

## Velocities in rotating objects

Everyone is familiar with the concept of the *frequency* of rotation of a wheel or other rotating system: the frequency  $f$  is the number of revolutions the wheel makes in a given time interval (usually 1 second). Since every point on a wheel rotates together, every part of the wheel rotates at the same frequency (if it didn't, and some parts rotated faster than others, the wheel would be coming apart).

If you actually measure the speed that a piece of the wheel is moving, though (i.e. look at some given location on the wheel and measure how fast it is moving through space), you'll get different values depending on where you look. Near the hub of the wheel, velocities are very small – a small wheel doesn't have to move very far in a single revolution. The further out you go, the bigger the distance that has to be traveled in a single revolution, so the greater the velocity.

Think of a point on the outside edge of a wheel of radius  $R$ . In a single revolution, that point has to travel all around the circumference of the wheel, i.e. it moves around a circle of diameter  $2\pi R$  in a time period  $1/f$ . That means its velocity must be

$$v = 2\pi R \cdot f$$

This relationship will hold for any point on the wheel a distance  $r$  away from the center, i.e.  $v = 2\pi r \cdot f$ , where  $r$  can be any distance from 0 (at the center of the hub) to  $R$  (at the rim of the wheel).

Often we simplify by defining the "angular velocity" ( $\omega$ ), the number of degrees (radians) rotated in a given time. Since there are  $2\pi$  radians in one revolution,  $\omega = 2\pi f$ . That means velocity is conveniently

$$v = \omega \cdot r$$

Logically enough, the speed at which a part of the wheel travels depends on the rate the wheel is rotating ( $\omega$ ) and how far out from the hub that point is.

This relationship sets a limit for how fast big things can rotate. If speeds get too large, a turbine can shake itself apart. In general, the bigger the turbine, (bigger  $r$ ), the slower rotation ( $\omega$ ) must be to keep "tip speed" ( $v$ ) manageable.