Geosci 232 Take-Home Final Exam
Fall Quarter 2013

December 8, 2013

Instructions: This exam is open book and open notes, and you may also use any computer resources, including web searches and Python scripts distributed as part of the class or problem set solutions. You should not consult with any humans or other sentient sources, though. Turning in this exam constitutes your agreement to the statement: *I will not discuss the contents of this exam with any person, nor solicit advice from any person as to methods of solution, until after the end of the University's exam period. I will turn in my answers to this exam within 48 hours of the time I begin work on the problems.*

Further instructions: Feel free to use Python to help you compute things when you reach the point that algebra won't get you further or will take too long (this point will vary from one person to another). Thermodynamic constants you may need can be found in the table in Chapter 2; they are also in `phys.py`. Don't panic! There will be lots of partial credit. If you get this exam even half right you are doing adequately, provided you have made a reasonable effort on the problem sets. We want to see how much you've learned about how to think about the climate of planets. If you need to make a choice, it's better to do a few problems thoroughly than all problems poorly. Remember to justify your answers. We need to be able to understand your reasoning. Be sure to explain what your reasoning is as you do the problem, and justify your answers. If you can't get an answer all the way, you can still get significant credit by explaining clearly what you would need to know in order to calculate the answer, and what steps you are hung up on.

Each problem counts for the same number of points, even though some are harder than others. It is to your advantage to spot the problems which are easiest for you.

8 Final Exam Problems

Problem 8.1 An energy conscious home-owner has brought her electricity usage down to an average rate of 1 kilowatt. She wishes to design a compressed air storage tank able to store one day's worth of energy, as a way to store energy (to run a generator) as a buffer against fluctuations in renewable energy supply. To keep things simple, you may assume the generator converts stored energy into electricity with 100% efficiency. Note that the tank still contains energy when the pressure is equal to the outside air pressure (10^5 Pa), but that this energy cannot be extracted by running the stored air through a turbine, so this part of the energy doesn't count toward storage.

As a function of the maximum pressure p_o the tank can stand without exploding, determine the volume the tank needs to have in order to store the required amount of energy. Do this for two cases: (1) The tank is un-insulated, so the temperature of the contents stays constant at the ambient temperature (say, 280K) both during storage and withdrawal. (2) The tank is very well insulated, so that there is no heat exchange between the interior of the tank and the outside; in this case, also estimate the interior temperature the tank attains when fully charged.

Hint: The First Law of thermodynamics is the key to solving problems relating to energy.

Problem 8.2 A cylindrical space station is in orbit around Mercury. The air inside is stirred well enough that the entire interior is at the same temperature, and the shell is a good enough thermal conductor that it has the same temperature as the interior. What should the albedo be in order to keep the interior temperature at 280K? Give your answer in the form of a formula relating the required albedo to the dimensions (length and radius) of the cylinder, and note any cases in which additional heating would need to be supplied in order to maintain the desired temperature. Do the problem for the following two orientations: (a) The cylinder is oriented such that the axis is perpendicular to the line joining the center of the cylinder to the center of the Sun, (b) The cylinder is oriented with its axis pointed directly at the Sun.

Problem 8.3 Consider a planet made of a substance whose density is independent of temperature, but which is very slightly compressible so that the density is given by the equation of state

$$\rho = \rho_0 + \kappa p$$

where the compressibility factor κ is small enough that $\kappa p_c \ll \rho_0$, with p_c being the pressure at the center of the planet. Write down the set of equations that determine the mass-radius relation for such a planet. Then give an approximate solution for the mass-radius relation taking into account the effect of small (but nonzero) κ. *Hint:* First compute the pressure distribution you would get for $\kappa = 0$, then use that pressure to re-do the calculation with a slightly modified density.

Problem 8.4 Titan's atmosphere is a mix of N_2 and CH_4, but suppose that the atmosphere were pure N_2, with a surface pressure of 2 bars and a surface temperature of 100K.

(a) If the atmosphere were rapidly stirred by convection and there is no condensation of any substance, what would the temperature be at the 100mb level?

(b)At what level, if any, whould you expect N_2 to condense?

Problem 8.5 The planet GJ1214b orbits an M-dwarf star with a photospheric temperature of 3026K. The star has a radius of 0.211 times the radius of the Sun. The planet orbits its star at a distance of 0.0143 AU (1 AU = the mean distance of the Earth from the Sun. Assuming the planet to have an albedo of 30% and to be isothermal, what is its radiating temperature? Finally, suppose that the planet is instead an airless rocky body like the Moon or Mercury. What is the maximum surface temperature, assuming the planet to be tide-locked to its star?

Hint: The solar flux at Earth's orbit is 1367 $\mathrm{W/m^2}$. *This, (apart from a few universal physical constants) is the only additional data you need in order to do the problem.*

Problem 8.6 Estimate the total power radiated by the Earth at wavelengths in the Ultraviolet A range (.4 to .315 microns) and the Ultraviolet B range (.315 to .280 microns). Estimate the number of photons per year emitted in each range (recall Planck's law, $\Delta E = h\nu$). For the purposes of this problem, assume that the Earth has a uniform radiating temperature of $255K$.

Problem 8.7 A typical well-fed human in a resting state consumes energy in the form of food at a rate of $100W$, essentially all of which is put back into the surroundings in the form of heat. An astronaut is in a spherical escape pod of radius r, far beyond the orbit of Pluto, so that it receives essentially no energy from sunlight. The air in the escape pod is isothermal. The skin of the escape pod is padded with a good insulator, so that the heat flux through the skin is $k \cdot (T_i - T_o)$, where T_i is the interior temperature and T_o is the temperature of the surface of the pod, and k is a measure of the effectiveness of the insulation. The surface radiates like an ideal blackbody.

Find an expression for the temperature in terms of r, and k, and determine the necessary value of k for a few reasonable values of r. Is it better to have a bigger pod or a smaller pod? Is it ever possible to do without insulation? ($k = \infty$ corresponds to no insulation).

To what extent is the escape pod behavior analogous to the way a planet's atmosphere affects the surface temperature?

Problem 8.8 Venus has a surface pressure of $92bar$, and a surface gravity of $8.87m/s^2$. 3.5% of the atmosphere (by mole fraction) consists of N_2. Compute the mass of N_2 per unit surface area of Venus, and compare with the corresponding number for Earth's atmosphere.

Problem 8.9 Consider an atmosphere made of a gas which is transparent to infrared radiation, and which is well-stirred so that it's temperature profile is described by the dry adiabat. The surface pressure is p_s. The temperature of the ground equals the temperature of the air in contact with the ground. The atmosphere surrounds a planet with uniform surface temperature T_s, in an orbit where the stellar constant is L_*. The surface is perfectly absorbing throughout the spectrum, but the gas of the atmosphere absorbs no shortwave (stellar) radiation.

Suppose that a cloud is placed in a very narrow range of pressures near the pressure $p_c < p_s$. The cloud acts like a blackbody in the infrared, but has shortwave albedo α_c. The cloud is tightly coupled to the gas, so that the temperature of the cloud is identical to the temperature of the atmosphere at the cloud level.

(a) Compute the temperature of the planet as a function of the parameters of the problem. Discuss how the temperature depends on the value of R/c_p for the gas that makes up the atmosphere.

(b) Compute the energy budget of the cloud. To maintain the temperature of the cloud do you have to add heat to it or take it away? What do you think is the physical cause of this energy transfer?

Problem 8.10 During a runaway greenhouse, all of a planet's ocean evaporates into the atmosphere, and eventually escapes to space. Estimate how long it would take the Earth's ocean to evaporate into the atmosphere if all of the absorbed solar energy were used to evaporate water and no energy were lost to space as OLR. (In reality the process would take longer, since some energy is radiated even during a runaway). The mass of the Earth's ocean is $1.4 \cdot 10^{21}$kg. Give your answer in years.