What we can learn about the ocean in a 60 x 60 x 60 cm$^3$ tank

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Outline

• Oceanic vortices
• Laboratory vortices:
  - Self propagating
  - Adveected by a background flow
• Vortex collision with a seamount
• Comparison of laboratory results with oceanic observations
Meddy collision with one island

Richardson and Tychensky (1998)
Fig. 2. Summary of historical Meddy observations listed by Richardson et al. (1991) and Shapiro and Meschanov (1996) and shown by Richardson and Tychensky (1998). The diameter of the dots in this figure is approximately 50 km, somewhat smaller than the diameter of a typical Meddy which is around 100 km. Contours of the salinity anomaly of the Mediterranean Water relative to 35.01 psu near a depth of 1100 m are based on a figure by Needler and Heath (1995) and shown by Joyce (1981).
Fig. 12. Time sequence (20.9.1993 12UTC to 26.9.1993 12UTC) of ECMWF analysis fields. SLP (shaded, interval 4 hPa; max: 1002 hPa; 980 and 990 hPa bold) and temperature on 850 hPa (contour interval 4 K, from 0 to 12°C.).

Schwierz & Davies (2003)
Vortex generation: the “ice cube” technique

Whitehead et al. (1990)
Experimental apparatus: self-propagating

**SIDE VIEW**

- **A** to **A'**
- Depths: shallow and deep
- **61 cm**

**TOP VIEW**

- **D** = 1-16 cm
- **d** ≈ 4-12 cm
- **f** = 0.25 s\(^{-1}\)
- **h_0** = 10 cm
- **s** = 0.5
- **Obstacle**
- **Ice cubes**
- **Video camera**

**Note:**
- NORTH to SOUTH
- Orientation and dimensions indicated.
Self propagating vortex

\[ f = f_0 + \beta y, \quad \text{where} \quad f_0 = 2\Omega \sin \lambda_0 \quad \text{and} \quad \beta = \frac{2\Omega \cos \lambda_0}{R} \]

\[ PV = \frac{f + \zeta}{H + \eta} \quad \text{in absence of motion} \Rightarrow \]

\[ PV_\beta = \frac{f_0 + \beta y}{H_0}, \quad PV_s = \frac{f_0}{H_0 \left(1 + \frac{sy}{H_0}\right)} \approx \frac{f_0 \left(1 + \frac{sy}{H_0}\right)}{H_0} \quad \text{when} \; s \ll 1 \]

\[ \beta = \frac{f_0 s}{H_0} \]

Exact dynamical equivalence between the variation of the Coriolis parameter with latitude, the \( \beta \)-effect, and variation of topography in the presence of a constant \( f \)

N.B.: for Meddies a \( \beta = 1.57 \times 10^{-13} \, \text{cm}^{-1} \text{s}^{-1} \) is equivalent in the lab to \( s = 0.05 \)
Vortex westward drift

\[
PV = \frac{f + \zeta}{H + \eta}
\]

Cushman-Roisin (1994)
Self propagating: Barotropic flow
Experimental apparatus: background flow

**SIDE VIEW**

- Motor
- Cylinder
- Ice cubes

**TOP VIEW**

- \( d_c = 1, 3, 5, 6, 9 \text{ cm} \)
- \( d_v \approx 5-13 \text{ cm} \)
- \( f = 0.25 \text{ s}^{-1} \)

**Dimensions**

- \( 61 \text{ cm} \)
- \( h_0 = 10 \text{ cm} \)

**Flow Conditions**

- \( U \approx 2 \text{ cm s}^{-1} \)
Background flow
Other applications of a background flow

Aguilar and Sutherland (2006)
The “Reynolds number issue”

*Sorocco Island*

http://www.earthisart.gsfc.nasa.gov

![Image of Sorocco Island]

*“Lab” Island*

http://www.engineering.uiowa.edu/~cfd/gallery/vortex.html

*Re ~ 10^7*

![Image of Lab Island]
Tanabe & Cenedese (2007)

islands chain

\( G/d \)


cross sectional geometry

right vertical cylinder

\( D/d, Y/R \)

Cenedese (2002)

two islands

\( G/d, y/g \)

Cenedese et al. (2005)

\( G/d, y/g \)

height

\( h/h_0 \)

sloping sidewalls
Influence of sloping side walls: motivations

Conservation of the ratio of relative vorticity gradient and planetary vorticity gradient

\[
\frac{U_{lab}}{\beta_{lab} R_{lab}^2} = \frac{U_{oce}}{\beta_{oce} R_{oce}^2}
\]

Topographic beta

\[
\beta = \frac{sf}{h_0}
\]

\[
\begin{align*}
U_{lab} & \approx 0.2 \text{ cm s}^{-1} \\
R_{lab} & \approx 5 \text{ cm} \\
f_{lab} & \approx 0.25 \text{ s}^{-1} \\
h_{0lab} & \approx 10 \text{ cm}
\end{align*}
\]

\[
\begin{align*}
U_{oce} & \approx 5 \text{ cm s}^{-1} \\
R_{oce} & \approx 100 \text{ km} \\
f & \approx 10^{-4} \text{ s}^{-1} \\
h_{0oce} & \approx 1000 \text{ cm}
\end{align*}
\]

\[
\begin{align*}
\alpha_{lab} & = 83^\circ \implies s_{oce} = 5.0 \cdot 10^{-3} \\
\alpha_{lab} & = 70^\circ \implies s_{oce} = 1.7 \cdot 10^{-3}
\end{align*}
\]

\[
\begin{align*}
s_{lab} & = 64 \quad \Leftarrow \quad s_{oce} = 0.04 \\
\alpha_{lab} & = 89.1^\circ
\end{align*}
\]
South and Central collisions: $Y/R \leq 0$
South and Central collisions: \( \frac{Y}{R} \leq 0 \)
Circulation around a circular cylinder

\[ \frac{\partial \mathbf{u}}{\partial t} + (\zeta + f) \hat{k} \times \mathbf{u} = -\nabla \left( \frac{p}{\rho} + \frac{1}{2} |\mathbf{u}|^2 \right) + \text{Diss} (\mathbf{u}) \]

\[ \frac{\partial}{\partial t} \oint_C \mathbf{u} \cdot \hat{\mathbf{i}} ds = \oint_C \text{Diss} (\mathbf{u}) \cdot \hat{\mathbf{i}} ds \]

\[ \oint_C \text{Diss} (\mathbf{u}) \cdot \hat{\mathbf{i}} ds = \oint_C \nabla^2 \mathbf{u} \cdot \hat{\mathbf{i}} ds = 0 \]

\[ \oint_C \nabla^2 \mathbf{u} \cdot \hat{\mathbf{i}} ds = \oint_C \nabla \cdot R \, d\theta = 0 \]

\[ \oint_C v \, d\theta = 0 \]

\[ v_e \, \theta_e = v_s \, \theta_s \]

(Godfrey, 1989; Pedlosky et al., 1997; Cenedese, 2002)
Circulation around a circular and elliptical cylinder

\[ v_s E(\gamma_s, k) + v_e E(\gamma_e, k) = 0 \]

\[ v_e \theta_e = v_s \theta_s \]
Bifurcation occurs for: \[ 0.2 \leq \frac{D}{d} \leq 1.0 \quad \frac{Y}{R} \leq 0 \]
Bifurcation occurs for: \(400 \leq Re \leq 1100\)

\[ Re = \frac{v_s L_{\text{max}}}{v} \]

Adveced by background flow (diamonds):

\[ Re = \left( \left( \Omega_e \frac{\theta_e}{\theta_s} \frac{r}{R} \right) + U \right) \frac{L_{\text{max}}}{v} \]

Self-Propagating (triangles):

\[ Re = \frac{\Omega_e \frac{\theta_e}{\theta_s} \frac{rL_{\text{max}}}{v}}{\frac{\Omega_e \frac{\theta_e}{\theta_s} \frac{r}{R} \frac{RL_{\text{max}}}{v}}{v}} \]

(Cenedese, 2002)
Comparison with observations

\[ 0.2 \leq \frac{D}{d} \leq 1.0 \]

\[ d = 150 \text{ km} \]

\[ D_{cr} = 18 \text{ km} \]

\[ D_{i} = 36 \text{ km} \]

Cruiser
\[ \frac{D}{d} = 0.12 \]

\[ \frac{h}{h_0} = 0.82 \]

Irving Seamount
\[ \frac{D}{d} = 0.24 \]

\[ \frac{h}{h_0} > 1.0 \]

Richardson and Tychensky (1998)
North Brazil Current ring collision with multiple islands

Fratantoni and Glickson (2002)
Vortex collision with two islands: 
Geometry of the interaction

(Cenedese et al, 2005)
Dipole formation

anticyclone
cyclone
cylinder 1
cylinder 2
islands

cyclone
Dipole formation
Comparison with observations
Comparison with observations

\[ \frac{G}{d} = \frac{120 \text{ km}}{200 \text{ km}} = 0.4 \]

(Cenedese et al, 2005)
Conclusions

Throw an ice cube in a rotating tank…..it is fun!
… and … you can learn something about the ocean….

THANK YOU!