6.1 Workbook

Problem 6.1.1 Using Eq. **??**, show that for an optically thin grey gas the emission and absorption coefficients e_a,$e_{a,top}$ and a_+ defined in Section **??** are given by $a_+ = \tau_\infty$ and $e_a = e_{a,top} = \tau_\infty(\bar{T}/T_{sa})^4$, where $\bar{T}$ is the mean atmospheric temperature given by Eq **??**. Compute $(\bar{T}/T_{sa})^4$ and $a_+/(e_a+e_{a,top})$ for the following three cases:

- An all-tropophere ideal gas atmosphere on the dry adiabat, without pressure broadening.
- The same with linear pressure broadening
- The same with linear pressure broadening, but with the temeprature patched to in isothermal stratosphere at high altitudes

Under what circumstances is the radiatively-driven temperature jump at the ground unstable to convection?

Problem 6.1.2 Consider a Titanlike situation in which the dominant balance at the surface is between latent and sensible heat fluxes, and all other terms are negligible. Allow for stable boundary layer effects, so that C_D is a function of the Richardson number. Show that if C_D is nonzero, the solution becomes independent of $C_D U$, and derive an expression for the strength of the surface inversion. Show that there is another solution, with $Ri = Ri_c$ and $C_D = 0$, and derive an expression for the strength of the inversion in this case.

Put in numbers corresponding to a CH_4/N_2 atmosphere on Titan, assuming $h_{sa} = .7$, $U = 5m/s$, $z_1 = 10m$ and $z_* = .001m$, for temperatures in the vicinity of $95K$. Do the same for an Earthlike water/air atmosphere with temperatures in the vicinity of $350K$. How valid is the neglect of infrared radiation in the Earthlike case?

Problem 6.1.3 Consider the surface energy budget in a cold climate, in which evaporation can be neglected and the budget consists only of solar absorption, infrared, and sensible heat flux terms. Include the Monin-Obukhov theory for stable surface layers, and set the parameters to Earthlike values.

Plot the surface energy budget as a function of T_g for fixed T_{sa} so as to determine the equilibrium surface temperature. Try this for various values of the absorbed solar radiation $(1 - \alpha_g)S_g$, and discuss the strength of the inversion and the effect of turbulence suppression. Are there multiple equilibria? Are there sharp transitions between strong-inversion and weak-inversion cases as the solar absorption is changed? Repeat this calculation for some much colder and much warmer values of T_{sa}, so as to alter the importance of the infrared radiation term.

To keep things simple, you may assume that e_a is held constant at a value of 0.2 throughout this problem, and that the ground has unit emissivity. *Hint:* You should first get a feel for the problem by making a set of energy balance graphs analogous to Fig. **??**, but at some point you may wish to write a simple Newton's method routine to solve the surface budget equations iteratively. You can use the iteration to implement a function `Tg(Tsa)` using your favorite programming language.

Problem 6.1.4 Re-do the graph in Fig. **??** for parameters which make the surface layer more stable: smaller U, smaller relative humdity, and/or smaller solar absorption

Problem 6.1.5 Sublimation steals energy that could be more effectively used for melting. Study this by computing the latent heat flux for the cases in Fig. **??** and determining how much additional melting could be sustained if this energy went into melting instead. Study the issue further by re-computing the figure with lower and higher relative humidity. Increasing the wind speed increases both sensible and latent heat fluxes; does the net effect enhance or retard melting?

Problem 6.1.6 Use the linearized form of the surface energy budget to compute $T_g - T_{sa}$ in the weak evaporation limit, and use the resulting expression to discuss how the latent heat flux increases with temperature. Assume the boundary layer to be governed by neutrally stratified theory, so that C_D can be regarded as independent of temperature. For simplicity, you may also assume $e^* = .8$ throughout this problem, and neglect variations with temperature.

Problem 6.1.7 This problem explores the behavior of the weak evaporation limit when the surface layer becomes stable enough to significantly suppress turbulence. You may assume that the atmosphere is also cold enough that the buoyancy is determined by temperature alone, and is unaffected by the concentration of the condensible vapor. First determine the ground temperature $T_{g,o}$ at which turbulence is completely suppressed. You do this by setting $C_D = 0$ in the surface balance equation and finding an expression for the resulting radiatively-determined T_g. Then use this value to determine the circumstances under which $Ri = Ri_c$ for this temperature. This determines when the assumption $C_D = 0$ is consistent. Finally, assuming this condition to be satisfied for some given air temperature $T_{sa,o}$, discuss how T_g varies as T_{sa} is increased beyond $T_{sa,o}$ (holding other parameters fixed), retaining terms out to second order in $T_{sa} - T_{sa,o}$. Use the result to say how the evaporation behaves as a function of temperature in the weak evaporation limit. Illustrate your result by putting in some numbers characteristic of a cold Earthlike climate with weak absorbed solar radiation, with an atmosphere consisting of air and water vapor. For simplicity, you may assume $e^* = .8$ throughout this problem, and neglect variations with temperature.

Problem 6.1.8 As discussed in the text, the limiting strength of the inversion in the strong-evaporation limit is given by the temperature T_o where $F_L(T_{sa}, T_o) = 0$, i.e. where the latent heat flux vanishes. Using the expression for F_L, show that this occurs when $p_{sat}(T_o) = h_{sa} p_{sat}(T_{sa})$, where h_{sa} is the relative humidity of the air at the upper edge of the surface layer. Use the simplified exponential form of Clausius-Clapeyron to derive an expression for $T_{sa} - T_o$. How rapidly does this increase with T_{sa}? Use the results to put an upper bound on the growth of evaporation with temperature in the strong evaporation limit. Put in some numbers corresponding to Titan, and to a very hot Earth.

Problem 6.1.9 Determine equilibrium ice thickness for a slab of ice which allows solar penetration into the ice as discussed in text, but with the following extensions: (1) Assume that a geothermal heat flux F_i is applied at the base of the ice, and (2) Allow for the possibility that the ice is so thin or transparent that some solar radiation penetrates the ice and escapes into the ocean below.

First do the calculation assuming that solar radiation that leaves the bottom of the ice is carried away in the ocean and does not need to re-escape through the ice. Then say what happens if the energy is absorbed locally and heats the water below the ice, leading to the heat being delivered to the base.

For the purposes of this problem, you may neglect the effects of sublimation and accumulation at the top of the ice