
A simple carbon cycle representation for economic and policy analyses

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Abstract Integrated Assessment Models (IAMs) that couple the climate system and the economy require a representation of ocean CO₂ uptake to translate human-produced emissions to atmospheric concentrations and in turn to climate change. The simple linear carbon cycle representations in most IAMs are not however physical at long timescales, since ocean carbonate chemistry makes CO₂ uptake highly nonlinear. No linearized representation can capture this dual-mode behavior, with initial rapid uptake and then slow equilibration over ~10,000 years. In a business-as-usual scenario followed by cessation of emissions, the carbon cycle in the 2007 version of the most widely used IAM, DICE (Dynamic Integrated Climate-Economy model) produces errors of ~1000 ppm CO₂ and ~6°C by the year 3500. We suggest here a simple alternative representation that captures the relevant physics and reproduces carbon uptake in more complex models to within the inter-model uncertainty. The scheme involves little additional complexity over the DICE model, making it a useful tool for economic and policy analyses.

Keywords DICE · carbon cycle · ocean uptake · integrated assessment model · IAM

1 Introduction

All Integrated Assessment Models (IAMs) that couple the climate system and the economy to evaluate the impacts of climate change require some representation of the global carbon cycle. Anthropogenic climate change is driven primarily by CO₂ emissions produced by human economic activity, but CO₂ does not simply accumulate in the atmosphere, and instead evolves according to the balance between emissions and ocean uptake. IAMs must represent this uptake to translate emissions into atmospheric CO₂ concentrations and in turn climate change. Because state-of-the-art climate models are too computationally expensive for use in economic analyses, IAMs use simplified representations of the physical climate system.

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(See e.g. Goodess et al, 2003). Care must be taken however to ensure that simplifications do not produce inaccurate behavior that can affect damage estimates.

Many simple IAMs use a linearized representation of ocean carbon uptake (for review, see Hof et al, 2012; van Vuuren et al, 2011), but linearized representations cannot fully reproduce the ocean response to increased atmospheric CO₂: rapid initial uptake followed by a slow ‘long-tail’ equilibration stage. In state-of-the-art climate models, about half of a CO₂ perturbation is lost in 30 years (Denman et al, 2007) but most of the remainder persists for thousands of years (Archer et al, 2009). Linearized models that match initial carbon uptake will necessarily produce too-rapid removal of atmospheric CO₂ over the long term and therefore underestimate aggregate climate impacts. We use as an example the carbon cycle in the most widely used simple IAM, the Dynamic Integrated model of Climate and the Economy (DICE) (Nordhaus, 1993, 2008, 2010). In a business-as-usual scenario to 2300 followed by cessation of emissions, the 2007 version of DICE produces errors in CO₂ concentration of ~ 1000 ppm and in temperature of $\sim 6^\circ\text{C}$ on millennial timescales (Figure 1; for comparison of many IAMs see van Vuuren et al (2011)). One alternative approach to capturing nonlinearity is representing ocean carbon uptake as a combination of multiple exponential decays (Maier-Reimer and Hasselmann, 1987). Some IAMs adopt this practice, but it is not robust across emissions scenarios as decay timescales depend on the magnitude of CO₂ perturbations (Archer et al, 2009; van Vuuren et al, 2011). Deficiencies in simple IAM carbon cycle parameterizations have been frequently discussed (Hof et al, 2012; Joos et al, 1999; Schultz and Kasting, 1997; Warren et al, 2010), and authors have called for improved representations that better capture known physics (e.g. van Vuuren et al, 2011).

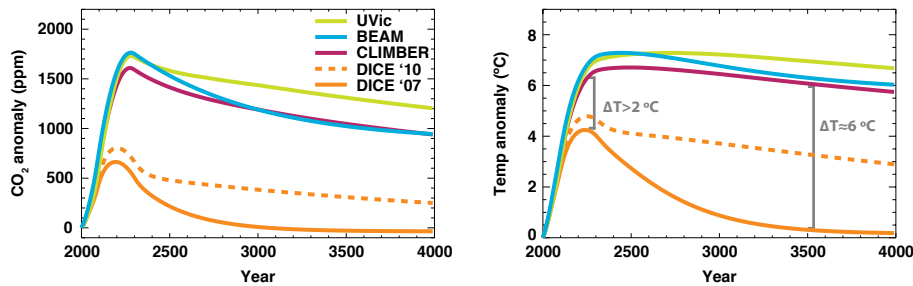


Fig. 1 Atmospheric CO₂ anomaly for BEAM and DICE (2007 and 2010) carbon models compared to intermediate complexity UVic and CLIMBER-2 models for the A2⁺ scenario (left). (See Section 5 for descriptions.) BEAM matches output of the more complex models well for the duration of the simulation, leading to an accurate temperature anomaly projection (right). The DICE carbon model performs well for the first several decades but then draws down CO₂ too rapidly, producing a long-term temperature discrepancy of $\sim 6^\circ\text{C}$.

The main cause of nonlinear ocean uptake is carbonate chemistry, well-understood since the 1950’s (Revelle and Suess (1957), or see e.g. Sarmiento and Gruber (2006) for review). After any increase in atmospheric CO₂, uptake will occur until the atmosphere and ocean come into equilibrium. In a simple solution equilibrium, the number of gas-phase molecules leaving the atmosphere would equal the number of molecules appearing in solution. In the present-day ocean, about ten times as many CO₂ molecules are removed from the atmosphere as the corresponding increase in aqueous CO₂, because dissolved inorganic carbon species are partitioned between CO₂, bicarbonate (HCO₃⁻), and carbonate (CO₃⁼) in proportions fixed by the ocean’s acidity. Uptake becomes nonlinear because it increases acidity,

slowing further uptake by reducing the ocean's ability to store carbon. Models without this nonlinear chemistry cannot reproduce removal of atmospheric CO₂ over long timescales.

We present here a simplified global carbon cycle representation that we term the "Bolin and Eriksson Adjusted Model" (hereafter "BEAM") that captures the nonlinear chemistry of ocean carbon uptake. The scheme provides a computationally inexpensive means of capturing the known physics and chemistry of the global carbon cycle. We describe the model below and compare it both to DICE and to state-of-the-art climate models. Equations, parameter values, and initial conditions are listed in full in Appendices A.1, A.2 and A.3.

2 DICE carbon model

The DICE carbon cycle representation is a simple "box diffusion" model of the atmosphere, upper ocean, and lower ocean that assumes constant fractional transfer of CO₂ from each of the three reservoirs. After any CO₂ perturbation, concentrations reach equilibrium when the reservoir concentrations have adjusted so that their transfers are equal. Transfers are described in a system of three linear first-order ordinary differential equations (Eqs. 1-3), which in DICE are specified discretely assuming 10-year timesteps. All versions of DICE use this same functional form, but parameter values have changed across versions.

$$M_{AT}(t) = CE(t) + \phi_{11} \cdot M_{AT}(t-1) + \phi_{21} \cdot M_{UP}(t-1) \quad (1)$$

$$M_{UP}(t) = \phi_{12} \cdot M_{AT}(t-1) + \phi_{22} \cdot M_{UP}(t-1) + \phi_{32} \cdot M_{LO}(t-1) \quad (2)$$

$$M_{LO}(t) = \phi_{23} \cdot M_{UP}(t-1) + \phi_{33} \cdot M_{LO}(t-1) \quad (3)$$

where the ϕ 's are the transfer coefficients, $CE(t)$ is cumulative emissions of CO₂ over the timestep t , and M_{AT} , M_{UP} , and M_{LO} are the mass of inorganic carbon (in gigatons) in the atmosphere, upper, and lower ocean, respectively. DICE does not explicitly describe the speciation of inorganic carbon, but the ocean reservoirs M_{UP} and M_{LO} can be thought of as comprising dissolved CO₂, HCO₃⁻, and CO₃⁼.

Because the model has only four distinct fluxes between reservoirs, only four of the seven transfer coefficients are independent. (For example, $(1 - \phi_{11}) \cdot M_{AT}$ is the loss of carbon from the atmosphere to the upper ocean in a given timestep, and $\phi_{12} \cdot M_{AT}$ is that same carbon arriving in the ocean.) To conserve mass, coefficients must be related by:

$$\begin{array}{lll} \text{atmosphere to upper ocean:} & \phi_{12} & = (1 - \phi_{11}) \\ \text{upper ocean to atmosphere and lower ocean:} & \phi_{21} + \phi_{23} & = (1 - \phi_{22}) \\ \text{lower ocean to upper ocean:} & \phi_{32} & = (1 - \phi_{33}) \end{array}$$

3 BEAM carbon model

The three-reservoir carbon cycle model was first outlined by Bolin and Eriksson (1959), who used a 2-layer ocean specification from Craig (1957) and derived transfer coefficients for CO₂ anomalies by considering carbonate chemistry. Bolin and Eriksson (1959) used fixed parameter values, but their model can be readily extended to allow coefficients to change as CO₂ uptake progresses. The three differential equations of BEAM carbon transfer are essentially identical to those of DICE:

$$\frac{dM_{AT}}{dt} = E(t) - k_a \cdot (M_{AT} - A \cdot B \cdot M_{UP}) \quad (4)$$

$$\frac{dM_{UP}}{dt} = k_a \cdot (M_{AT} - A \cdot B \cdot M_{UP}) - k_d \cdot (M_{UP} - \frac{M_{LO}}{\delta}) \quad (5)$$

$$\frac{dM_{LO}}{dt} = k_d \cdot (M_{UP} - \frac{M_{LO}}{\delta}) \quad (6)$$

where $E(t)$ is the emissions rate. The four independent parameters describing the fluxes between reservoirs are now the two k 's, which are inverse exchange timescales between atmosphere-ocean (k_a) and upper-lower ocean (k_d), and two dimensionless parameters: δ , the ratio of volume of lower to upper ocean (~ 50), and $A \cdot B$, the equilibrium ratio of atmospheric to upper ocean inorganic carbon. The term $M_{AT} - A \cdot B \cdot M_{UP}$ is the *disequilibrium* between atmospheric and ocean inorganic carbon, and is eroded with time constant $1/k_a$. The term $M_{UP} - M_{LO}/\delta$ is the disequilibrium between upper and lower atmospheric inorganic carbon, eroded with time constant $1/k_d$.

The only variation in coefficients over time occurs in $A \cdot B$, which we separate to distinguish factors with different dependences on environmental conditions. A is the ratio of atmosphere to ocean CO_2 concentration at equilibrium, which is weakly dependent on temperature: a warmer ocean holds less dissolved CO_2 . B is the ratio of dissolved CO_2 to total ocean inorganic carbon at equilibrium (see Section 4), a strong function of acidity: more acidic seawater stores less inorganic carbon. Variation in B in particular alters uptake rates dramatically. In the business-as-usual scenario of Figure 1, atmospheric CO_2 rises 5.2 times over present-day concentrations by year 2200, but acidification and warming simultaneously raise $A \cdot B$ by a factor of 4.8. That is, the ocean's ability to hold inorganic carbon relative to atmospheric CO_2 drops nearly as fast as atmospheric CO_2 rises. This near-cancellation lowers average carbon uptake ~ 50 times below that expected from initial coefficient values.

equiv. coefficients		timescales (years)			
DICE	BEAM	$\tau_{\text{DICE},07}$	$\tau_{\text{DICE},10}$	$\tau_{\text{BEAM},1995}$	$\tau_{\text{BEAM},2200}$
ϕ_{12}	k_a	50	(80)	5	5
ϕ_{21}	$k_a \cdot A \cdot B$	100	(200)	3.8	0.8
ϕ_{23}	k_d	200	(2,000)	20	20
ϕ_{32}	k_d/δ	3,000	(13,000)	1000	1000
<i>BEAM parameters from model year</i>		<i>1800</i>		<i>1995</i>	<i>2200</i>
	A	220		220	260
	$1/B$	220		170	41
	$A \cdot B$	1.0		1.3	6.3

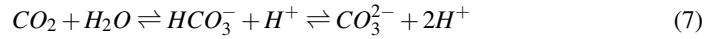
Table 1 Equivalence of coefficients in DICE and BEAM and corresponding timescales (inverses of transfer coefficients, $\times 10$ for DICE because the model uses 10-year timesteps, consistent to 1 significant figure with continuous representation). Because its parameters cannot evolve in time, DICE in both 2007 and 2010 versions approximates real-world CO_2 uptake primarily by lengthening exchange timescales. All parameters are shown to 1 or 2 significant figures; see Appendix A.2 to derive BEAM parameters more precisely.

DICE, with fixed parameters, could not match present-day atmospheric CO_2 uptake rates were it to use realistic values for reservoir sizes and exchange timescales. To approximate

uptake slowed by changing acidity, DICE raises exchange timescales substantially above plausible physical values (Table 1). Real-world coefficients continue to evolve as ocean carbon storage drops in an acidifying ocean, however, so DICE is unable to reproduce uptake rates over all time. In practice, DICE parameter values are set by fitting against relatively short simulations¹, sacrificing accuracy at longer timescales.

4 Carbonate chemistry in BEAM

Ocean carbonate chemistry. Aqueous carbonate chemistry is well understood; we review it briefly only to explain its treatment in BEAM. See any aqueous chemistry textbook (e.g. Sarmiento and Gruber, 2006) for more thorough review. Uptake of CO_2 proceeds when concentrations in the atmosphere and upper ocean are out of equilibrium ($M_{AT} \neq A \cdot B \cdot M_{UP}$). As CO_2 dissolves, we assume instantaneous repartitioning of inorganic carbon species:



Partitioning is set by the dissociation coefficients k_1 and k_2 and the concentration of hydrogen ions $[\text{H}^+]$, i.e. the acidity (pH) of seawater². The ‘‘carbon storage factor’’ $1/B$, the equilibrium ocean total inorganic carbon relative to dissolved CO_2 , is then:

$$\frac{1}{B} = \frac{[\text{CO}_2] + [\text{HCO}_3^-] + [\text{CO}_3^{2-}]}{[\text{CO}_2]} = 1 + \frac{k_1}{[\text{H}^+]} + \frac{k_1 k_2}{[\text{H}^+]^2} \quad (8)$$

Higher acidity (higher $[\text{H}^+]$, lower pH) reduces the ocean’s ability to store carbon (Fig. 2). As equation 7 indicates, dissolved CO_2 itself acts a weak acid. Any ocean uptake of CO_2 therefore intrinsically reduces the efficiency of future uptake.

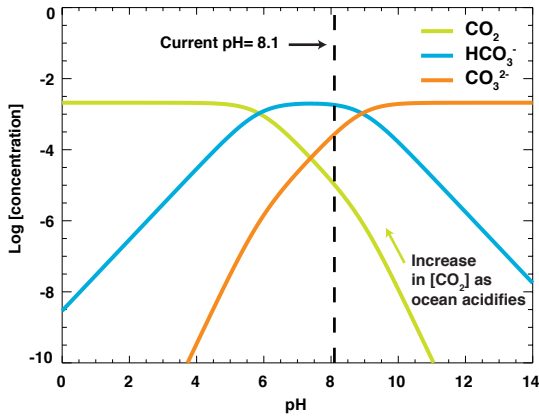


Fig. 2 Partitioning of dissolved inorganic carbon species in seawater (DIC) as a function of pH. Total of all species is held constant at $2100 \mu\text{mol/kg}$. At present, bicarbonate dominates DIC and the carbon storage factor is ~ 170 . In a more acidic ocean (lower pH), CO_2 becomes more significant and the carbon storage factor drops. At pH below 5 (1000 times increase in $[\text{H}^+]$ over present-day conditions), CO_2 dominates and the carbon storage factor approaches 1. Present-day seawater contains strong bases that raise pH; pure water interacting with current atmospheric CO_2 would have pH ~ 5.6 .

Solving for $[\text{H}^+]$ is complicated by the fact that ocean acidity is not simply governed by atmospheric CO_2 . Seawater contains strong bases (and acids) whose combined effect

¹ Nordhaus (2008) tuned carbon cycle parameters to best match those of a simple but more physical carbon cycle (MAGICC, which is similar to BEAM) using an emissions trajectory from 1750-2100 that is a combination of historical emissions and the IPCC A1FI scenario (Nordhaus, 2007).

² The pH scale is $-\log_{10}([\text{H}^+])$, i.e. the ocean’s current pH of ~ 8.1 means that $[\text{H}^+] = 10^{-8.1} \text{ mol/kg}$.

raises pH above that for a pure water/CO₂ system. The carbonate system then acts as a buffer against further changes in acidity. To estimate those changes, we assume constant acid-neutralizing capability, or “alkalinity”, approximated as the amount of H⁺ that would have to be added to convert all bicarbonate and carbonate to CO₂:

$$Alk = [HCO_3^-] + 2[CO_3^{2-}] = \left(\frac{k_1}{[H^+]} + \frac{2 \cdot k_1 \cdot k_2}{[H^+]^2} \right) \cdot M_{UP} \cdot B \quad (9)$$

where $M_{UP} \cdot B$ is the concentration of upper ocean CO₂. We determine Alk by assuming equilibrium in the pre-industrial ocean with pH=8.28 (see Appendix A.3). BEAM solves for $[H^+]$ at each timestep using eqs. 8-9 and assuming constant alkalinity.

Constant alkalinity is a reasonable assumption for several thousand years (Archer et al, 2009; Zeebe and Wolf-Gladrow, 2001), but eventually dissolution of calcium carbonate would help return pH to its original value, increasing drawdown of atmospheric CO₂. BEAM will therefore underpredict CO₂ uptake on ~10,000-year timescales.

Temperature dependence of coefficients: The discussion above concerned only changes in ocean carbon storage capacity due to changing acidity. To a lesser degree, temperature affects both the carbon storage factor $1/B$ and CO₂ solubility $1/A$. The magnitudes of temperature-induced changes are absolutely smaller than acidity effects, and temperature-dependent changes in A and B partially counteract each other: the solubility of CO₂ decreases in a warmer ocean, but the dissociation constants k_1 and k_2 grow with temperature, raising the carbon storage factor (see Appendix A.2 for equations).

Constant	25°C	27°C	30°C	Percentage change 25-30°C
$1/A$	$4.5 \cdot 10^{-3}$	$4.3 \cdot 10^{-3}$	$4.0 \cdot 10^{-3}$	-11%
k_1	$1.00 \cdot 10^{-6}$	$1.02 \cdot 10^{-6}$	$1.05 \cdot 10^{-6}$	5%
k_2	$7.53 \cdot 10^{-10}$	$7.95 \cdot 10^{-10}$	$8.56 \cdot 10^{-10}$	14%

Table 2 Representative values for the temperature-dependent BEAM parameters $1/A$, k_1 , and k_2 at selected temperatures. The dissociation and solubility effects act in opposite directions on ocean carbon storage.

5 Model validation and comparison to DICE

We test the BEAM and DICE (2007 and 2010) carbon models against two intermediate complexity Earth system models used to study evolution of atmospheric CO₂: the University of Victoria Earth system climate model (UVic) (Eby et al, 2009) and the CLIMate and BiosphERE 2 model (CLIMBER-2) (Petoukhov et al, 2000). These models capture complex ocean physics, but have relatively low computational demands that enable multi-millennial simulations. The UVic model consists of a full three-dimensional, 19-layer ocean model and an energy-moisture balanced model of the atmosphere (Weaver et al, 2001). CLIMBER-2 consists of a simpler three-reservoir ocean that includes biogeochemistry (Brovkin et al, 2002) and sedimentation (Archer et al, 1998) coupled to a two-dimensional atmospheric model. The models differ in some feedbacks: CLIMBER-2 does not include terrestrial carbon dynamics while UVic omits ocean biology and weathering sources of CaCO₃³. In

³ Lack of biology in UVic should reduce the rate of carbon transfer to the deep ocean. Lack of terrestrial vegetation dynamics in CLIMBER-2 may have some effect on CO₂ evolution, but the terrestrial feedback on CO₂ is highly uncertain even in sign (Joos et al, 2001; Sitch et al, 2005)

comparisons shown here, both models were forced with an A2⁺ CO₂ emissions scenario (Montenegro et al, 2007), which reproduces the business-as-usual A2 CO₂ emissions scenario for 100 years (Nakicenovic et al, 2000)⁴ followed by linearly declining emissions that reach zero after another 200 years. Both models were run for 10,000 years (starting at pre-industrial using historical emissions), although we compare only the first 2000 years.

CO₂ evolution in BEAM is consistent with that in UVic and CLIMBER-2 throughout the simulation period (Figure 1 shows 2000 years and Figure 3 highlights the first 300 years). CO₂ evolution in both DICE versions is consistent for only the first few decades. By 100 years, DICE atmospheric CO₂ is markedly lower than that of the other models (Fig. 3). After 300 years, when emissions cease, rapid drawdown of atmospheric carbon in DICE 2007 returns climate to near pre-industrial levels, while BEAM and the intermediate complexity models retain persistent high CO₂ and temperatures elevated by $\sim 6 - 7^\circ\text{C}$ for millennia (Fig. 1). (DICE 2010 retains a moderate anomaly.) By the end of the simulation period, BEAM does begin to diverge in behavior from the more realistic models: BEAM CO₂ concentrations begin to asymptote while slow CO₂ uptake continues in the intermediate complexity models. For very long simulations, the assumption of constant alkalinity in BEAM must be relaxed to allow restoration of pH and continued uptake.

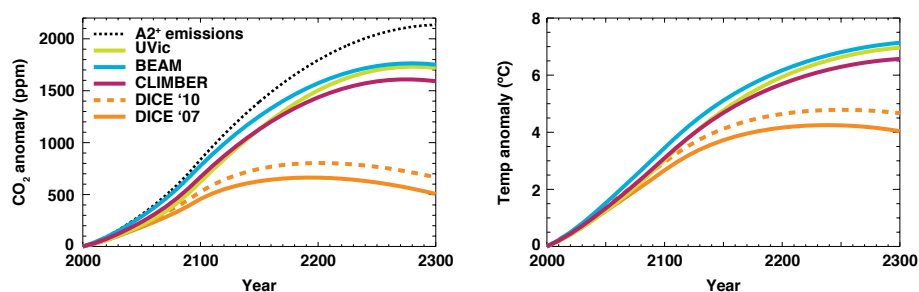


Fig. 3 2000-2300 atmospheric CO₂ anomaly for BEAM and DICE carbon models as compared to intermediate complexity UVic and CLIMBER-2 models for the A2⁺ emissions scenario (left). Dotted black line shows cumulative emissions, i.e. atmospheric CO₂ concentration if no ocean uptake occurred. BEAM matches the more complex model output well, leading to an accurate projection of warming (right). The DICE carbon model performs adequately for the first several decades, then diverges rapidly. DICE removes 3/4 of all anthropogenic carbon from the atmosphere within 300 years while more realistic models remove only 1/4. Too-low CO₂ in DICE produces underestimation of warming by over 2 °C after 300 years.

The dominant driver of difference between DICE and the more realistic carbon cycle models, including BEAM, is the changing carbon partitioning as the ocean acidifies. Figure 4 shows atmospheric CO₂ evolution for 2007 and 2010 DICE compared to BEAM with and without temperature-dependent coefficients. Omitting the temperature dependence of CO₂ solubility and dissociation constants produces negligible changes for the first few hundred years and only small changes even at millennial timescales.

For perspective, Figure 4 also shows the original Bolin and Eriksson (1959) linearized model, which is very similar to 2010 DICE. While 2007 DICE quickly draws down any anthropogenic CO₂ perturbation to near pre-industrial levels, both 2010 DICE and the original Bolin and Eriksson representation retain some anthropogenic CO₂ in the atmosphere for the duration of the simulation. Both achieve this in part by effectively reducing the total ocean

⁴ The SRES A2 scenario was specified by the Intergovernmental Panel on Climate Change as part of the modeling exercises for their Fourth Assessment Report (Parry et al, 2007).

volume. The equilibrium ratio of total ocean to atmospheric carbon, $(\delta + 1)/A \cdot B$, is ~ 39 in present-day BEAM and ~ 32 in 2007 DICE but has been reduced to ~ 20 in 2010 DICE and was ~ 5 in Bolin and Eriksson (1959).⁵

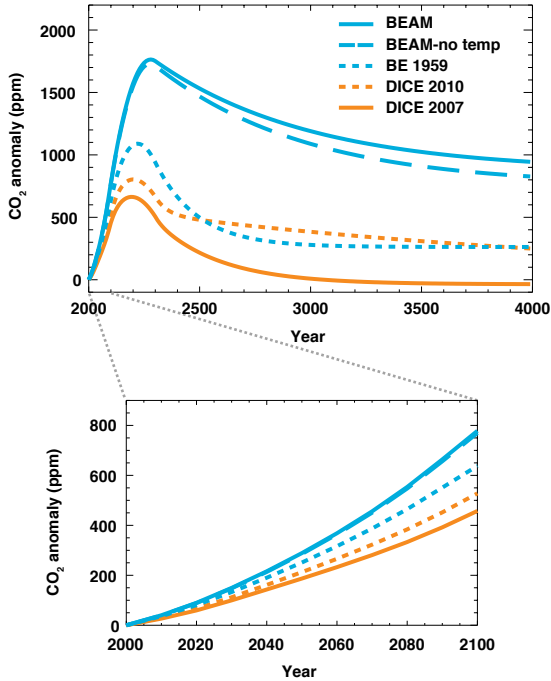


Fig. 4 Atmospheric CO₂ anomaly for different versions of BEAM and DICE carbon cycle models for the A2⁺ scenario. *BEAM* (solid blue) is the full model described here; *BEAM-no temp* (dashed blue) omits temperature dependence; and *BE 1959* (dotted blue) is the original linear Bolin and Eriksson model. Temperature dependence in BEAM coefficients has a relatively minor effect and some users may wish to neglect it. Long-term, 2010 DICE retains more CO₂ in the atmosphere than does 2007 DICE because it has a longer ocean equilibration timescale and effectively a smaller ocean. Atmospheric CO₂ in the Bolin and Eriksson model is fully equilibrated by the end of the simulation; in 2010 DICE it is still evolving toward a lower equilibrium level with a time constant of $\sim 13,000$ years.

6 Discussion and conclusions

Numerous authors have pointed out that the carbon cycle representations in DICE and other simple IAMs do not accurately reproduce the response of more physical models (e.g. Hof et al, 2012; van Vuuren et al, 2011), though no previous studies have examined multi-millennial timescales. While updates of the DICE carbon cycle since 2007 have reduced discrepancies, all versions of DICE diverge from predicted real-world behavior within decades. We confirm that discrepancies occur because linearized models cannot capture the changing ocean carbon storage potential due to changing ocean acidity. These discrepancies can be largely eliminated for thousands of years by adding to the DICE framework a single equation describing acidity evolution. Linearized carbon cycle representations with long-term errors may be acceptable in some cases, because applying discount rates of even a few percent essentially disregards climate-related harms after several centuries. Accurate representation of the carbon cycle is especially important if climate damages substantially reduce economic growth rates and low or even negative discount rates are appropriate (e.g. Moyer et al, 2013). The BEAM model described here offers a simple, computationally tractable carbon cycle representation that retains fidelity over millennial timescales.

⁵ Although 2010 DICE and Bolin and Eriksson (1959) are mathematically equivalent, they are not exactly equivalent in intent: Bolin and Eriksson described evolution of CO₂ anomalies rather than total reservoirs. The reduction in ocean carbon content in 2010 DICE exceeds that produced by any plausible choice of pH.

A BEAM model equations, parameter values, and initial conditions

A.1 Equations

Without temperature dependence, the model consists of four independent equations: three that track total carbon in each layer (atmosphere, upper, and lower ocean- Eq. 10), and one that tracks acidity (Eq. 13). The three temperature-dependent parameters are given in Eqs. 14-16, but because the temperature dependence of parameters is relatively minor, some users may wish to omit it. Carbon transfers in BEAM are described by:

$$\frac{d}{dt} \begin{pmatrix} M_{AT} \\ M_{UP} \\ M_{LO} \end{pmatrix} = \begin{pmatrix} -k_a & k_a \cdot A \cdot B & 0 \\ k_a & -(k_a \cdot A \cdot B) - k_d & \frac{k_d}{\delta} \\ 0 & k_d & -\frac{k_d}{\delta} \end{pmatrix} \begin{pmatrix} M_{AT} \\ M_{UP} \\ M_{LO} \end{pmatrix} + E(t) \quad (10)$$

where the M s represent the mass of carbon (in CO_2 or dissolved inorganic carbon) in the atmosphere (AT), upper ocean (UP), and lower ocean (LO); and $E(t)$ is rate of anthropogenic CO_2 emissions. (Emissions units must match those of concentrations and are therefore specified in mass of carbon in CO_2 .)

The parameter A is the ratio of mass of CO_2 in the atmosphere relative to dissolved CO_2 in the upper ocean, i.e. A is inversely proportional to CO_2 solubility. Solubility is set by ‘Henry’s law’, which prescribes that in equilibrium, the concentrations of CO_2 in the atmosphere and ocean are related by a coefficient dependent only on temperature. Henry’s Law may be written in various forms; for convenience we define the coefficient k_H as a dimensionless ratio of the molar concentrations of CO_2 in atmosphere and ocean. The parameter A is then

$$A = k_H \cdot \frac{AM}{OM/(\delta + 1)} \quad (11)$$

where AM are OM are the number of moles in the atmosphere and ocean, respectively, and $OM/(\delta + 1)$ signifies the upper ocean only. B is the ratio of dissolved CO_2 to total oceanic carbon, a function of acidity:

$$B = \frac{1}{1 + \frac{k_1}{[H^+]} + \frac{k_1 k_2}{[H^+]^2}} \quad (12)$$

where k_1 and k_2 are dissociation constants. Alkalinity, Alk , is used to determine $[H^+]$ by solving:

$$\frac{M_{UP}}{Alk} = \frac{1 + \frac{k_1}{[H^+]} + \frac{k_1 k_2}{[H^+]^2}}{\frac{k_1}{[H^+]} + \frac{2k_1 k_2}{[H^+]^2}} \quad (13)$$

A.2 Parameter values

<i>parameter</i>	<i>representative value</i>	<i>source</i>
k_a (years ⁻¹)	.2	Bolin and Eriksson (1959)
k_d (years ⁻¹)	.05	Bolin and Eriksson (1959)
δ	50	Bolin and Eriksson (1959)
k_H	$1.91 \cdot 10^3$	Weiss (1974)
k_1 (mol/kg)	$1.00 \cdot 10^{-6}$	Mehrbach et al (1973)
k_2 (mol/kg)	$7.53 \cdot 10^{-10}$	Mehrbach et al (1973)
AM (mol)	$1.77 \cdot 10^{20}$	Warneck (1999)
OM (mol)	$7.8 \cdot 10^{22}$	Garrison (2009) – (for conversion see ⁶)
Alk	662.7 Gt C	(for conversion to $\mu\text{mol}/\text{kg}$ see ⁷)

Table 3 BEAM parameters. Temperature-dependent parameters k_H , k_1 , and k_2 are calculated here for $T=25^\circ\text{C}$. For convenience we state alkalinity in units of Gt C; our value is equivalent to $\sim 2002 \mu\text{mol}/\text{kg}$, slightly below standard literature values, e.g. $\sim 2364 \mu\text{mol}/\text{kg}$ (Sarmiento and Gruber, 2006).

⁶ $OM \approx 1.37 \cdot 10^9 \text{ km}^3 \text{ ocean} \cdot \frac{10^{15} \text{ cm}^3}{\text{km}^3} \cdot \frac{1.027 \text{ g seawater}}{\text{cm}^3 \text{ seawater}} \cdot \frac{1 \text{ mol water}}{18 \text{ g}} \approx 7.8 \cdot 10^{22}$ moles

⁷ $Alk = 662.7 \text{ Gt C} \cdot \frac{1 \text{ mol}}{12 \text{ g}} \cdot \frac{10^{15} \text{ g}}{\text{Gt}} \cdot \frac{1}{OM/(\delta+1)} \cdot \frac{1 \text{ mol water}}{18 \text{ g}} \cdot \frac{1000 \text{ g}}{1 \text{ kg}} \approx 2002 \mu\text{mol}/\text{kg}$

Most parameter values in Table 3 are well established. Time constants k_a , k_d , and the ratio of upper to lower ocean (δ) are not well constrained; we use reasonable values from Bolin and Eriksson (1959). We determine alkalinity by assuming equilibrium in the pre-industrial ocean at pH=8.28 (see Appendix A.3). It is also possible to specify Alk and adjust pre-industrial pH, k_a , k_d , and δ for best fit to more complex models. For maximal accuracy, use the temperature-dependent forms of Eqs. 14-16 for k_1 , k_2 , and k_H .

CO₂ solubility (Henry's law) (Weiss, 1974)

$$\mathbf{k}_H = k_0 \cdot \frac{1.027 \text{ kg}}{\text{liter seawater}} \cdot \left(\frac{\text{liter}}{55.57 \text{ mol}} \right) \quad \text{with} \quad (14)$$

$$k_0 = \exp \left(\frac{9345.17}{T} - 60.2409 + 23.3585 \cdot \text{alog} \left(\frac{T}{100} \right) + 35 \cdot (.023517 - .00023656 \cdot T + .0047036 \cdot \frac{T}{100})^2 \right)$$

where T is temperature in K , k_0 has units $\frac{\text{mol C}}{\text{kg seawater} \cdot \text{atm}}$, and k_H is dimensionless.

First and second dissociation constants (Mehrbaeh et al, 1973)

$$\mathbf{k}_1 = 10^{-pK_1} \quad \text{and} \quad \mathbf{k}_2 = 10^{-pK_2}$$

$$\text{with} \quad pK_1 = -13.721 + (0.031334 \cdot T) + \frac{3235.76}{T} + 1.3 \cdot 10^{-5} \cdot S \cdot T - (0.1031 \cdot S^{0.5}) \quad (15)$$

$$\text{and} \quad pK_2 = 5371.96 + (1.671221 \cdot T) + (0.22913 \cdot S) + (18.3802 \cdot \log(S)) - \frac{128375.28}{T} \quad (16)$$

$$- (2194.30 \cdot \log(T)) - (8.0944 \cdot 10^{-4} \cdot S \cdot T) - (5617.11 \cdot \frac{\log(S)}{T}) + 2.136 \cdot \frac{S}{T}$$

where k_1 and k_2 have units of mol/kg seawater , T is temperature in K , and S is salinity: $S \sim 35 \text{ g/kg seawater}$.

A.3 Initial conditions and model implementation

	Atm. CO ₂ (ppm)	M_{AT} (tC)	M_{UP} (tC)	M_{LO} (tC)	pH
BEAM Pre-industrial	280	596	591	29,574	8.28
BEAM present-day	380	809	604	29,595	8.17
DICE '07 present-day	380	809	1,255	18,365	(NA)
DICE '10 present-day	370	787	1,600	10,010	(NA)

Table 4 BEAM, DICE initial conditions. 'Present-day' is year 2005 for DICE 2007 and 2000 for DICE 2010.

We set pre-industrial carbon content of the ocean reservoirs by assuming that the ocean is in equilibrium with atmospheric CO₂ at 280 ppm (IPCC, 2007) and pH of 8.28. The pre-industrial pH is chosen so that after running forward with historical emissions (Boden et al, 2010), BEAM present-day pH matches that of UVic and CLIMBER-2 (~ 8.16 , Montenegro et al (2007)). Matching pH is needed for a valid comparison because pH values markedly affect CO₂ uptake rates. Both pre-industrial and present-day pH levels are uncertain by ± 0.05 (Feely et al, 2009). We define 'present-day' as that point where atmospheric CO₂ concentration matches 380 ppm, the 2007 DICE initial condition, which occurs near year 1995 for BEAM. The exact timing is not significant for subsequent evolution of CO₂ anomalies.

Because the atmosphere/upper ocean exchange timescale is short in BEAM, the present-day upper ocean is nearly in equilibrium with the atmosphere ($M_{AT}/M_{UP} \sim A \cdot B \sim 1.3$), though the lower ocean is slightly out of equilibrium ($M_{LO}/M_{UP} \sim 49$ while $\delta = 50$). Both DICE versions begin with excess atmospheric CO₂ relative to the upper ocean. (In 2007 DICE, $M_{AT}/M_{UP} \sim 0.6$ but $A \cdot B \sim 0.5$.) The larger upper ocean carbon reservoir in DICE (2007 and 2010) than in BEAM reflects a larger equilibrium upper ocean/atmosphere carbon ratio and can be thought of as a deeper mixed ocean layer. The smaller total ocean reservoir in DICE can be thought of as a smaller total ocean volume or lower initial pH that reduces the carbon storage factor.

The more realistic BEAM representation does have one drawback, that the strong sensitivity of coefficients mandates fine timesteps to avoid instability during numerical integration. The figures shown here were generated using 0.01 year timesteps. Timesteps as coarse as $1/10^{\text{th}}$ year can produce oscillation in pH and uptake/release of CO₂ from the ocean when emissions are changing rapidly. Resulting error in atmospheric CO₂ anomaly for the emissions scenario used here would reach several percent.

B Temperature model

For completeness, we describe the 2007 DICE temperature model, which appears to adequately capture temperature evolution (Fig. 5). Just as the ocean takes up CO_2 in response to atmospheric CO_2 perturbations, it also takes up heat in response to surface warming, with a long equilibration time because of the large thermal inertia of the ocean. Heat uptake in DICE is represented by a linear model similar to that used for carbon uptake. Because heat uptake is in reality largely linear, this representation adequately reproduces climate behavior. As with the carbon cycle, many of the coefficients in the DICE 2007 temperature model given below are calibrated to the MAGICC model (Wigley et al, 2007) or taken from the IPCC (2001) and IPCC (2007).

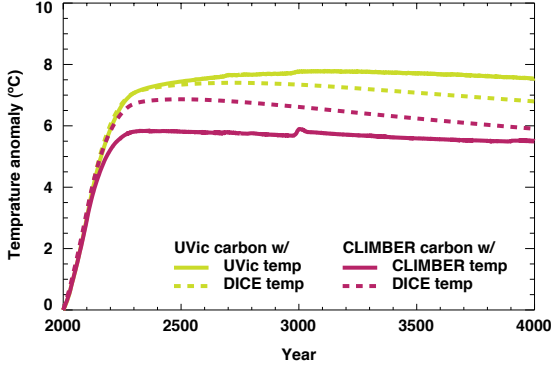


Fig. 5 The two-box DICE temperature model appears to adequately capture temperature evolution in more complex models. We drive the 2007 DICE temperature model with atmospheric CO_2 anomalies from UVic (green) and CLIMBER-2 (maroon) from the A2⁺ emissions scenario and compare atmospheric temperatures from DICE (dashed) to the other models' own temperature representations (solid). DICE temperature evolution differs somewhat in behavior but lies within the range of uncertainty.

The DICE temperature model uses only two layers, the atmosphere and lower ocean; the upper ocean is assumed to follow atmospheric temperature (Eqs. 17-18, but note that we have re-organized parameters to be more intuitive). Radiative forcing F due to increased atmospheric CO_2 warms the atmosphere (and upper ocean), producing a disequilibrium with the lower ocean that is eroded with timescales $1/\mu$:

$$T_{AT}(t) = T_{AT}(t-1) + \mu_{AT} \cdot [A \cdot (T_{eq}(t) - T_{AT}(t-1)) - \gamma \cdot (T_{AT}(t-1) - T_{LO}(t-1))] \quad (17)$$

$$T_{LO}(t) = T_{LO}(t-1) + \mu_{LO} \cdot \gamma \cdot (T_{AT}(t-1) - T_{LO}(t-1)) \quad (18)$$

where T s are atmospheric and lower ocean temperature changes (in $^{\circ}\text{C}$) since pre-industrial; γ relates atmosphere-ocean heat transfer to temperature anomaly ($\gamma = 0.3 \text{ W/m}^2/^{\circ}\text{C}$); A is climate sensitivity ($1.3 \text{ W/m}^2/^{\circ}\text{C}$); $F(t)$ is the increase in radiative forcing since pre-industrial, in W/m^2 ; and $T_{eq}(t)$ is the equilibrium temperature that would be produced by that forcing: $T_{eq}(t) = F(t)/A$. Note that equilibration timescales need not be equal since temperature is not a conserved quantity: $\mu_{AT} = 0.22 / 10$ years so $\tau_{AT} \sim 45$ years while $\mu_{LO} = (1/6) / 10$ years so $\tau_{LO} \sim 60$ years. Because DICE timesteps are long relative to these timescales, coefficient values differ from those of the continuum representation. For timesteps of 1 year or shorter, we recommend $\tau_{AT} \sim 36$ years and $\tau_{LO} \sim 140$ years to replicate the 2007 DICE temperature model.

Forcing $F(t)$ is assumed to be linear with the binary logarithm of the fractional change in CO_2 since pre-industrial times, a standard assumption in climate science⁸:

$$F(t) = \alpha \cdot \log_2(M_{AT}(t)/M_{AT}(PI)) \quad (19)$$

where $M_{AT}(PI)$ is the mass of pre-industrial atmospheric carbon (596.4 Gt, equivalent to ~ 280 ppm CO_2) and α is the assumed forcing increase per doubling of CO_2 ($\alpha = 3.8 \text{ W/m}^2$). The climate sensitivity A is derived by dividing α by β , the assumed equilibrium warming after doubling of CO_2 ($\beta = 3.0^{\circ}\text{C} / \text{doubling}$).

The 2010 DICE temperature model uses the same equations with small adjustments to three coefficients: $\gamma = 0.31 \text{ W/m}^2/^{\circ}\text{C}$, $\mu_{AT} = 0.208$ ($\tau_{AT} \sim 48$ years), and $\beta = 3.2^{\circ}\text{C} / \text{doubling}$.

⁸ In 2007 DICE, radiative forcing is specified as a function of CO_2 at timesteps (t) and ($t+1$), but that assumption can be relaxed without significant difference. 2007 DICE also adds a constant of 10^{-6} to M_{AT} to preclude $F(t) = 0$, presumably for numerical reasons. We neglect forcing from other greenhouse gases to compare with models driven only by changing CO_2 , but additional forcings can be added to $F(t)$.

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