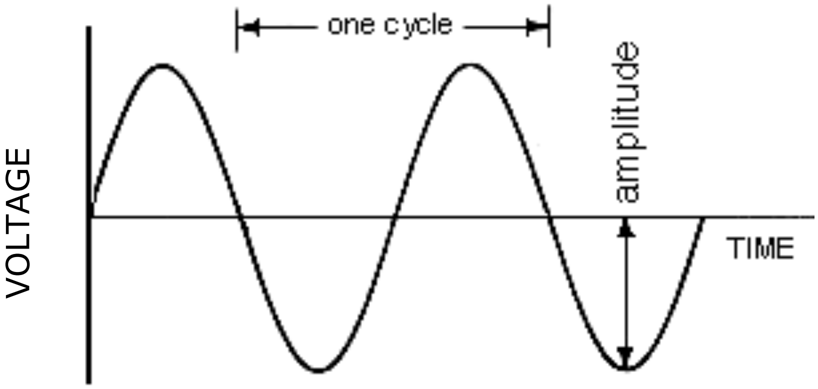
GEOS 24705 / ENST 24705
Problem set #8
Due: Tues. April 27

Problem 1: Generating AC voltages
AC generating systems produce voltage in a current loop that rises and falls in a sine wave, as in the picture below. The voltage in turn drives an alternating current that moves back and forth at the same frequency. ("AC" means "alternating current").

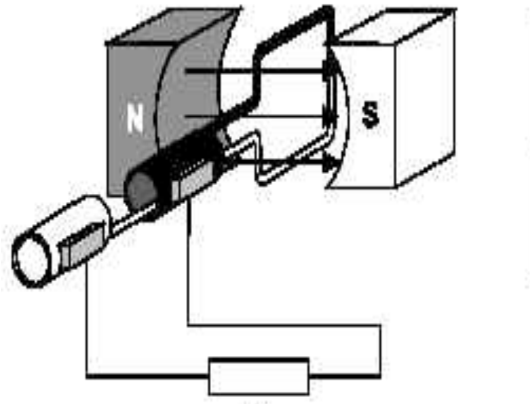


The voltage is produced from a magnetic field varying in a current loop, or by a current loop rotating in a magnetic field (so that the magnetic field it "captures" is varying. Faraday's law expresses this as (in our notation):

$$\Delta V = d(A_\perp \cdot B)/dt$$

where ΔV is the voltage produced in the current loop, B is the amplitude of the magnetic field, $A_\perp$ is the area of the loop that is perpendicular to the magnetic field, and d( )/dt means a rate of change.

You can produce this voltage by the simplest possible generator, a single loop of wire rotating in the field of a permanent magnet:



For those not comfortable with calculus/dot products/sines and cosines:

A. Draw the system at various time points: t-0, t ¼ T, t=1/2 T, t=3/4 T, t=T. Yes, we did this in class as well, but it is good for everyone to re-think and do it themselves. If you want to make the problem somewhat different (and more like real generators), draw a magnet rotating in a loop instead.

B. Make a graph of both $A_\perp \cdot B$ and of the resulting voltage.

Assume that you are now rotating the loop (or magnet) twice as fast, so that you can complete two full revolutions in your original period T.

C. Draw the system again at the same time points (and add some intermediate time points now that you're going faster).

D. On your graph of $A_\perp \cdot B$, add the new $A_\perp \cdot B_{\_faster\ rotation}$

E. On your graph of voltage, add the new voltage generated by faster rotation. Make sure that you draw its amplitude correctly to scale…Think carefully about what maximum amplitude you'd produce if you rotated twice as fast.

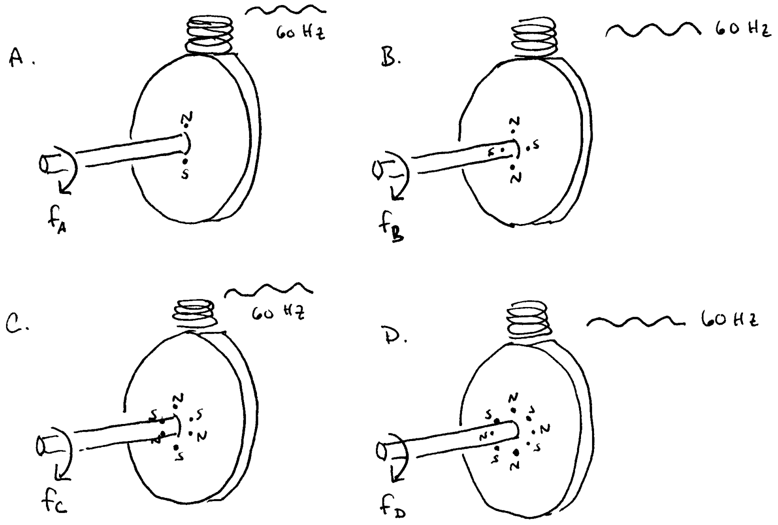
F. Is the voltage chart you just drew consistent with what we estimated in class: $|V|_{average} = B \cdot A \cdot N \cdot 4 \cdot f$?

The point of this problem is to get you realizing that the rotation rate of the mechanical system controls both the frequency and the amplitude of the resulting voltage.

For those comfortable with calculus/dot products/sines and cosines, instead:

A. Write down a formula for $A_\perp \cdot B$ as a function of time
B. Write the formula for voltage as a function of time
C. Write the formula for for $A_\perp \cdot B$ as a function of time with twice the rotation rate
D. Write the formula for voltage as a function of time with twice the rotation rate
E. Show by integrating the voltage that the quick and dirty derivation we did in class to get $|V|_{average} = B \cdot A \cdot N \cdot 4 \cdot f$ is actually correct.

Problem 2: AC generator and motor poles and frequencies



The drawing above shows four different synchronous generators, with different numbers of magnetic poles on the rotors.

A-D. For each generator, state how fast the rotor must turn if the generator is to produce 60 Hz power. Give your answer both in rotations per second (Hz) and in rotations per minute (rpm).

The answer you got above should suggest certain constraints on the speeds that synchronous generators can turn.

E. ***(Optional)***. Write an expression that relates rotor rotation frequency to the number of magnetic poles. (Remember that a single N-S magnet counts as two poles).

F. Check your understanding against the real world by looking online for existing turbo-generators (turbine + generator). Look up about 5, and write down all the rotation rates that are available. Do you see a pattern, and is it consistent with your expectations from A-D??

AC motors that are driven by alternating voltages and current have the same constraints on their rotation rates as generators do. (The only difference is that AC motors come in two types, synchronous and induction, and induction motors, which we haven't discussed, have a slight frequency lag. Still, you should be able to see the same patterns in their rotation rates).

Go online shopping (one option is to Google and use the "shopping" tag) for AC motors that run off of ordinary household power. (That's 60 Hz and either normal 110-120 V or, for three-phase motors, something like "208-230"). Make sure the motors you choose have no gears and aren't "AC/DC" motors. Get about 10 different examples.

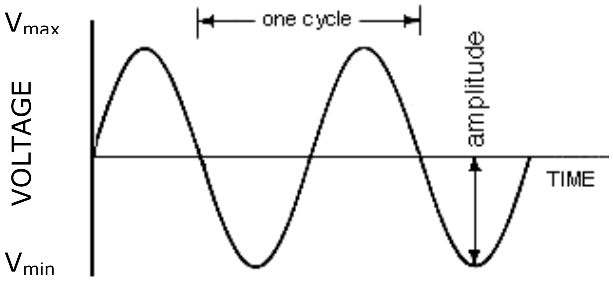
G. Write down the different rotation rates you could purchase. Do you see the same pattern as for generators? Can you get an AC motor at any speed you want?

H. Do the same online shopping for DC motors and write down the different rotation rates you can buy. Do you find the same pattern, or are DC motors more versatile?

The point of the problem is to make you understand how the 60 Hz requirement places constraints on both AC motors and generators.

Problem 3: Specifying AC voltages

The voltage in the U.S. electrical system is specified as between 110-120 Volts. But what does that really mean? Since voltage in an AC system varies from 0 to some maximum amplitude V_{max} back down through 0 and then to a negative $-V_{max}$, we have to pick some single value that represents that whole pattern.



A. Could the specified voltage be the average voltage? What is the average voltage?

The specified voltage could be the maximum voltage V_{max}, or it could be the average of the absolute value of the voltage $|V|_{average}$ that we derived in class… but it isn't. Instead it's the "root-mean-squared" voltage, i.e. the square root of the average of the square of the voltage:

$$V_{rms} = \sqrt{\overline{V^2}(t)}$$

Where the horizontal bar represents a time average.

B. Determine V_{max}, the peak voltage you get on your household 110-120 V electrical system, from your known V_{rms}. *For anyone with good calculus, integrate to find V_{rms}. For others, use the formula $V_{rms} = V_{max} / \sqrt{2}$*

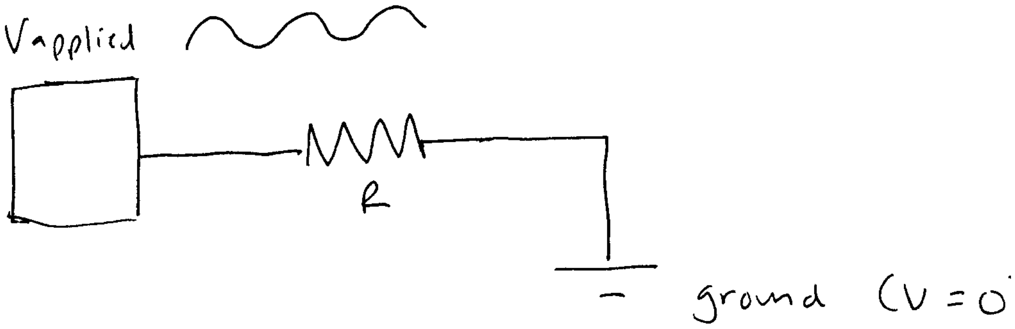
Why do we use V_{rms}? What's so special about squaring the voltage? As you've seen in the readings, power is the product of current and voltage:

$$P = I \cdot V$$

where current I has units of charge/time and V has units of energy/charge, so that $I \cdot V$ has units of energy/time, or power. And the current that flows in a resistive system is proportional to a voltage drop (by Ohm's law: $\Delta V = I \cdot R$, where ΔV is the voltage drop).

The diagram below describes a simple system, with current flowing from an applied voltage V to ground through a resistive load R.

C. Write an expression for the power dissipated in the resistive load in terms of both I and R and also in terms of V and R.



The point of the problem is that by knowing V_{rms}*, we can readily know the average power delivered by the system.*

Problem 4: Power draw of household appliances.
In Problem 3 we ignored the possibility of heating due to resistance in the wires carrying current, and in general in our electrical transmission system those losses are small. If however in your house you tried to pull huge current through your household wiring, you could produce enough heating in the wires to be a fire danger. For this reason houses have built-in protection systems to prevent too much current from flowing and melting the wires. The wiring in your house is arranged in several individual circuits, each of which has thick enough wire to safely carry 15A or sometimes 20A of current, and each circuit is connected to a fuse or circuit breaker that will trip and cut off all current from flowing if you try to carry any more than that. Probably at some time you have plugged in too many appliances and blown a breaker – nearly everyone has had that experience.

A. Calculate how many space heaters or hairdryers you can plug in to a single household circuit before you blow a breaker.

Info: a space heater or hairdryer typically draws ~ 1500 W of power.

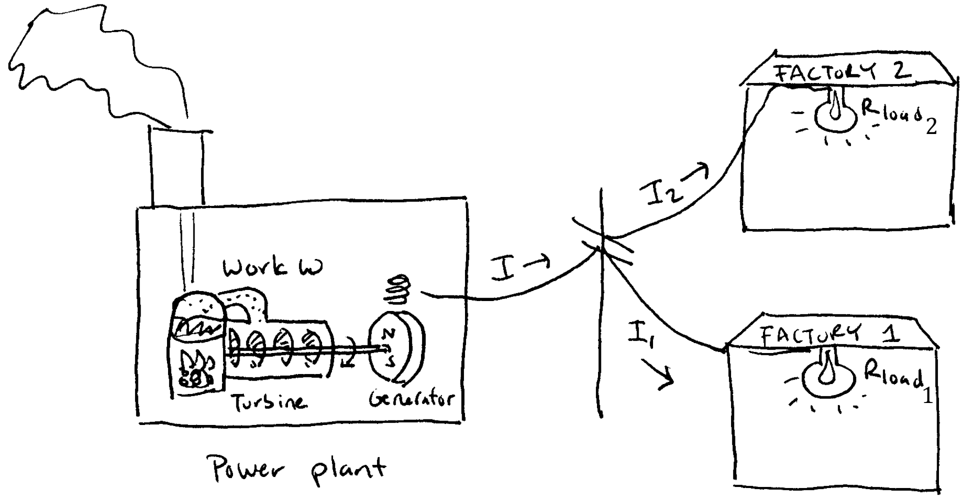
Note that to use an electric stove, which draws more power than hairdryer, you have to have a separate, high-current circuit installed for it.

B. For extra credit (VERY optional), test your hypothesis and report the results.

Make sure you know where your breaker box is first! And make sure you have breakers rather than fuses, which are more of a pain to change. Resetting a breaker is just flipping a switch, so there is no harm done by this experiment, but you don't want to waste time hunting around for the box with a flashlight.

Problem 5: Stability of power generation.
In this problem you are the facility manager of a power plant that provides power to two factories only. You'll try to maintain the quality of your output power as the demand for electricity changes. You have to maintain your voltage near 120 V and your frequency at 60 Hz or lose your job.

You have a single 2-pole synchronous turbo-generator, as in the pictures below. From Problem 2, you know your turbo-generator is supposed to be running at 3600 rpm. (Make sure you understand this). It's properly designed so that at this operation it generates alternating voltage at 120 V. *(In this problem, we'll ignore voltage transformation – assume you're producing at 120V. This means that the currents that are flowing might feel unrealistically high at the generator itself, but that really is how much current is eventually distributed to homes and factories).*



Both components of your system have limits. The turbine can put out a maximum of 60 MW of work (beyond that it just can't go; no matter how fast you dump coal into your firebox – you can't get steam pressures any higher). The generator will obediently source as much current as you ask for, but the manufacturer says that it will overheat dangerously if you try to generate more than 50 MW of power, because of resistive heating if currents get too large. Being lazy, you've installed no protective systems – you assume you can manage all operations by hand.

At time 0, your power plant is running comfortably. Only a Factory 1 is "asking" for power, and it is demanding 20 MW, driving a resistive load. In the drawing above, Factory 2 has at this time not plugged in their equipment so no current I_2 is flowing.

A. What is the current the generator is putting out? What must the resistance of the Factory 1's load be? (Check your units to make sure your answer is correct).

A short time later, the second, identical factory starts up with the same power demand.

(So $R_{load2} = R_{load1}$. From this point on in the problem, in this problem the two factories will always have identical loads).

B. Just from immediate common sense, how much current will now “want” to flow to the second factory?
C. Again, from immediate common sense, how much total power are you now trying to put out?
D. What must the effective resistance of the two-factory system be? Does more demand mean higher or lower effective resistance?

Due to a rapid growth in the U.S. economy, both factories simultaneously ramp up production and suddenly “ask” for more power - they both drop their resistance to 2/3 of the original values.

E. What is the total power they are demanding now? Can your turbine provide it? What happens to your generator? (Hint: rather than calculating numbers out, try to just scale from the previous situation, using the relationships of Problem 3).

Somehow you’ve managed to hold on to your job, the insurance covers everything, and you’ve used the insurance settlement to purchase a new generator to cope with the increased demand – this one can handle 100 MW of power. You think you’re fine.

The economy continues growing, and one day both factories again simultaneously step up production and drop their effective resistance now to ½ their original values from D.

F. If you could hold your voltage to 120 V, what would your power output be? Can your turbine supply enough power to meet that demand?

What happens now? Your turbine can only do what it can do. Assume you’ve desperately increased the steam pressure until the turbine is putting out as much power as it can.

G. What voltage must your generator be running at?
H. What speed is the rotor turning?
I. What frequency are you putting out?
J. Do you keep your job now?