A First Course in Climate

Earth and Elsewhere

Volume I: Thermodynamics and radiation

Volume II: Dynamics of the Atmosphere

(with just enough oceanography to get by)

R. T. Pierrehumbert
Department of Geophysical Sciences
University of Chicago
Chicago, IL

Preface

[**Course from standpoint of somebody who comes in from outside. My experience on moving to U. of. C. Envious of colleagues working on sweeping problems like Cretaceous warmth. What's a poor dynamicist to do? For that matter, what's a poor paleoclimatologist to do? (to get background in the necessary climate physics).]

[** "Just enough." i.e. just enough radiation, just enough thermodynamics, etc.]

[**Interweave data, theory and models. Include source code and data for just about everything used in book, on accompanying CD-ROM.]

1. Introduction – The Big Questions

Central (to us!): Robust equable climate of earth (all three phases of water co-exist). Has been like this for about 3. billion years, despite massive changes in continents, atmospheric composition and solar output. Below we will give an overview of some of the major outstanding problems for climate theory.

The Faint Young Sun

History of presumed solar output, and reasons (NB. recent speculations about early solar mass loss). Implies ice-covered, and perhaps ice-locked early earth (will go into ice-albedo feedback in more detail later in the course. Evidence for liquid water in Archaean and PreCambrian times. Many impacts 4.2-3.9Ga, may have brought oceans to boiling; high temperature crust, temperatures as high as 1200C. Crustal formation 3.8-4Ga. Microcontinents 3.8-2.5Ga, but large continents after 2.5Ga. Banded iron 2.5-2Ga, indicating oxygen and photosynthesis; expansion of blue-green algae (stromatolites -- carbonate evidence). In fact, earliest stromatolites are found 3.3-3.5Ga, and are coeval with the first carbonates (formed by weathering, and indicative of removal of CO_2 from atmosphere). Note life didn't have much time to get started! Life implies liquid water. Oldest glacial rocks are 2.7Ga. [**NB: for classes where some students have familiarity with energy balance models, use this as a vehicle for reviewing the material. Ignore atmosphere.]

Warm climate ages

Present kind of climate has cold ice-covered poles. Much of Earth History, have had warm, ice-free climates (or perhaps only seasonal ice). Formerly thought that these climates represented 90% of record, but re-examination of evidence, esp. last 100Myr, has thrown this into doubt. Early warm climates: Ice free earth from 4.6 to 2.5 Ga, and from 2.5 - .9 Ga despite faint sun. (But how good is evidence? Mostly comes from *lack* of certain ice-related deposits. Note, though, evidence that at least *some* liquid water existed throughout this time is quite firm.) Also generally mild from .6 to .1Ga. Note esp. mid-cretaceous (100Ma) and eocene (55Ma). Strong evidence that there was little or no permanent polar ice in the mid-Cretatious, though surface temperature estimates somewhat uncertain; not clear whether tropics was warmer than now. Dinosaurs up to Arctic circle. Eocene warm climate is the best documented. During eocene, tropics were about as warm as now, but poles were believed to be 10-15^{0}C warmer, and also without much of a winter. Alligators and lemurs as far north as 78°. Problem: How to warm up poles without warming up tropics?

Glaciation

From time to time, factors maintaining warm climates break down. First glaciation about 2.5Ga. Pre-cambrian glaciation: 3 major glaciations between .9 to .6 Ga. Particularly interesting because of evidence of ice even in paleo-tropics. The generally mild climate from .6 to .1Ga, had 2 major glaciations: Late Ordovician ca 440Ma and Permian, ca 280-225 Ma (length not well fixed), but there may also have been more seasonal ice than previously thought. Cooling between 50 and 3 Ma, leading into the present glacial era.

Effect of Bolide impact

Dust veil. Evaporation of water? Note esp. mass extinctions at KT boundary. Similar problems for major volcanic eruption eras. Possible future applications — what happens if Swift/Tuttle or something similar hits us?

Pleistocene ice ages

During the past 2M years, have good time resolution on fluctuations in extent of glaciers. During the whole time, there has been ice at the poles, but the ice-sheet has repetitively advanced and retreated. δO^{18} record (ice volume effect vs. sea surface temperature). Dominant period is 100Kyr, recently, but earlier was more like 40Kyr. Where we are now: just past the "climatic optimum" following previous ice age. Details of temperature and glacial extent over past 20Kyr. Problem of rapid deglaciation.

Comparative Climatology

Earth-Mars-Venus divergent climate evolution. Loss of water from atmosphere to space. Loss of other atmospheric constituents. "Water vapor traps?" Iceteroids? Evidence for early water & warm climate on Mars.

Planetary circulation patterns & jet structure. Venus, Earth, Mars, Jupiter, Saturn, Uranus & Neptune. Multiple jets and persistent eddies on the giant planets. Note that the most weakly thermally driven planets (Neptune, Uranus) have the strongest winds. Why?

Basic problems in present climates

Vertical structure of atmospheres: Troposphere & stratosphere, tropospheric lapse rate. Why is atmosphere stably stratified at all? Solar radiation heats atmosphere from below; why don't we get a "stellar type" convective circulation. [**NB for students that know basic atmospheric thermodynamics, use this as a vehicle for reviewing potential temperature and hydrostatic balance. This topic should perhaps be moved to next section.]

Determination of pole-equator temperature gradients (NB: Venus has essentially no P-E gradient), and meridonal heat fluxes in atmospheres and oceans.

Amplitude of the seasonal cycle. Seasonal cycle of lapse rate (will be interesting to compare with Mars!)

Determination of how much incident sunlight the Earth reflects; role of clouds.

Moisture distribution in the Earth's atmosphere. Note radiative importance of water vapor, esp. mid-tropospheric water vapor.

Note: The currently important issue of human-induced "greenhouse warming" represent small effects in the grand scheme of things (2-4ºK averaged over the globe and over the year.), which is why they are hard to predict precisely. But note also: Crowley points out that the combination of glaciated poles and warm extratropics may be unique in Earth history.

[**Pick out some of the above issues (e.g. faint young sun, CO2 needed for martian water), and keep coming back to it more and more quantitatively as the necessary tools are developed.]

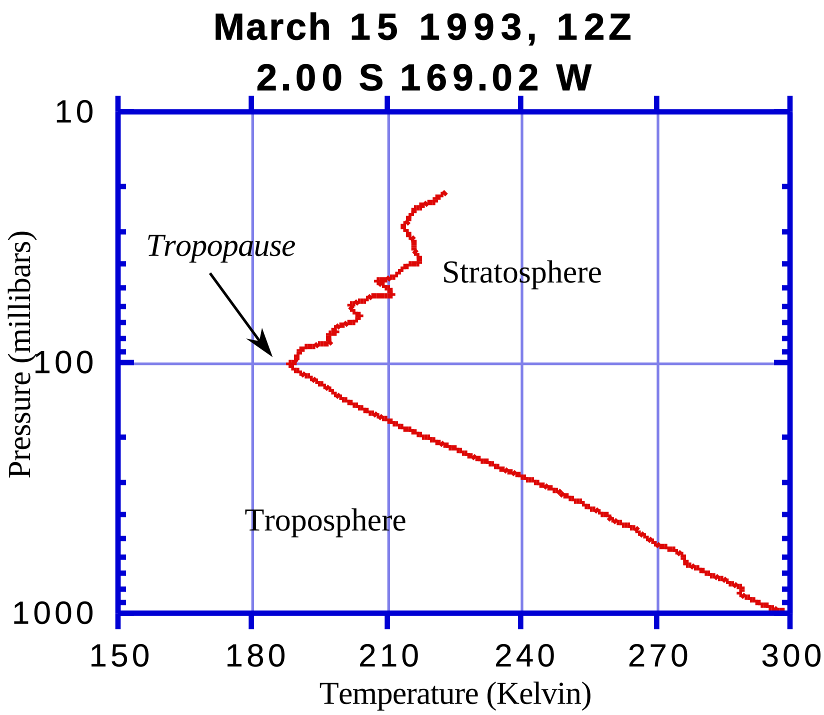
2. Vertical structure of compressible atmospheres

or, "All the thermodynamics I need to know, I learned in kindergarten."

The atmospheres which are our principal objects of study are made of compressible gases. The compressibility has a profound effect on the vertical profile of temperature in these atmospheres. As things progress it will become clear that the vertical temperature variation in turn strongly influences the planet's climate. To deal with these effects it will be necessary to know some thermodynamics — though just a little. This chapter does not purport to be a complete course in thermodynamics. It can only provide a summary of the key thermodynamic concepts and formulae needed to treat the basic problems of planetary climate. It is assumed that the student has obtained (or will obtain) a more fundamental understanding of the general subject of thermodynamics elsewhere.

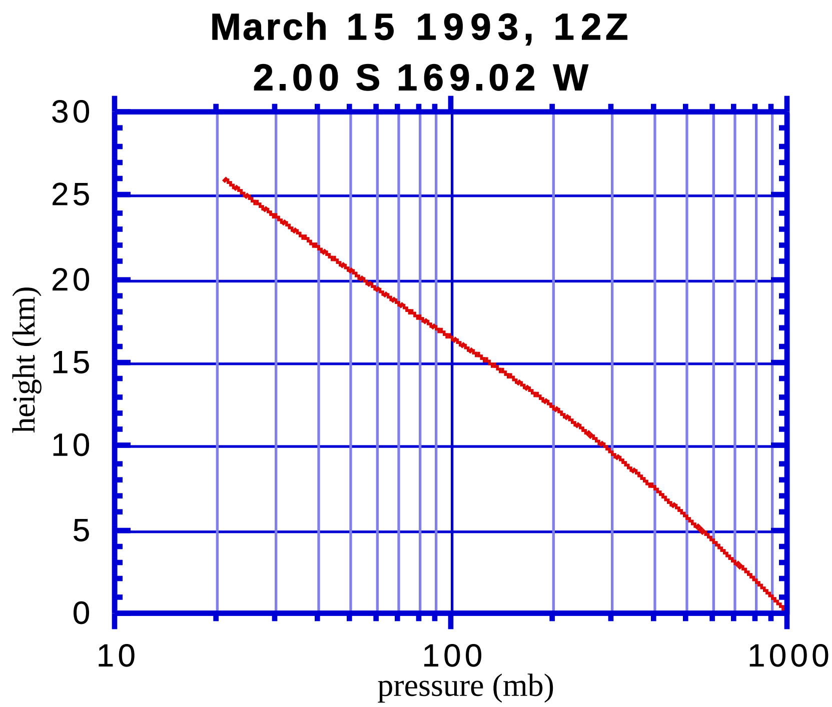
2.1 A few observations

The following temperature profile measured in the Earth's tropics introduces most of the features that are of interest in the study of general planetary atmospheres. It was obtained by releasing a "weather balloon" (more properly, a "radiosonde") which floats upwards from the ground, and radios back data on temperature and pressure as it rises. Pressure goes down monotonically with height, so the lower pressures represent greater altitudes.



Pressure is a very natural vertical coordinate to use. Many devices for measuring atmospheric profiles directly report pressure rather than altitude, since the former is generally easier to measure. More importantly, most problems in the physics of climate require knowledge only of the variation of temperature and other quantities with pressure; there are very few cases for which it is necessary to know the actual height corresponding to a given pressure. Atmospheres in essence present us with a thermodynamic diagram conveniently unfolded in height. Throughout, we will use pressure (or its logarithm) as our fundamental vertical coordinate.

However, for various reasons one might nevertheless want to know at what altitude a given pressure level lies. By altitude tracking of the balloon, or using the methods to be described in Section 2.3, the height of the measurement can be obtained in terms of the pressure. This relation is shown in the graph below. One can see that the height is very nearly linearly related to the log of the pressure. This is the reason it is often convenient to plot quantities vs. pressure on a log plot. If p_o is representative of the largest pressure of interest, then $-\ln(p/p_o)$ is a nice height-like coordinate, since it is positive and increases with height.



With the prelimaries regarding the choice of vertical coordinate out of the way, we can now return to our discussion of the critical aspects of the temperature profile. The most striking feature of the temperature sounding is that the temperature goes down with altitude. This is a phenomenon familiar to those who have experienced weather in high mountains, but the sounding shows that the temperature drop continues to altitudes much higher than sampled at any mountain peak. This sounding was taken over the Pacific Ocean, so it also shows that the temperature drop has nothing to do with the presence of a mountain surface. The temperature drop continues until a critical height, known as the *tropopause*, and above that height (100mb, or 16 km in this sounding) begins to increase with height. The portion of the atmosphere below the tropopause is known as the *troposphere*, whereas the portion immediately above is the *stratosphere*. "Tropo" comes from the Greek root for "turning" (as in "turning over"), while "Strato.." refers to stratification. The reasons for this terminology will become clear only later. The stratosphere was discovered by the French balloonist Montgolfier ([**track down date.]), who discovered the layer by noting that at a certain altitude his balloon precipitously ran out of buoyancy and came to a stop as if it had hit a ceiling.

The sounding we have shown is qualitatively typical, and raises a number of questions. Why does the temperature decrease with height in the lower layer? Why isn't the situation with cold air over warm air unstable? Why does the temperature increase with height in the stratosphere? What determines the tropopause height? How does it vary with time and position? Providing physically based answers to these questions, for the atmospheres of Earth and other planets, will be one of our main preoccupations in subsequent chapters.

[**Add some extratropical soundings, to give an idea of how tropopause varies with latitude. Do this as a composite page of several tropical and several extratropical soundings, with extratropical soundings over land and ocean, and at various seasons.]

[**Put in Galileo probe for Jupiter, and some Mars profiles. Phenomena not specific only to Earth.]

2.2 Dry thermodynamics

The three thermodynamic variables with which we will mainly be concerned are: temperature (denoted by T), pressure (denoted by p) and density (denoted by ρ). Temperature is a measure of the amount of kinetic energy per molecule in the molecules making up the gas. We will always measure temperature in degrees Kelvin, which are the same as degrees Celsius (or Centigrade), except offset so that absolute zero — the temperature at which molecular motion ceases — occurs at zero Kelvin. In Celsius degrees, absolute zero occurs at about -273.15 C, which is then 0K by definition.

Pressure is a measure of the flux of momentum per unit time carried by the molecules of the gas passing through an imaginary surface of unit area; equivalently, it is a measure of the force per unit area exerted on a surface in contact with the gas. In the mks units we employ throughout this book , pressure is measured in Pascals (Pa); 1 Pascal is 1 kg/(m s). For historical reasons, atmospheric pressures are often measured in "bars" or "millibars." One bar, or equivalently 1000 millibars (mb) is approximately the mean sea-level pressure of the Earth's current atmosphere. We will often lapse into using mb as units of pressure, because the unit sounds comfortable to atmospheric scientists. For calculations, though, it is important to convert millibars to Pascals. This is easy, because 1 mb = 100 Pa. Hence, we should all learn to say "Hectopascal" in place of "millibar." It may take some time.

Density is simply the mass of the gas contained in a unit of volume. In mks units, it is measured in kg/m^3.

For a *perfect gas,* the three thermodynamic variables are related by the perfect gas equation of state, which can be written

$$p = \rho \frac{R^*}{M} T$$

where R* is the *universal gas constant*, which is independent of the gas in question, and M is the molecular weight of the gas. We can define the gas constant $R = R^*/M$ pertinent to the gas in question. For example, dry Earth air has a mean molecular weight of 28.97, so $R_{dry\ air} = 287$, in mks units. Generally speaking, a gas obeys the perfect gas law when it is tenuous enough that the energy stored in forces between the molecules making up the gas is negligable. Deviations from the perfect gas law can be very important for the dense atmosphere of Venus, but for the purposes of the current atmosphere of Earth or Mars, or the upper part of the Jovian atmosphere, the perfect gas law can be regarded as an accurate model of the thermodynamics— in fact, "perfect," one might say.

An extension of the concept of a perfect gas is the *law of partial pressures*. This states that, in a mixture of gases in a given volume, each component gas behaves just as it would if it occupied the volume alone. The pressure due to one component gas is called the *partialpressure* of that gas. For a mixture of air and CO_2, for example, the partial pressures of the two gases are

$$p_{CO2} = R_{CO2}\, \rho_{CO2} T$$
$$p_{air} = R_{air}\, \rho_{air}\, T$$

where the two temperatures are taken to be identical, in accordance with thermodynamic equilibrium. Concentrations of atmospheric gases are often quoted in terms of ratios of partial pressure, and are referred to as "volumetric concentration." Thus, when it is said that the concentration of CO_2 in present Earth air is 330 ppmv (parts per million volume), what is meant is that $p_{CO2}/p_{air} = 330\ 10^{-6}$. The terminology has to do with the fact that a given number of molecules of any gas always occupies the same volume at a standard temperature and pressure. If we want to convert to the *mass mixing ratio* ρ_{CO2}/ρ_{air}, we simply take the ratio of the two gas laws, obtaining

$$\rho_{CO2}/\rho_{air} = (M_{CO2}/M_{air})\, (p_{CO2}/p_{air})$$

in which we have used the definition of the gas constants in terms of the universal gas constant. It is useful to keep in mind that there is enough atmospheric mixing to keep the mixing ratio of noncondensible components independent of height up to a great height in the atmosphere of Earth and other planets. This near-constancy applies in the stratosphere as well as the troposphere, and holds up to the *homopause* (about **km on Earth).

Condensible substances, like water vapor, have a sink internal to the atmosphere and therefore do not have uniform mixing ratios.

The specific heat at constant volume, c_v is the amount of heat that has to be added to a unit mass in order to raise the temperature by 1 degree K, while keeping the volume constant. For atmospheric purposes, specific heat at constant pressure, c_p, is a generally more useful quantity. The two specific heats may be related by noting that

$$\delta q = c_v dT + p\, d(\rho^{-1}) = c_v dT + d(p/\rho) - \rho^{-1} dp$$
$$= (c_v - R) dT - \rho^{-1}\, dp$$

whence we conclude that the specific heat at constant pressure is given by $c_p = (c_v - R)$. [**Remarks on how c_p depends on temperature. On pressure.]

Next, dividing both sides by T and using the perfect gas law again, we find that

$$\delta q/T = c_p\, dT/T - R\, dp/p = c_p\, d\,[\,\ln(\,T\;\; p^{-R/c_p}\,)\,]$$

assuming that c_p is constant. In essence, we have just derived the expression for the *entropy* of an ideal gas. For an *adiabatic process* $\delta q = 0$, which leads to the definition of *potential temperature*

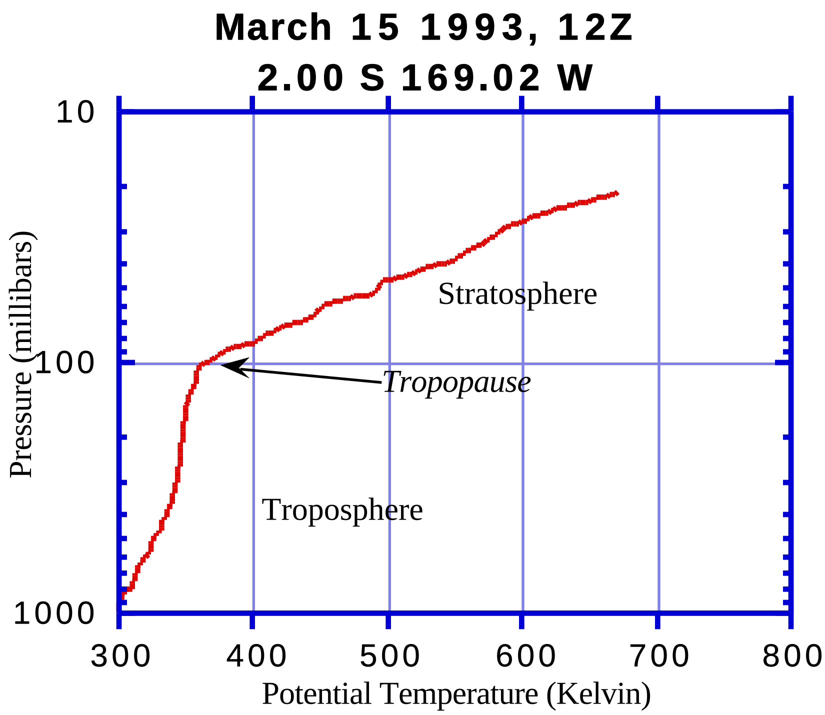
$$\theta = T\,(\frac{p}{p_o})^{-R/c_p}$$

The potential temperature is the temperature a parcel of air would have if compressed (or expanded) adiabatically to the reference pressure p_o. For Earth problems, we generally take p_o to be 1000mb. For dry adiabatic processes, it is the potential temperature θ, rather than T, which we expect to become uniform if a system is well mixed. The *dry adiabat* , or temperature profile corresponding to a constant potential temperature θ_0, can be written in log-pressure coordinates as

$$T(\,\ln(p_o/p)) = T_o e^{-(R/c_p)\ln(p_o/p)}$$

[**Discuss what determines R/c_p. Gives some values for various atmosphere.] A system in which θ is independent of height is *neutrally stable*. One in which θ increases with height is *staticallystable*, since one must fight buoyancy to displace air parcels in the vertical. One with θ decreasing with height is *staticallyunstable*, and should rapidly succumb to overturning motions which mix the system towards constant θ.

Below, we re-plot our temperature sounding in terms of potential temperature, instead of ordinary temperature.



Now we can see where the stratosphere gets its name; in this layer, the potential temperature increases sharply with height. Thus, it is strongly stratified. The stratospheric profile indicates that there has been little vertical mixing, and that the atmosphere will tend to resist vertical displacements. In contrast, the troposphere shows evidence of relaxation towards constant θ — though, importantly, it doesn't get all the way there. The mixing in question is primarily due to buoyancy driven turbulence, or *convection.* In the case of a planet with a solid surface, this arises primarily from strong solar heating of the ground by absorbed sunlight, but in other cases the convection can arise from absorption of energy in the lower atmosphere itself.

The upper troposphere in this sounding shows some evidence of approaching constant θ, but there is considerable remaining stable stratification in the lower troposphere. The determination of the degree of stable stratification in the troposphere, is an important and not completely solved problem in the study of climate. In Section 2.4 we will see that condensation of water vapor may be a significant factor in maintaining the tropospheric stratification, but this is unlikely to be the whole story. As more data become available, a study of the static stability of the Martian troposphere, for which condensation is thermodynamically unimportant, will provide important hints as to the essence of the phenomenon.

[***CDROM Data lab*: Study T(p) and θ(p) in Earth soundings given on CDROM. Need to include some extratropical soundings.]

2.3 Hydrostatics

If the vertical accelerations of the atmosphere are not too large, the vertical balance of forces reduces to a balance between the pressure gradient force and the force of gravity, just as if the fluid were at rest. This is known as the *hydrostaticapproximation,* and can be written

$$\frac{dp}{dz} = -\rho g$$

The hydrostatic approximation says that the pressure at altitude z is given by the net weight of atmosphere above z.

By integrating the hydrostatic relation from the ground to outer space, we can find the mass per unit area of the atmosphere in terms of the surface pressure, p_o. Specifically,

$$M_{atm} = p_o/g$$

For well-mixed substances like CO_2, this can be combined with the mass mixing ration measured at the ground, to get the entire atmospheric inventory of the substance. More generally, mass per unit area δM in a layer of thickness δp is given by $\delta M = \delta p/g$. This will prove very useful to us in our studies of absorption and emission of radiation by the atmosphere, both of which are proportional to δM.

Substituting the perfect gas law into the hydrostatic relation, we obtain

$$\frac{d}{dz} -\ln(p/p_o) = \frac{g}{RT}$$

If the temperature is constant, then this implies that pressure and density decay exponentially with height, with an e-folding scale H = RT/g. This is known as the *isothermal scale height*. Even if T is not strictly constant, the equation indicates that -ln(p/p_o) will be approximately proportional to height. [**Give examples of H for various atmospheres.]. In any event, the hydrostatic relation in this form allows us to reconstruct the altitude given only pressure and temperature data.

We can also use the hydrostatic relation to obtain the temperature profile T(z) for an adiabatic (i.e. constant θ) atmosphere. Taking the derivative of $T(\ln(p_o/p))$ with respect to the log-pressure coordinate, and using the chain rule and the hydrostatic relation, we obtain

$$\frac{d}{dz} T_{ad} = -\frac{g}{c_p}$$

This is known as the *dry adiabatic lapse rate*, and is constant if c_p is constant. The dry adiabatic lapse rate for Earth is about 10K/km, whereas observed tropospheric soundings typically have a lapse rate closer to 6.7K/km. This is just another way of saying that the troposphere has some residual static stability.

[**Dry static energy. Derivation from expression for δq, using hydrostatic relation to change dp to dz. Integration in case of constant c_p.]

[**Static stability, Brunt-Vaisala frequency.]

2.4 Moist thermodynamics

Many planetary atmospheres have a component that undergoes a phase change from gaseous to solid or liquid form at some places and times. This is the case for water vapor on Earth. It also occurs for carbon dioxide on Mars, for methane on Titan, and for ammonia, water and a few other substances on Jupiter and Saturn. [**Nitrogen condensation on Triton.]. The influence of condensation on atmospheric structure can be very important. Therefore, we need to know how to handle the thermodynamics of systems containing a component that undergoes a phase change. By analogy with water vapor, the familiar condensible substance on Earth, we shall refer to this subject as "moist thermodynamics," even though the condensible substance involved is often not water.

The first important concept from moist thermodynamics is *latentheat*. When a unit mass of substance at a given temperature undergoes a change to another phase at the same temperature, a certain amount of energy is released in the transition, owing to differences amongst the phases in the amount of energy stored in the form of intermolecular interactions. For example, when one kilogram of water vapor condenses into a liquid, it releases about $2.5\ 10^6$ Joules of energy. The amount of energy per unit mass involved in making the transition from one phase to another (keeping temperature fixed) is known as the *latent heat* of the transition. [**Latent heat of fusion. Of vaporization. Of sublimation. Examples and worked problems.]

The next important concept we take up is that of *saturation vapor pressure.* [**Give definition and discuss meaning. Uses definition of partial pressure, if condensible gas is in a mixture.] Note that condensation is a phase transition problem; the saturation vapor pressure is just the critical pressure for condensation.

Very general thermodynamic arguments imply that the slope of the saturation vapor pressure curve $p_s(T)$ is given by

$$\frac{dp_s}{dT} = \frac{1}{T}\frac{L}{1/\rho_v \ - \ 1/\rho_c}$$

where ρ_v is the density of the vapor phase and ρ_c is the density of the condensed phase. This is the Clausius-Clapeyron equation. Typically, $\rho_v << \rho_c$, so that the second term in the denominator can be neglected. If we further stipulate the perfect gas law for the partial pressure of the condensing substance, we have $1/\rho_v = R_v T/p_s$, so

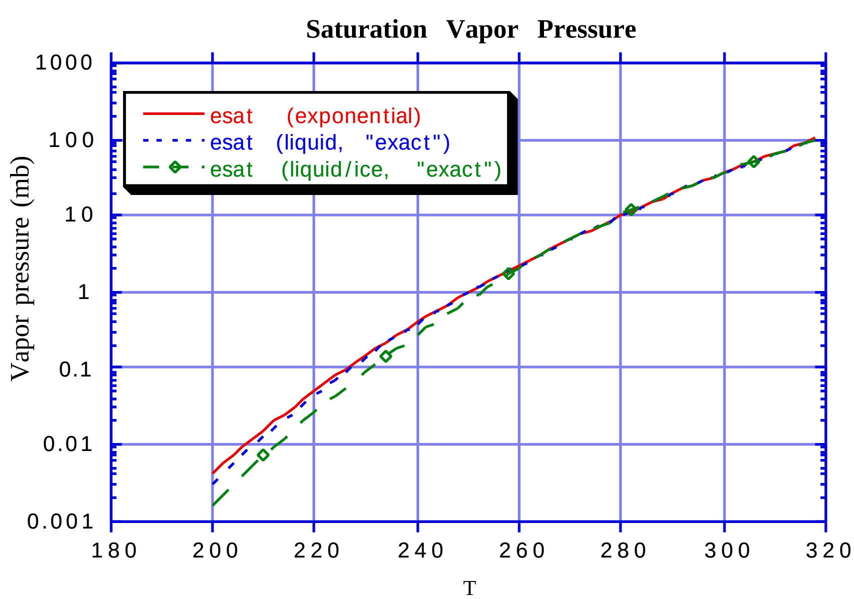
$$\frac{d}{dT}\ln(p_s) = \frac{L}{R_v T^2}$$

If, further, L is independent of temperature, we can integrate this relation analytically, obtaining

$$p_s = p_\infty e^{-L/(R_v T)}$$

For water, L/R_v = 5423K at room temperature. This equation implies that the quantity of condensible substance that the gas can hold goes down precipitously as temperature is decreased. This has profound implications for atmospheric composition.

Compare approximate $\exp(-L/R_w T)$ form, and compare with more exact empirical formula. Do comparison with and without ice; this is a good chance to point out the consequences of rh over ice vs. rh over water. Discuss some typical values for water near ground and aloft.



Discuss CO_2 condensation (and its relevance to present and early mars). Discuss condensation for methane and ammonia. What about Nitrogen and Oxygen? Why don't these (and CO_2) condense out of the Earth's atmosphere?

The moist adiabat (various condensible substances). Basic idea: as parcel is lifted, it cools. Condensation sets in, and the latent heat released warms the atmosphere and hence reduces the lapse rate to below the dry adiabatic value. Two limiting cases: (a) dilute condensible substance, like water vapor on Earth. (b) Case where primary component is condensible (e.g. CO2 on Mars, or steam in runaway greenhouse atmosphere.). Re-do atmospheric sounding given above, in terms of θ_e . Note that most of the Earth's atmosphere is not undergoing a moist adiabatic process at any given time, so it's not clear why the moist adiabat should govern the static stability.

3. Simple radiation balance models

3.1 Elementary properties of radiation

Climate is a problem in the interaction of electromagnetic radiation with matter — interaction of light from the Sun with matter making up the planet's atmosphere (if it has one) and surface (if it has one)[1]. [**We deal largely with something called "blackbody radiation", which is radiation in thermodynamic equilbrium.] While blackbody radiation can be conceived of as a gas of photons in thermodynamic equilbrium, characterized by a temperature T, in normal circumstances photons do not interact with each other strongly enough to distribute the energy amongst each other and bring the system to equilbrium. [**think of a gas of photons at T_1, another at T_2, and mix together. They will just carry out their ghostly co-existence and never relax toward a single temperature T.] Hence, in dealing with blackbody radiation, we mostly think of a photon gas in thermodynamic equilibrium with a blob of matter at temperature T. The absorption and re-emission of photons by the matter provides the scrambling necessary to maintain equilibrium.

[**Understanding BB radiation is one of the great triumphs of 20th century physics. See Pais book on Bohr for an intellectual history of the subject. Involves (indeed led to) quantum theory. In some ways, it was a more important development than relativity, since the effects of the quantum world are revealed through blackbody radiation in the macroscopic world. Quantum effects on radiation are critical to such things as the habitability of the universe.] [**Spectrum of blackbody radiation. The Planck function:]

$$B(\nu,T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT} - 1}$$

B = energy/(area time frequency). ν is the frequency [**In Hz? Check.]. h is Planck's constant ($6.625\ 10^{-34}$ Joule-sec), c the speed of light ($2.99725\ 10^{8}$ m/s), k the Boltzman thermodynamic constant ($1.38054\ 10^{-23}$ J/(^{o}K)). The Planck formula gives the intensity of the energy radiated in each direction. For blackbody radiation, the intensity is independent of direction. If we draw a plane, and want the total energy in a given

[1] We assume the planet has one of these. Jupiter basically has no surface, and the Moon basically has no atmosphere, but outside of Zen cosmology, it is hard to conceive of a planet with neither an atmosphere nor a surface. An ocean is just a special case of an atmosphere, which happens not to be very compressible.

frequency band going through the plane, we need to multiply the Planck function by π. Note limiting form for $h\nu/kT >> 1$, and its interpretation. For T=300K, the crossover frequency is $\nu = 6.4\ 10^{12}$ Hz, corresponding to a wavelength of $\lambda = c/\nu = 47\ 10^{-6}$ m = 47 μ (microns). This is in the deep microwave range. [**Check these numbers out for missing factors of π, etc.]. [**The maximum of B is at $h\nu/kT = 2.821439$, i.e. $\nu = 0.587711 \times 10^{11}$ T, or ν (gigahertz) = 58.77T

[**Remark on spectral energy density in wavelength space. Max emission wavelength. $(hc/(kT\lambda)) = 4.965114$, i.e. $\lambda T = 2.896880 \times 10^{-3}$. e.g for T = 300K, λ is about 10 microns. Peak of spectral energy density fn not a very meaningful quantity. Just provides a point of reference on the curve. More meaningful to do spectral energy density in log units (octaves), which are independant of how the spectrum is represented.]

Stefan-Boltzman law. Integrate Planck function w.r.t. frequency. Total flux per unit are of surface of a body, integrated over all forward directions. $F = \sigma T^4$. $\sigma = 5.6696\ 10^{-8}$ Watts/(m^2 $^oK^4$). In terms of the fundamental constants of the universe, $\sigma = 2\pi^5 k^4/(15c^2h^3)$. To derive this it is useful to know that

$$\int_0^\infty \frac{x^3 dx}{e^x - 1} = \frac{\pi^4}{15}$$

[**Some remarks about what the universe would be like if h were smaller or bigger. Quantum effects in everyday life.]

[**Solar radiation. Solar "constant." $L = I_o/r^2$, where I_o is the luminosity (like the wattage rating on a light bulb) and r is the distance of the planet from the Sun.]

3.2 Global ("zero dimensional") radiation balance models.

Consider a spherical planet with *no atmosphere*. Let a be the radius of the sphere, and L be the Solar constant. Assume (*mirabile dictu)* that the temperature of the surface is constant over the whole sphere. Then, the net radiation budget of the sphere implies

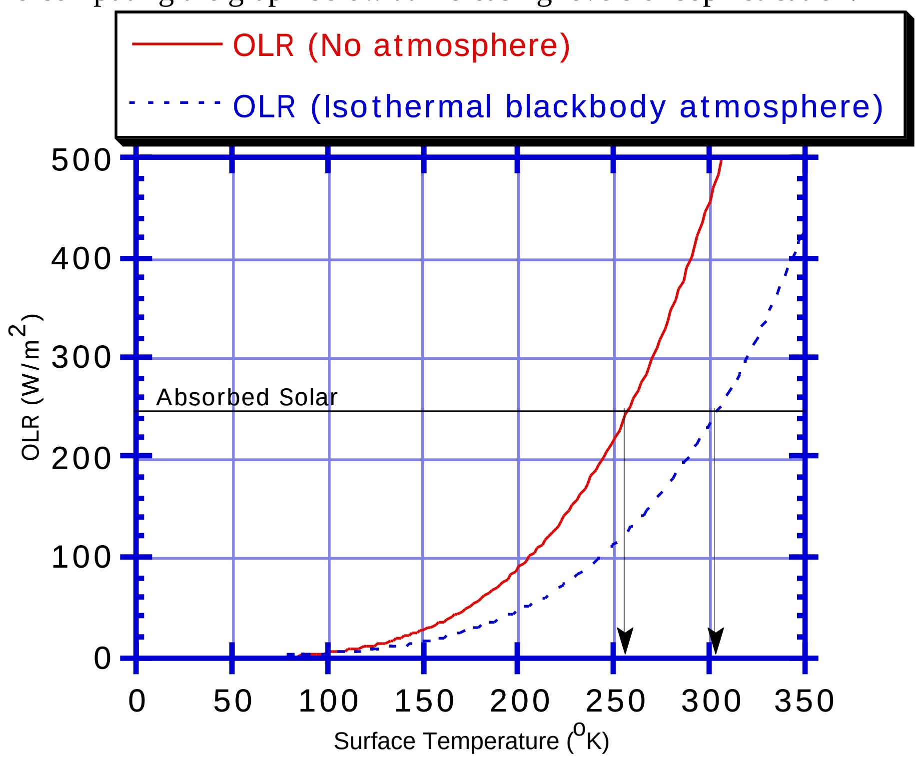
$$(1-\alpha)\ \pi a^2 L = 4\ \pi\ a^2 \sigma\ T^4$$

where α is the *albedo*, i.e. the proportion of the incident radiation that is reflected back to space. The factor πa^2 reflects the fact that the sphere intercepts a disk of radiation. Its shadow is a circle, but the sphere radiates over its whole surface, whence the $4\pi a^2$ factor on the right hand side. Equivalently,

$$\frac{1-\alpha}{4} L = \sigma\ T^4$$

The right hand side of this expression is the radiation to space, or "Outgoing Longwave Radiation." ("OLR"). [** Introduce notation "S" for absorbed solar flux per unit area of radiating surface. Relation to L depends on geometry.]

[**Basic temperature calculation is to match absorbed solar radiation (the LHS) to OLR.] An example of this procedure is shown in the graph below. Much of this course consists of re-computing the graph below at increasing levels of sophistication.



L = 1380 at mean radius of Earth's orbit. Importance of albedo. For terrestrial conditions with α = 0, T = 279ºK. For α = .3, T = 255ºK. Actual mean surface temperature of Earth is **ºK. For Mars, T= . For Venus, T = . Distance from sun is one important factor governing climate. (Note L scales like r^2, where r is distance from sun, so T scales like $\sqrt{r}$, i.e. not too sensitive to distance.). Albedo is an even more important one. Note role of clouds and ice in determining albedo.

Planetary distances (A.U.)

Mercury	.387
Venus	.723
Earth	1.00
Mars	1.524
Jupiter	5.203
Saturn	9.539
Uranus	19.218
Neptune	30.06
Pluto	39.44

Some Planetary albedos

Moon	.07
Mercury	.06
Mars	.15
Earth	.35
Jupiter	.51
Saturn	.50
Uranus	.66
Neptune	.62
Venus	.76

Visible albedos of some surfaces

Fresh snow	.75-.95
Old snow	.4 - .7
Dry sand	.35 - .45
Dark soil	.05 - .15
Water	.06 - .2
Forest	.05 - .2
Cumulus	.7 - .9
Stratus	.59-.84
Cirro-stratus	.44 - .5

[**Remarks on geometry and heat transport. The above is a "copper earth" example. We did a global radiation budget, with uniform surface temperature. Rather unrealistic to assume temperature uniform without atmosphere/ocean, since atmos/ocean would be needed to move heat around and keep the temperature uniform. Rotation, and "thermal inertia" help even out day/night cycle. Contrasting example of noontime equatorial temperature. Equatorial temperature for no-atmosphere case, with rapid enough rotation to average out the temperature. around the Equatorial belt. So, even without radiative effects, atmosphere/ocean is having a profound effect on the global temperature, by evening out such contrasts in time and space. Remark: T^4 is a nonlinear function. Effects of temperature variations on OLR. Give latitudinal temperature range of earth, and say what effects on OLR would be based on T^4. Point out that OLR(T) for real earth may be rather different (come back to this later). Temperature variations act to increase OLR (for a fixed global mean temperature), and hence give a cooler planet. Conversely, a planet with a more uniform temperature will tend to have a warmer mean temperature than one with large seasonal and/or latitudinal gradients. Note implications for positive mixing feedback in warm climates like the Cretaceous. Will return to this subject in Chapter **.]

Now let's clothe our bare planet with an atmosphere. At this point, we need to consider the spectrum of solar and terrestrial radiation. There is little atmospheric absorption of solar radiation (on Earth 19% is absorbed in atmosphere, 51% at the ground, the rest is scattered back to space). There is little scattering of outgoing IR. The basic picture is that the Earth absorbs solar radiation (mostly at the surface of the planet) and re-radiates energy as IR, some of which is absorbed by the atmosphere and re-radiated (as IR) upward to space and downward back to the surface.

Often, in this course, it will be sufficient to idealize the atmosphere as being transparent to solar radiation. A notable exception will be the need to consider absorption of ultraviolet radiation as an explanation of the thermal structure of the Earth's stratosphere. We will be able to get by primarily with IR radiative transfer theory. Solar scattering by the atmosphere will not be treated in detail; we incorporate it only through a reflection coefficient, to be thought of as primarily due to clouds.

Consider an "atmosphere" which is a black body transparent to the solar radiation but which absorbs (and re-emits like a black body) all terrestrial IR. A key point is that the atmosphere has *two sides*, a bottom and a top, and therefore radiates from both sides. For simplicity, assume both IR albedo is zero. Then

$$\frac{1-\alpha}{4} L + \sigma T_a^4 = \sigma T_e^4$$

$$\sigma T_e^4 = 2 \sigma T_a^4 \quad , \quad \text{i.e. } T_a^4 = T_e^4 / 2$$

Hence, the planetary surface is warmer than the no-atmosphere case by a factor of $2^{1/4}$. (i.e. with zero albedo, get T_e = 332ºK (Hot! Even with 30% albedo, get T_e = 303.5). Actual mean surface temperature of the Earth is about 285K, currently. Note T_a = 279ºK, the "old" temperature of the earth. Combine the two equations. OLR is σT_a^4. Or, could use second equation, and get OLR as function of T_e. This is another instance of "OLR thinking," and is depicted in the [**blackbody graph] above.

Consistency problem: Atmosphere is colder than the planetary surface. Hot air will form at the bottom and rise, giving vertical mixing. Simple "radiative-convective" (box) model.

$$\frac{1-\alpha}{4} L + \sigma T_a^4 = \sigma T_e^4 - k(T_a - T_e)$$

$$\sigma T_e^4 = \sigma T_a^4 + k(T_a - T_e)$$

[**Solve for T_e and sub in 2.2.xa; surface flux drops out. T_a stays fixed, but surface temperature varies. This shows that thermal coupling between the atmosphere and surface is a key factor in governing surface temperature. If coupling is strong, then the "greenhouse effect" is not very potent (at least for an isothermal atmosphere.)]

[**Ice-albedo feedback. Equilibrium equatorial and global average temperature of ice-covered earth. Multiple equilibria. How to escape from ice-covered earth?]

3.3 Partial absorption: Emissivity and Kirchoff's Laws

[**Dealing with partial absorption of IR. Why a black body is called "black." Emissivity. Kirchoff's law: absorption coefficient = emissivity. Thermodynamic derivation. Derivation via time-reversibility.]

[**Applications of Kirchoff's law. Black-body atmospheric greenhouse effect for emissivity < 1. Skin temperature and the stratosphere. Effect of solar heating on the skin temperature — why a small solar heating has a big warming effect on the stratosphere.]

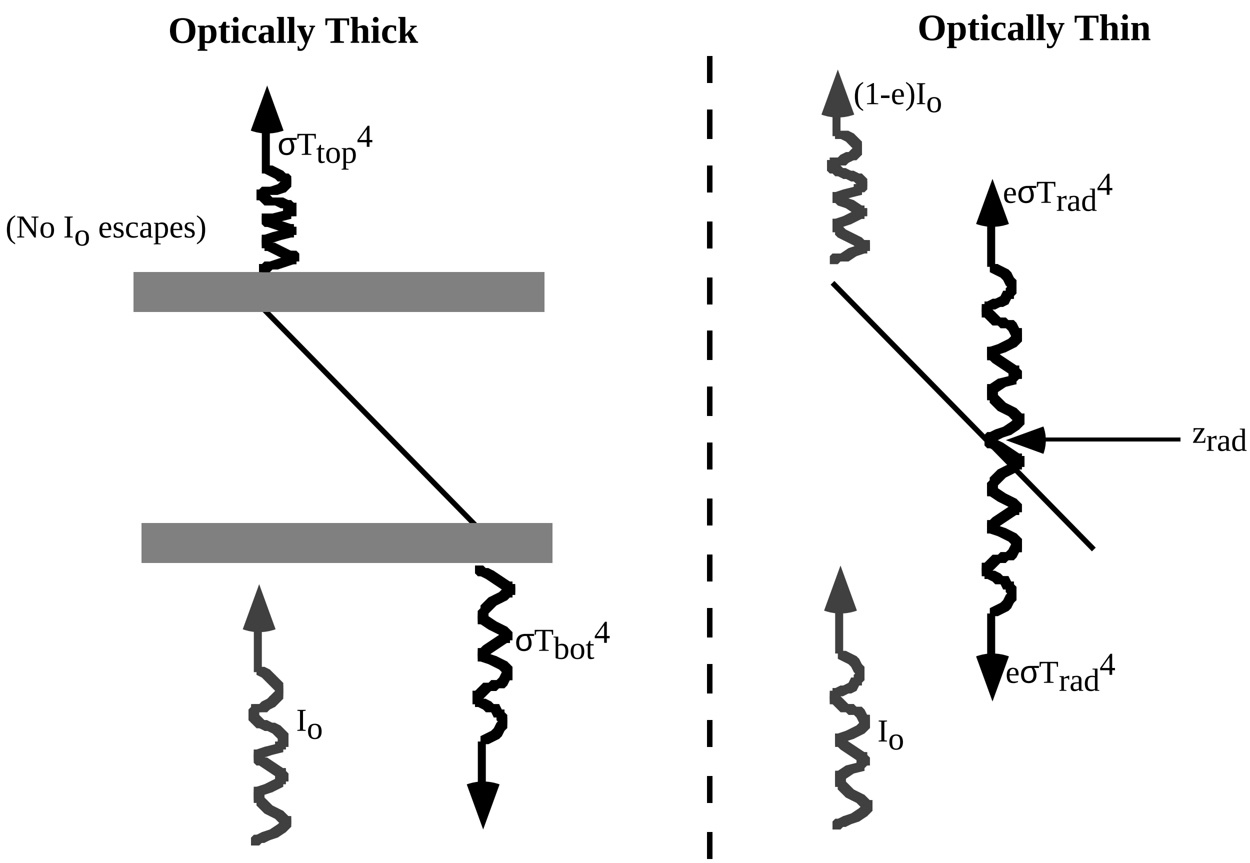
3.4 Making sense of the isothermal slab-atmosphere model

[**Basic question is whether we can make any sense of the slab atmosphere model, when applied to a real atmosphere which is manifestly non-isothermal. Can't discard it, because it seems to give reasonable temperatures for e=1.]

[**Difficulty of explaining Venus temperature with isothermal slab model, especially in view of its high albedo. Venus surface temperature is 753ºK, which is actually hotter than Mercury. What is missing? Compressibility effects of atmosphere (implies T decreases with height even if atmosphere is "well mixed" in vertical), combined with unequal radiation from "top" and "bottom" of atmosphere. Thick atmosphere of Venus: 88 atmospheres of mostly CO_2. Basic model for optically thick case. Radiating level H. $\sigma T(H)^4$ balances S. Then extrapolate along adiabat (because there is a troposphere, which is mixed) to get the surface temperature. Venus has so much absorber that the radiating level is moved high up on the adiabat.]

[**Some "handwaving" estimates of greenhouse warming for optically thick vs. optically thin cases, assuming temperature goes down linearly with height. For optically thin case, assume radiation out top and bottom of atmosphere is the same, and is given by an effective radiating temperature comparable to a mid tropospheric value. This also yields an estimate of the tropopause height: basically the atmosphere adjusts the mean atmospheric temperature by adjusting the tropopause height. Does this also eliminate the drive to instability at the ground? Will see later.

For optically thick case, put a black body at the top and at the bottom, and radiation is asymmetrical. Behavior is not approximated by an isothermal model. Will see in Section 4 that temperature decrease with height is critical to the way the Greenhouse effect operates. Earth is in the middle ground between Mars and Venus with regard to how the greenhouse effect operates.]



3.5 The Earth's observed radiation budget

The Top of Atmosphere Budget

[**Use this to motivate successes and failures of the slab-atmosphere models.]

[**Note that a good way of illustrating the effect of the atmosphere is to show a histogram of the OLR (with and without clouds), and compare it to a histogram of σT_s^4. The former bunches up at around 300 W/m^2, which is suggestive of the water vapor feedback.]

The surface radiation budget

[**Discuss downwelling IR back-radiation. Asymmetry between TOA and surface back radiation. Tropics vs. midlatitudes/poles. This shows that something is wrong with the slab atmosphere model. Discuss CEPEX IR back-radiation data from the Vickers ship. Means there is a big missing term in surface energy budget. (could show that surface temperature in data doesn't much exceed overlying air temperature, whereas radiative equilibrium would give much larger surface temperature); this is latent and sensible heat flux, which we will discuss later. How to close the imbalance? Defer full discussion of surface energy balance for later.]

[**This is a good place to point out that the observed surface temperature is usually close to the overlying air temperature, so thermal coupling is tighter than radiation would imply. Main exceptions are over dry deserts. Suggests evaporation is the culprit ("steaming gun.") Will return to this in Section **.]

3.6 Ice Albedo Feedback

[**Three great feedbacks: CO2 thermostat, water vapor, and ice albedo. In this section we will consider a simple model of the ice-albedo feedback. Simply put, the idea is that if you make a planet a bit colder, more of it is covered by ice. Then, since ice has a higher albedo than water or land, less solar energy is absorbed and the planet cools further. This can lead to many interesting phenomena. For simplicity, we will illustrate the working of the ice-albedo feedback in a planet with an atmosphere that has no greenhouse effect.]

Suppose that the planet is ice covered from latitude φ_i to the poles, in each hemisphere. Then the fractional area covered by ice is $1-\sin\varphi_i$. If α_o is the albedo of the ice-free surface and α_i is the albedo of the ice covered surface, the planetary albedo is

$$\alpha = \alpha_o \sin\varphi_i + \alpha_i (1 - \sin\varphi_i)$$

Next define a mean surface temperature by $\sigma \overline{T}^4 = (1-\alpha)L/4$. The meridional temperature gradient profoundly affects the ice-albedo feedback, so rather than assuming the planet to be isothermal, we allow for a specified meridional temperature profile

$$T = \overline{T} + \Delta T \left(\frac{1}{2} - \sin\varphi\right)$$

where φ is the latitude and ΔT is a specified constant. This profile has an area-weighted mean of $\overline{T}$. The ice margin is determined by the latitude where $T = T_f$, where T_f is the freezing temperature (273.15 for fresh water). From this we infer that the ice margin is at

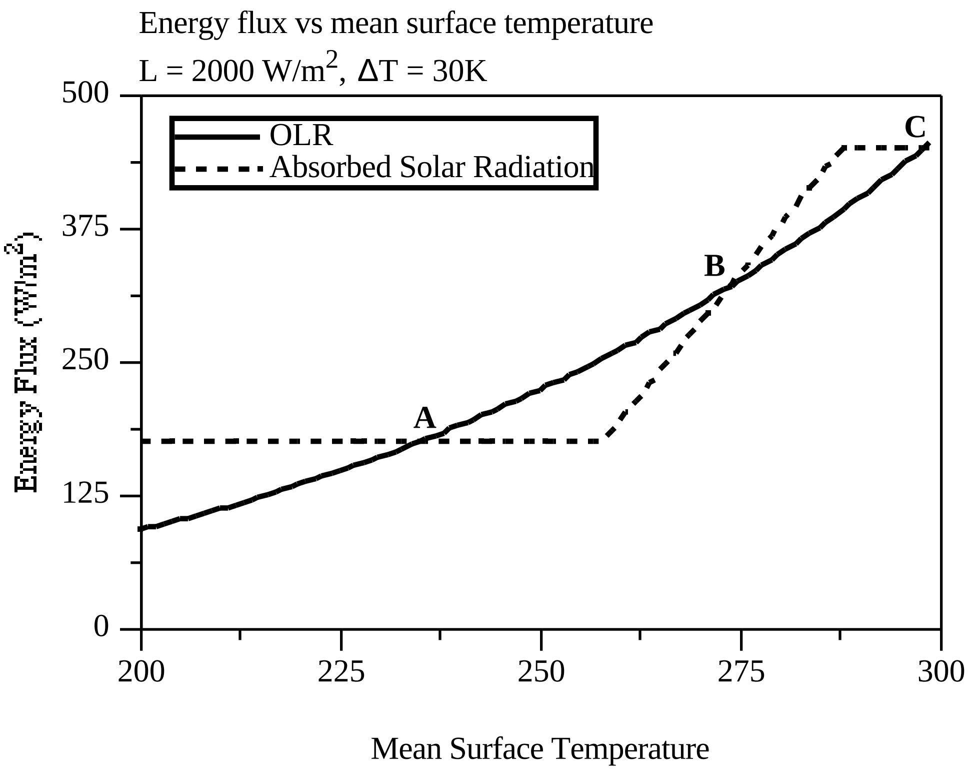
$$\sin\varphi_i = \frac{1}{2} + \frac{\overline{T} - T_f}{\Delta T}$$

From this we compute the coalbedo as a function of mean surface temperature

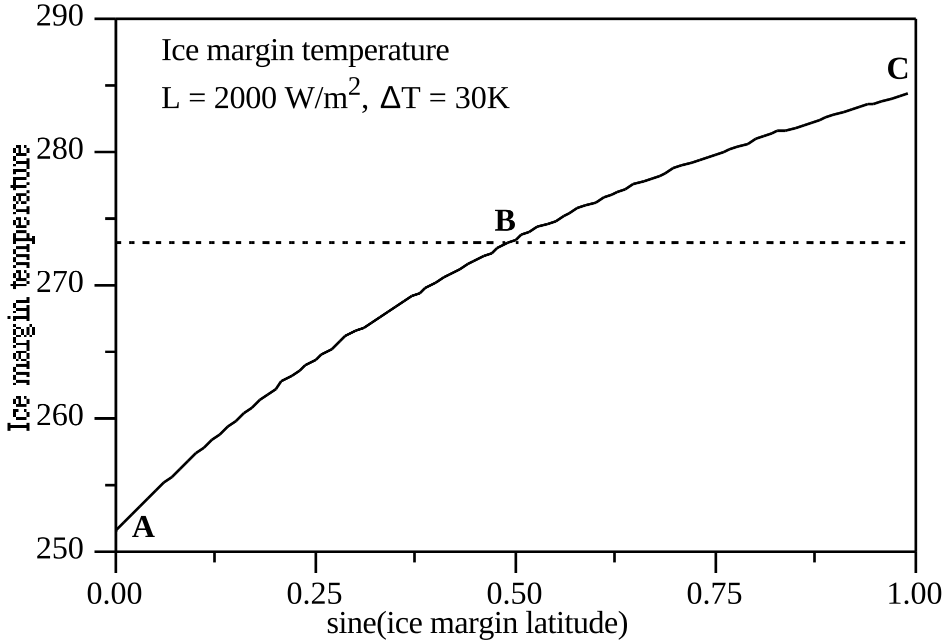
$$(1-\alpha) = \left(1 - \frac{1}{2}(\alpha_o+\alpha_i)\right) + (\alpha_i-\alpha_o)\frac{\overline{T} - T_f}{\Delta T}$$

[**Cut this off at $(1-\alpha_i)$ at cold temperatures, and $(1-\alpha_o)$ at warm temperatures so as to keep $\sin\varphi_i$ in its allowable range.]

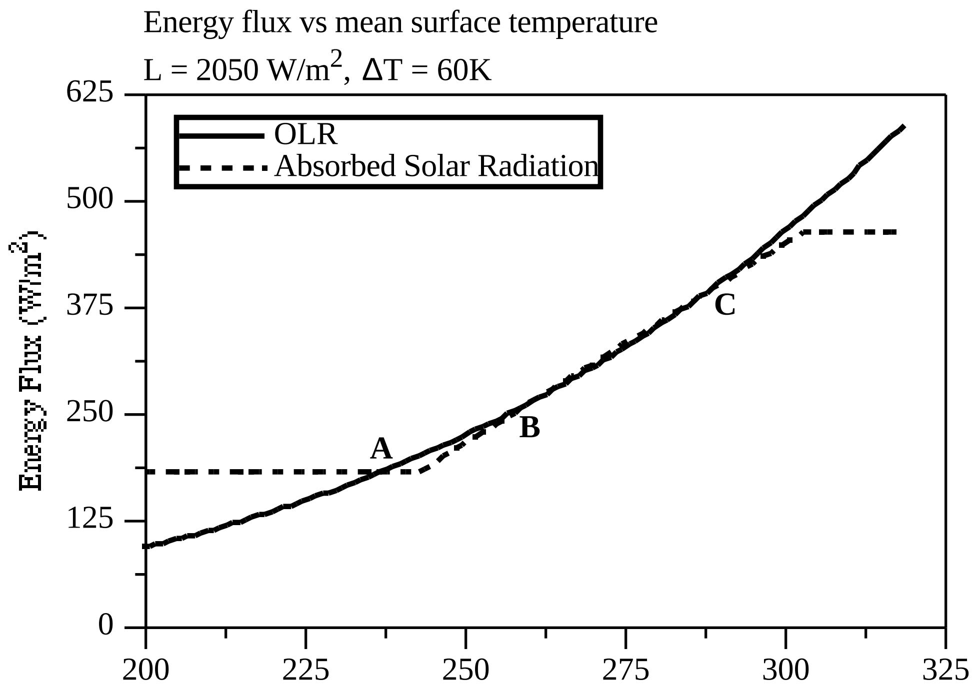
To find the equilibrium, we just plot the infrared cooling $\sigma\overline{T}^4$ on the same graph as the absorbed solar radiation $(1-\alpha)L/4$, and where the two curves intersect, we have a balanced state. An example of this for one particular set of parameters is shown below.

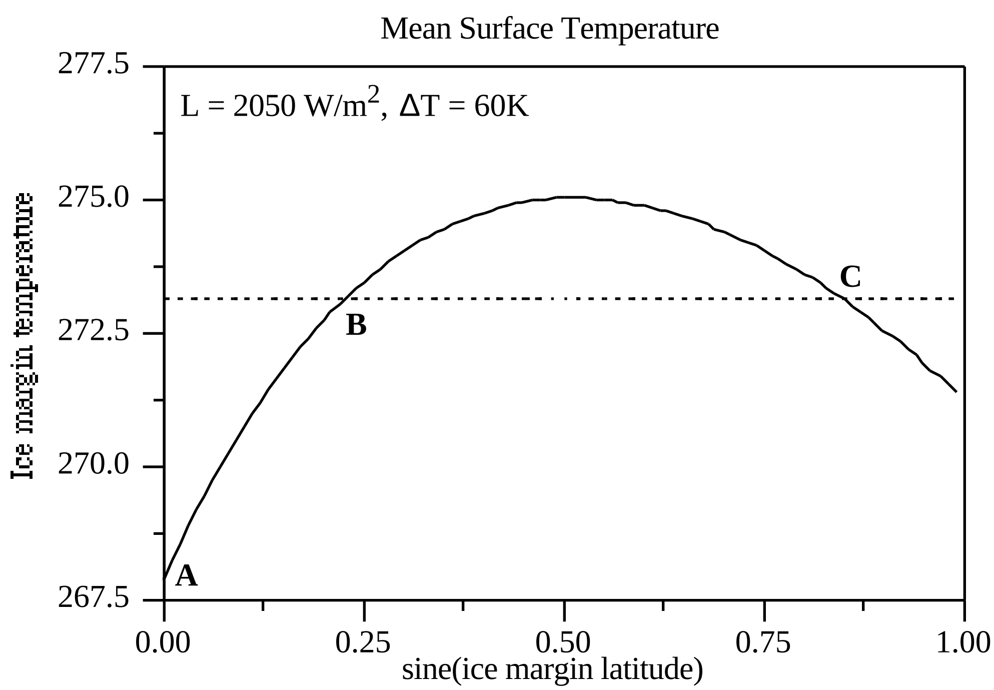


For the same solar constant, there are three possible states: an ice-covered state "A," a partly ice-covered state "B," and an ice-free state "C." [**Suggestive of Hothouse vs. Icehouse bimodality in Earth's climate history.] [**Issue of stability of ice margin. State B is unstable, as is determined by looking at the temperature at the ice margin, shown in the following graph.]



The state B is unstable, since a slight displacement of the ice-margin poleward causes the ice to melt, whereas a slight displacement equatorward causes the ice margin to fall below freezing, whereupon the equatorward water freezes and the ice margin advances until it covers the whole globe. The present state of the Earth is partly ice covered, and seems to be relatively stable. It is possible to replace the ice-free state by a stable partially ice-covered state by changing the parameters, as shown in the following two graphs:





The state B is unstable, but the state C is now stable. With the present model it is possible to get a stable state C to exist, but it is not easy, since the OLR curve and the straight-line solar absorption curve are nearly parallel. A more realistic expression for T(φ) can change the latter, whereas the inclusion of the greenhouse effect can change the shape of the OLR curve. Both effects can help make the stable state with glaciated polar regions more robust.

[**Do a third case, with small dT. In this case, the partially ice covered state can be the stable one.]

[**Fragility of the polar ice state suggests just a bit of change could cause it to be lost, leading to an ice-free state. Suggestive of Cretaceous and other hothouse climates.]

[**Discuss climate senstitivity for state C. Change L, and look how T changes, with and without ice feedback. Relevance to global warming.]

4. One-dimensional radiative-convective models

4.1 IR radiative transfer in a continuously stratified grey-gas atmosphere

[**Definition of optical depth, and what it means.] Usually, we choose z_o such that τ=0 at the ground. Since the emissivity/absorptivity is more or less proportional to the mass of absorbing matter encountered by the light beam, it is most natural to express the specific emissivity in terms of the density of the medium, so

$$\tau(z) = \int_{z_0}^{z} \rho(z)\ \kappa(z)\ dz = \int_{p}^{p_o} \kappa(p)\ dp/g$$

where κ is now the emissivity expressed in units of 1/(mass/area) ,i.e. it is a kind of mass cross-section. The second equality holds by virture of the hydrostatic relation. Note that if κ is bounded, then the atmosphere has a finite optical depth, since it has a finite mass. [**Note: The following expressions are for parallel beam radiation. In reality, IR radiation is *diffuse,* i.e., it comes in equal intensities at all angles. This can be handled by just re-defining optical depth with an apropriate proportionality constant out front.]

Schwartzschild equations for IR radiation, in terms of optical depth (grey atmosphere):

$$\frac{dI^+}{d\tau} = - I^+ + \sigma T^4$$

$$\frac{dI^-}{d\tau} = I^- - \sigma T^4$$

The general solution for arbitrary T(z) (re-expressed as a function of τ) is

$$I^+ = \int_{\tau_o}^{\tau} \sigma\ T(\tau')^4\ e^{-(\tau-\tau')}\ d\tau' + A e^{-\tau}$$

and *mutatis mutandum* for I^-. *A* is a constant of integration used to satisfy one boundary condition on I^+, which is usually that I^+ at the ground is equal to the blackbody radiation flux emitted by the planetary surface. There is a similar constant of integration and boundary condition (usually applied at the "top" of the atmosphere, stating that there is no *incoming* IR, i.e. $I^- = 0$ at the "top") for I^-.

The formula for the intensities states that the radiation is a weighted average of the blackbody radiation, taken over a couple optical depths. If the atmosphere is "optically thick", i.e has a large difference in τ between the top and bottom, then the atmosphere radiates to space at the temperature of the "top" of the atmosphere, and to the ground at the temperature of the (usually colder) "bottom" of the atmosphere. If the atmosphere is "optically thin", then there isn't so much asymmetry between the outgoing flux and the flux radiated to the planetary surface, and both are connected to the "mean temperature" of the atmosphere as a whole. [**Implies difference between Venus and Earth. Optical thickness of Venus atmosphere is why it can support such a strong greenhouse effect.]

[**Radiation flux convergence and heating rates.] For a given profile of $I^+(\tau)$ and $I^-(\tau)$, the system will not ordinarily be in *radiativeequilibrium*; that is, the net amount of radiation going into the slab between τ and τ+dτ is not equal to the amount leaving. Thus the slab will heat up or cool down. The net heating rate (in units of energy absorbed per unit optical depth) is

$$H = -\frac{d}{d\tau}(I^+ - I^-)$$

How to turn this heating into more physical units (degrees/day).]

Now, let's do a few calculations to exercise our facility with the Schwartzschild equations, and get a feel for their consequences.

[**Show that an isothermal slab radiates like a black body when it gets optically thick. Compute cooling profile for such a slab. Note relevance to cloud top cooling in stratus decks. An alternate form of this calculation is an isothermal slab in contact with a surface of the same temperature.]

[**Discuss net radiation in and out for isothermal body and for body with T(z). Asymmetric downward and upward radiation for T(z). Discuss optically thick limit for $T(z) = T_0 - \gamma z$. This is a more precise derivation of the picture we sketched out in the previous chapter.] [**Surface greenhouse effect for general optically thick atmosphere with T(z) decreasing with height. Decrease of T with height due to vertical mixing and to compressibility. This might be a good lead-in to discussion of why surface heating creates a troposphere. Surface must get rid of S, as well as the incident IR back radiation.] Suppose T(z) is decreasing with height, at least near the ground, and that the atmospheric temperature at the ground, T(0) is equal to the ground temperature T_s. [**T decreasing because of tropospheric mixing, favoring an adiabatic lapse rate. T(0) equalizes to T_s because of thermal coupling to ground.] The downward IR flux at the ground is

$$I^-(0) = \sigma \int_0^\infty T^4 e^{-\tau} \, d\tau$$

and the surface energy balance is

$$F + I^-(0) = \sigma T_s^4 = \sigma T(0)^4$$

where F is the net solar flux reaching the ground (allowing for all albedo, absorption and geometric effects). Now, if the atmosphere is *optically thick*, then $I^-(0)$ is sensitive only to the temperature profile near the ground; thus we can write $T = T(0) - \gamma z$, where γ is the lapse rate. Further, T^4 can be linearized about T(0), since z is small over the range where the optical depth is order unity (this is in fact what we mean by *optically thick*). Thus,

$$I^-(0) = \sigma T(0)^4 - 4\sigma T(0)^3 \gamma z^*$$

where

$$z^* = \int_0^\infty z\, e^{-\tau}\, d\tau$$

The depth z^* is our measure of how optically thick the atmosphere is, and goes to zero as the atmosphere becomes thicker. Thus, optically thick atmospheres radiate to the ground at a temperature that is only slightly less than the surface temperature. [**If there were no convective heat transport, surface would warm to be much warmer than overlying atmosphere; strong convection would set in to rectify this. Write down surface energy budget. Convective heat fluxes would bring T_s to be similar to T(0). In that case, very little of the incoming solar energy is lost by reradiation to the atmosphere. The net IR cooling of the surface is weak (quote some typical values, for observations and for a dry atmosphere with present CO_2), and the dominant balance is between solar heating and convective heat flux (on Earth, mostly latent, at least in Tropics).]

[**Computation of OLR for an adiabatic greygas atmosphere. This is the first of three basic climate calculations: (1) "all troposphere" planet, (2) "all stratosphere" planet, (3) patching a troposphere into a stratosphere. Should this be here or further down? We haven't really discussed yet the way radiation drives the thermal instability that creates the troposphere.] Assuming the atmosphere to be optically thick, so that no radiation from the ground directly escapes to space, the OLR for an adiabatic atmosphere with (constant) potential temperature θ is

$$I^+(\tau_\infty) = \int_0^{\tau_\infty} \sigma\{\theta\, (\frac{p}{p_o})^{R/C_p}\}^4\, e^{-(\tau_\infty - \tau)}\, d\tau$$

We must eliminate p from this, to get a closed form expression. To do this, use $d\tau = -\kappa\, dp/g$, with constant κ The expression for OLR becomes

$$I^+(\tau_\infty) = \sigma\theta^4 (\tau_\infty)^{-4R/C_p} \int_0^\infty \zeta^{4R/C_p} e^{-\zeta} d\zeta$$

where we have introduced the dummy variable $\zeta = \tau_\infty - \tau$. We have also replaced the upper limit of integration with ∞ because we are assuming that $\tau_\infty$is large, so that given the exponential weighting in the integrand, it makes little difference if it is infinite or merely very large. The integral in the OLR expression is now only a function of R/C_p. [**Give value for 2/7 (air); 1.06. Will be close to unity. Note that for fixed temperature, OLR decreases to zero as atmosphere is made more optically thick. Note also that θ is the surface air temperature, by definition of p_o . Hence, balancing OLR against absorbed solar radiation, the low level air temperature increases without bound as the atmosphere is made more optically thick. Contrast with result for isothermal slab model of the greenhouse effect. Note that lapse rate is g/C_p, so by changing C_p with R fixed, one can examine the effect of lapse rate. For small lapse rate R/C_p becomes small, and the greenhouse effect vanishes (if the surface temperature equals the low level air temperature.)

Note that the upper part of the all-troposphere planet is hard to create or maintain. If there is any heating of it at all, then because of its low pressure it will acquire a very large θ. This will prevent low level air parcels from being able to penetrate, and a stratosphere will form to cap the atmosphere.]

[**Use formula to try to estimate surface temperature on Venus. Show what optical thickness would be needed to get surface temperature. What is the optical thickness for a 80 bar pure CO2 atmosphere? To answer this question, need to look in more detail at the way gases actually absorb IR radiation.]

[**Effect of clouds on OLR and surface temperature. Implications for remote sensing. Compute surface temperature for an adiabatic atmosphere, if we put clouds at the tropopause. Importance of cancellation between albedo effect of clouds and greenhouse effect of clouds.]

4.2 Some bad news and some good news: Non-grey effects

[**Reader should now be wondering: OK, just how optically thick is the Earth's atmosphere really? How does the emissivity depend on the amount of CO2? Answer is more complicated than one would like it to be.]

[**Water vapor and CO_2. Importance of window channels and saturation of absorption bands. Importance of the water vapor continuum.]

[**Non-grey gases. No longer have exponential decay of radiation. Discuss structure of band absorption. Discuss I(z) for average over a Lorentz shape. Discuss pressure dependence of band width (pressure broadening). For separated bands, net absorption (for constant mass and composition) is proportional to pressure. Discuss continua. Provide quantitative data on net absorption.]

[**Discuss Vickers FTIR data.]

These complications are bad news, because they mean that a very complicated radiation model is needed to compute the IR radiation profile [**Need to do calculation for separate τ for each ν, then sum up. In practice, do calc. for each band or group of bands]. This may be bad news for climate modelers, but it is good news for the life of the Earth, and for other would-be life supporting planets elsewhere in the Universe. [**Exponential absorption characteristic of a grey gas would make climate very sensitive to concentration of absorbers. Earth would get very warm easily. Estimate what doubling CO2 would do to temperature if CO2 were a grey gas. The saturation of the greenhouse effect is what makes CO2 a good thermostat.]

[**Using the NCAR radiation model, take a given (observed) T(p) and show OLR and surface back-radiation as a function of CO2 (for a dry atmosphere), and as a function of relative humidity (with dry stratosphere) for a no-CO2 atmosphere.]

[**Note, pressure corrected CO_2 in present earth atmosphere is about 1 m (check this). Thus, doubling or tripling CO_2 is in a range where emissivity is not changing much. Without water vapor and cloud feedback, the CO_2 warming would be pretty negligable. Difficulty of explaining warm Venus with CO2 alone.]

[**To do CO_2 absorption on Venus, the effects of myriad very weak absorption bands must be summed up. Not clear that we can account for sfc. temperature by CO_2 absorption alone. Is there water vapor on Venus? Could clouds be the answer? Note liquid droplets should act like black bodies. Putting clouds at a low pressure part of the atmosphere would give a warm surface temperature, by moving the emitting surface high up. How "blackbody" are the Venus clouds? What pressure level are they located at?]

4.3 Radiative equilibrium

Let's consider an atmosphere which is in local radiative equilibrium with the IR radiated from the planetary surface. We assume it is perfectly transparent to the incident solar radiation, so that the only heating of the atmosphere is due to IR absorption. [**First consider greygas case, which we can do analytically.] Local radiative equilibrium implies H =0, so $(I^+ - I^-)$ is a constant independant of τ. If τ_∞ is the optical depth of the top of the atmosphere ($z = \infty$), then $I^-(\tau_\infty) = 0$. Further, since the planet as a whole must be in radiative equilibrium with the incoming solar flux F, then $I^+(\tau_\infty) = S$. Thus, $(I^+ - I^-) = S$ everywhere. Further, upon adding the differential equation for I^+ and I^-, we obtain

$$\frac{d}{d\tau}(I^+ + I^-) = -(I^+ - I^-) = -S,$$

so (choosing a constant of integration to satisfy the upper boundary condition)

$$(I^+ + I^-) = -S(\tau - \tau_\infty - 1)$$

Finally, subtracting the equations for I^+ and I^- and using $H = 0$, we obtain the temperature as a function of τ:

$$2\sigma T^4 = (I^+ + I^-) = S(1 - \tau + \tau_\infty)$$

In radiative equilibrium, then, the atmospheric temperature decreases with optical depth. The rapidity with which the temperature decreases with *height* depends on the distribution of absorbers, which determines the function $\tau(z)$. [**Give some examples of $\tau(z)$ suggesting that T is roughly isothermal in the stratosphere. Discuss limiting temperature at the top of the atmosphere. Note that it is *colder* than the temperature of a no-atmosphere blackbody in equilibrium with S, by a factor $2^{-1/4}$. This is independent of whether the atmosphere is optically thick or optically thin. It is a characteristic temperature of a planet. Discuss values for Earth, Mars, Venus and Jupiter, and compare with observed upper-atmosphere values. On Earth, the comparison reveals the effect of ozone heating.]

With the above formulae, we have $I^-(0)$ and therefore can complete the surface energy budget to obtain the ground temperature T_s. Since $I^+(0) = \sigma T_s^4$, the surface energy budget becomes:

$$S + I^-(0) = \sigma T_s^4 \text{ , i.e. , } S + S(1+\tau_\infty) - \sigma T_s^4 = \sigma T_s^4$$

and the surface temperature is given by

$$T_s^4 = \frac{S}{\sigma}(1 + \tau_\infty/2)$$

When the atmosphere is optically thin ($\tau_\infty = 0$), then the surface temperature reduces to the no-atmosphere radiative equilibrium. The ground temperature increases without bound as the optical thickness of the atmosphere increases. For $\tau_\infty = 2$, we obtain the same greenhouse warming (i.e. surface temperature increases by a factor of $2^{1/4}$) as for the isothermal slab atmosphere; however, the surface temperature continues to increase as we make the atmosphere thicker, because the radiative equilibrium atmosphere is not isothermal. Its top is colder than its bottom, and so it radiates to the ground at a higher temperature than it radiates to space.

Note that

$$T(0)^4 = \frac{1+\tau_\infty}{2+\tau_\infty} T_s{}^4$$

Hence, as in the case of the isothermal slab atmosphere, there is a convective instability due to the surface being warmer than the atmosphere in contact with it. The strong convection inevitably ensuing will transport heat from the surface to the atmosphere, warming the atmosphere and cooling the surface until the lower atmosphere temperature becomes similar to the overlying air temperature. Such convection will also tend to mix up the atmosphere, though, leading to a roughly adiabatic troposphere. [**For consistency, need to patch together a troposphere with a stratosphere, which we will do shortly.]

[**Do optically thick and optically thin limit. Optically thin limit reduces to the isothermal slab model. For optically thick limit, surface temperature increases without bound.]

Heating at the ground is not the only possible source of instability though. Instability occurs if the temperature falls rapidly enough with height that the *potential temperature* decreases with height. Letting $\zeta = -\ln(p/p_s)$, then for an adiabatic atmosphere $d(\ln T)/d\zeta = -R/C_p$, and any atmosphere with a more negative gradient will be unstable. Since $d\ln(T)/dz = (d\ln(T)/d\tau)(d\tau/d\zeta)$, there can be convective instabilities interior to the atmosphere if $d\tau/d\zeta$ is large enough. Consider first the case of a uniform-composition atmosphere, ignoring the effects of collisional broadening [**NB have we defined collisional broadening yet?] For this case, we approximate $\tau = \kappa\,(p_s - p)/g$, where p_s is the surface pressure and κ is some constant. Then, $d\tau/dz = \kappa\,\rho$, using the hydrostatic relation. Further, we can evaluate $dT/d\tau$ using the formula for the radiative equilibrium temperature profile. Namely,

$$8\sigma T^3 \frac{dT}{d\tau} = -S$$

so

$$\frac{d}{d\tau}\ln(T) = -\frac{1}{4(1+\tau_\infty - \tau)}$$

Note that S has dropped out of this expression, so that the stability is independent of the insolation. Next multiply both sides by $d\tau/d\zeta$, and make use of the fact that $d\tau/d\zeta = \kappa p/g$. Thus,

$$\frac{d}{d\zeta}\ln(T) = -\frac{\kappa p/g}{4(1+\tau_\infty - \tau)}$$

So far, this expression is valid even if κ is a function of p. In the special case that κ is constant, then $\kappa p/g = \tau_\infty - \tau$, and the right hand side is a function of τ alond. This function of τ has a maximum at the ground, which approaches 1 in the limit of an optically thick atmosphere. It goes to zero at the top of the atmosphere, so that the radiative equilibrium is always stable sufficiently high up. Now lets take the "worst case" and ask whether the lapse rate is unstable (more negative than $-R/C_p$) near the ground in the limit of very large τ_∞. The magnitude of the lapse rate near the ground in this case is 1/4. Hence, the atmosphere is internally unstable if $1/4 > R/C_p$. Air, with $R/C_p = 2/7$, just misses being unstable. Hence, we must look to other means to generate the stirring to create the Earth's troposphere. [**Comment on other substances; pure water vapor, CO_2, hydrogen, helium.]

[**Discuss when a stable stratosphere is possible. Discuss effects of collisional broadening (via problem?). How do non-grey effects change this? On Jupiter and Saturn, where there is no surface, destabilization creating a troposphere must arise from internal destabilization, or from an internal heat source. Could conceivably be due to solar absorption, but since the available light decreases with depth (as in an ocean), this would tend to stabilize things (cf. problem on effect of absorption.)]

[** (moisture effects important in determining size of this factor, since moisture decreases strongly with height). Discuss role of moisture and clouds in making $d\tau/dz$ large and destabilizing the atmosphere. Discuss $d\tau/dz$ for a saturated radiative equilibrium atmosphere. Is it unstable?]

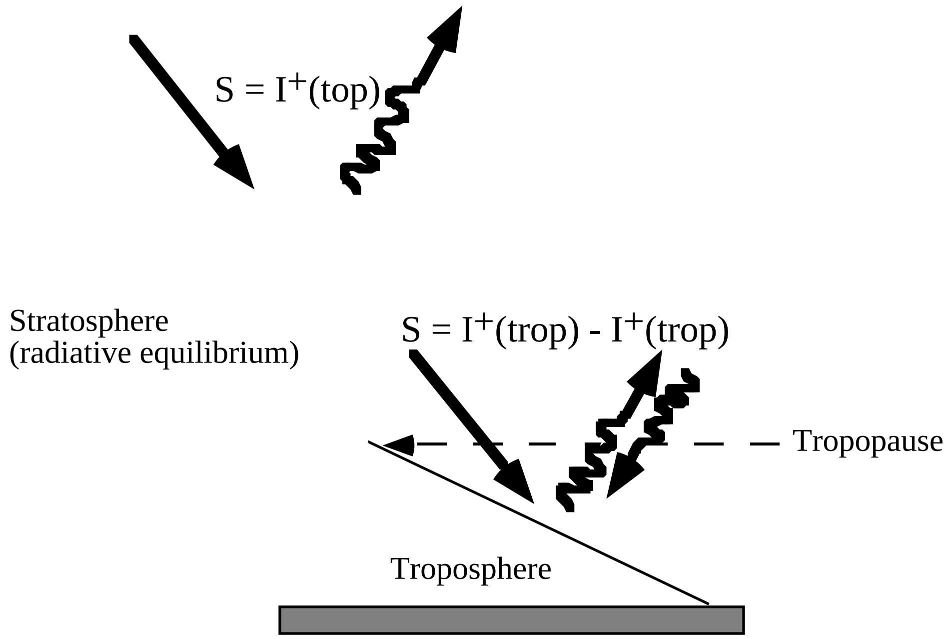
[**Approximation of convection by "convective adjustment." Troposphere (well mixed vertically, due to convection) and Stratosphere (not well mixed, roughly in local radiative equilibrium).]

[** Is there always a drive to convection from below? Have seen this twice now. For optically thick atmosphere, IR cooling of ground is weak, and ground will always heat up and stir convection. What happens in the optically thin limit?]

[**Nongrey effects. Give some radiative-equilibrium solutions using NCAR model, and compare with grey gas. Discuss optically thin vs. optically thick limit.]

4.4 Coupling radiation to convection: Making the Troposphere

[**Dry radiative convective model. Full problem: (a) $T(0) = T_s$ (finesses need to solve surface energy budget), (b) T continuous at tropopause, (c) Strat in radiative equilibrium, (d) Surface in radiative equilibrium, (e) Troposphere as a whole is in radiative equilibrium, though not locally at each height. Form of solution very dependent on the vertical distribution of the absorbers (for an example, do a pure CO_2 atmosphere).]



In the stratosphere, we have radiative equilibrium as before. Thus,

$$I^+ - I^- = S \qquad (I^+ + I^-) = S(1 + \tau_\infty - \tau) \qquad \sigma T^4 = \frac{1}{2}S(1 - \tau + \tau_\infty)$$

so

$$I^+ = \frac{1}{2}S(2 + \tau_\infty - \tau)$$

Now consider the troposphere. Unlike the stratosphere, the heat budget of the atmosphere need not be closed radiatively at each individual τ, because we also have turbulent mixing to move heat around. Instead, we only need to require that the energy budget of the troposphere *as a whole* be closed. This implies that

$S + I^- = I^+$ (evaluated at the tropopause)

However, in radiative equilibrium (which is valid just above the tropopause) $S + I^- = I^+$. Hence, the balance requirement will be automatically satisfied *provided that we require I^+ to be continuous across the tropopause.*

Let τ_{trop} be the optical depth of the troposphere. We finish the problem now by computing I^+ a second way, using the Schwartzschild equations within the troposphere, and setting the result equal to the stratospheric expression which is valid just above the tropopause. The continuity requirement on I^+ becomes:

$$\frac{1}{2}S\,(2+\tau_\infty-\tau_{trop}) = \int_0^{\tau_{trop}} \sigma T^4 \exp(-(\tau_{trop}-\tau))d\tau \;+\; \sigma T_s^4 \exp(-\tau_{trop})$$

We will assume that there is strong enough surface turbulent flux that $T_s = T(0)$. Further, since the troposphere is on the (dry) adiabat, we have (in the troposphere)

$$T = T_{trop}\left(\frac{p}{p_{trop}}\right)^{R/Cp}$$

Finally, $\sigma T_{trop}^4 = \frac{1}{2}S(1-\tau_{trop}+\tau_\infty)$. The problem is now closed once we specify that $\kappa p/g = (\tau_\infty - \tau)$. If we define the new dummy variable $\eta = \tau/\tau_{trop}$, and let $\delta = p_{trop}/p_s$, then the matching condition becomes

$$\frac{2+\delta\,\tau_\infty}{1+\delta\,\tau_\infty} = (1-\delta)\tau_\infty \int_0^1 (\eta+(1-\eta)/\delta)^{4R/Cp} \exp[-(1-\delta)(1-\eta)\tau_\infty]\,d\eta$$
$$+\;\delta^{-4R/Cp}\exp(-(1-\delta)\tau_\infty)$$

Note that S has dropped out of this equation. The magnitude of S affects the ground and tropopause temperatures, which can both be determined once δ is known. It does not affect the tropopause height (measured by δ), though. Upon solving the preceding equation numerically, one obtains δ as a function of τ_∞ and R/C_p. The following table gives some values for S= 300W/m^2 and $R/C_p = 2/7$.

τ_∞	p_{trop}/p_s	T_s
.01	0.5460	270.
.1	0.5527	272.
1.	0.6131	294.
2.	0.6666	315.
5.	0.7667	363

10.	0.8408	417.

[**General discussion. Note that surface temp gets large as atmosphere gets optically thick, but that the tropopause approaches the ground. Earthlike conditions for $\tau = 1$. Put in S and R/Cp for Venus, and see what optical thickness is needed to account for its surface temperature. Put in S for Mars. The behavior of the tropopause in the optically thick limit is very delicate, and whether the tropopause approaches the ground is sencitive to R/C_p, with the value for air (2/7) being near the crossover.]

[**Discuss how tropopause height compares with estimate of tropopause height based on level of neutral buoyancy of the surface temperature obtained in the pure radiative model. If they are the same, there must be some general energetics-based principle in operation.]

There are three important limits that need to be discussed.

Optically Thin Atmosphere

[**Stratospheric temperature = $(S/2\sigma)^{1/4}$, but ground temp = $(S/\sigma)^{1/4}$, which is the no-atmosphere limit. Tropopause height is given by finding where the adiabat starting at the ground temperature crosses the stratospheric temperature. Gives $\delta^{R/Cp} = 2^{-1/4}$, e.g. $\delta = .5453$ for air.]

[**In a worked problem, extend this to the case of a small emissivity atmosphere. Relate to the isothermal slab model, and give expression for "effective radiating level." Note, this is a pretty good conceptual model for Mars.]

[**Note surface budget is in radiative balance for the thin solution, with out the need for any turbulent fluxes. In fact, turbulent fluxes from the ground could create a problem, since the atmosphere can't get rid of the extra heat radiatively.]

Optically Thick Atmosphere

[**Look for solutions which become independent of the presence of the ground. Ones for which tropopause doesn't approach the ground. Of importance to Jupiter (with solar absorption occurring in the tropospheric gas, rather than at a surface), and to question of whether Venus has a "deep stratosphere" or a "deep troposphere." We are looking for solutions that have a deep troposphere, whose actual depth is immaterial, capped by a stratosphere of finite optical thickness. Basically, we are putting a stratospheric hat atop the all-troposphere planet we discussed in Section **.]

For these solutions (if they exist), the transmission term in the tropospheric energy balance is negligible. Since the I^+ coming up from the troposphere is sensitive only to the first few units of optical thickness in the troposphere, the optical depth of the atmosphere as a whole becomes irrelevant, and so we choose $\Delta\tau = \tau_\infty - \tau_{trop}$ as our basic parameter rather than τ_∞. Note that $\Delta\tau$ is the optical thickness of the stratosphere. We introduce a new dummy variable $\eta = \tau_{trop} - \tau$, whence $(p/p_{trop}) = 1 + \eta/\Delta\tau$. Because of the exponentially decaying weighting, we are also free to integrate to infinite optical depth into the troposphere. The resulting balance condition becomes

$$\frac{2 + \Delta\tau}{1 + \Delta\tau} = \int_0^\infty (1 + \eta/\Delta\tau)^{4R/Cp} \exp(-\eta)\, d\eta$$

The left hand side approaches 2 for small $\Delta\tau$, and becomes $1 + 1/\Delta\tau$ for large $\Delta\tau$. The right hand side becomes

$$\Delta\tau^{-4R/Cp} \int_0^\infty \eta^{4R/Cp} \exp(-\eta)\, d\eta$$

small $\Delta\tau$, which is proportional to Eq. (**) which we derived for the OLR of an all-troposphere optically thick atmosphere. For our purposes, the important thing is that this becomes infinite, so that the RHS curve of the energy balance starts out above the LHS. If the RHS is below the LHS at large $\Delta\tau$, than a crossing point is assured.

At large $\Delta\tau$, the RHS integral becomes

$$\int_0^\infty (1 + (4R/Cp)\, \eta/\Delta\tau) \exp(-\eta) = 1 + \frac{4R/C_p}{\Delta\tau}$$

Hence, crossing is assured whenever $4R/C_p < 1$. Remarkably, this is also the threshold for a deep radiative equilibrium atmosphere to go gravitationally unstable on its own without the necessity of surface heating. [**Nature is being kind and consistent. In just the case where a deep stratosphere is unstable, a solution with an infinite-depth troposphere becomes possible.] As for the instability criterion, dry air, with $R/C_p = 2/7$, just misses allowing a deep troposphere solution. [**Give values for pure CO_2 (relevant to Mars and Venus), and point out importance of variation of C_p with temperature. Give values for pure water vapor (steam atmosphere; runaway greenhouse). Give values for hydrogen/helium mix. (Jupiter and Saturn).]

[**Insert graph of LHS and RHS for 3 values of R/C_p .]

[**What happens in the case where there's no deep-troposphere solution: What is going on in the optically thick limit is that the tropopause radiates upward at very nearly the tropopause temperature, because of the optical thickness of the troposphere. However, the upward radiation in the stratosphere cannot be matched to this. As a result, the tropopause is forced downward until the troposphere is optically thin enough for its depth to affect the radiation. Another way of looking at it is that the zero order match for the optically thick case requires a high temperature (hence low) tropopause.]

[**Cautionary note: The delicacy of the tropospheric balance in the optically thick case means that the answer will be very dependent on other perturbations to the system, like collisional broadening or nongrey effects or solar absorption in the stratosphere. This makes the optically thick case very challenging. Note that for Venus, R/C_p gives a shallow strat for the upper air temperatures, but because of temp dependence of C_p crosses over to the deep strat value.]

[**Are there multiple equilibria? Specifically, when there's no deep-troposphere solution with a stratosphere, is the all-troposphere planet a possible solution? (yes -- this is always available). Which one is preferred?]

[**Are these results on the behavior of tropopause height in a thick atmosphere well known? Look up sources. There is a chance that this is a novel result. Goody doesn't seem to mention it. Check updated references, and Venus references.]

Small Lapse Rate

[**Change lapse rate by changing C_p with R fixed, since lapse rate is g/C_p. As lapse rate is reduced, the stratosphere gradually vanishes, and the solution reduces to the all-troposphere case. Note that in this limit, the greenhouse effect goes away. The upper atmosphere does get cold, but because this part is always optically thin (by virtue of having low density), it doesn't succeed in bringing down the OLR.]

[**The lapse rate dependence holds the key to tropopause height variations on Earth. We saw in Chapter 2 that the tropopause is higher in the tropics than in the midlatitudes. The increased absorbed solar cannot account for this. Water vapor makes the tropics more optically thick than the midlatitudes, but this effect would lower rather than raise the tropopause. However, condensation lowers the lapse rate. This is the main influence in raising the tropical tropopause.]

[**Defer discussion of T_s - T(0). For now, we'll just empirically say that it's mostly small. Discuss radiative balance of surface though, to show that the implied turbulent fluxes from the surface are indeed from the surface to the atmosphere.]

The static stability problem. It is "easy" to understand why there is a troposphere that is approximately well-mixed (adiabatic). But the Earth's troposphere, while *nearly* adiabatic is actually slightly statically stable. That small difference between slightly stable and slightly unstable makes a world of difference in the circulations. It is the key difference between terrestrial type (stably stratified) and stellar type (convective) circulations. On Earth, the static stability might (and probably does) have something to do with moisture. However, dry atmospheres (Mars), and dry GCM's still produce a statically stable atmosphere. (Show some data.) Why?

4.5 1D models of atmospheres with a condensible component.

[**Some illustrative examples concerning latent heat: (a) Time scale for unbalanced solar radiation to evaporate the ocean. Note that this is pretty much the rate of water flux out of the tropical ocean; however, the ocean doesn't disappear, because the water is quickly rained out back into the ocean (or snowed onto ice caps which flow and melt). (b) Amount of water that can be evaporated by the energy of a cometary impact. (c) Megaton equivalent of 1cm of precip over 1 km^2. (d) Heat balance of an atmosphere condensing out onto a polar night ice cap. Note that the darkside of a tide-locked planet is an extreme case of polar night. Atmosphere will tend to disappear to the nightside, but as the atmosphere moves there it carries heat (latent and sensible) with it. Constraints on the rate of mass flux into the polar night ($\sigma T^4 = A^{-1}$ dM/dt; however, if T gets too large or atmospheric pressure gets to low, condensation ceases (but then T goes down, in the absence of other heat transports)).]

[**Consider pure condensible atmosphere (like CO_2 for Mars). As if Earth had pure steam atmosphere. Tour of Ingersoll's "low mass atmospheres." Mars dry-ice cap is a good example. How to "terraform" Mars.]

Water vapor as a greenhouse gas. IR radiative effects of water vapor. Collisional broadening of absorption lines. Slab emissivities for water vapor; importance of non-grey effects: why emissivity is not exponential in slab thickness. Relative role of water vs. CO_2 as an IR absorber. Radiative properties of liquid water (continuum absorption) and ice; implications for cloud IR emissivity.

Radiative properties of liquid water and water vapor in the visible spectrum.

Why does a liquid ocean exist? Runaway greenhouse possibility. Surface pressure if oceans were evaporated. How long would it take for oceans to condense back? What would temperature be in the intermediate times?

[**Kombayashi - Ingersoll limit. Maximum OLR for a radiative equilibrium atmosphere.

$$2\sigma T^4 = (1 + \tau_\infty - \tau)OLR$$

$$2\sigma T_{trop}^4 = (1 + \frac{\kappa p_{trop}}{g})\ OLR$$

$$OLR = \frac{2\sigma T_{trop}^4}{(1 + \frac{\kappa p_{trop}}{g})}$$

but *if the tropopause is at saturation,* $p_{trop} = e_\infty \exp(-L/(RT_{trop}))$. Substituting in the above, this implies an upper bound on OLR (except for unrealistically large temperatures).

All-troposphere case. Suppose vertical structure given by $T = T_{sat}(p)$ (this is the "moist adiabat" for a pure condensible atmosphere).

$$OLR = \sigma T_s^4 e^{-\tau_\infty} + \int \sigma T^4 e^{-\tau}\, d\tau$$

but for a "saturated" pure condensible atmosphere,

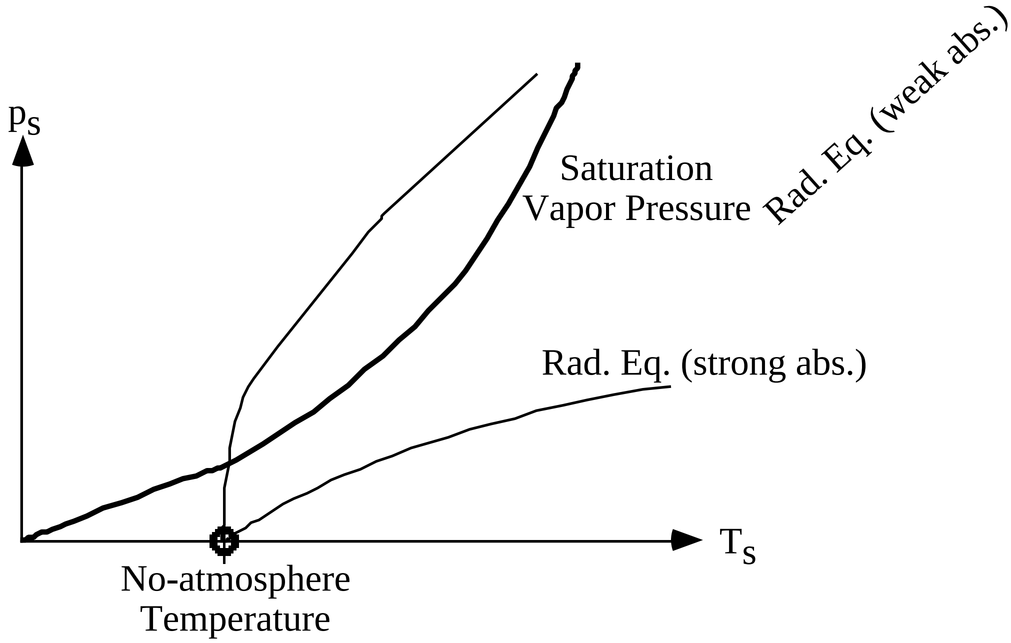
$$T = \frac{L}{R} \frac{1}{\ln(\tau) - \ln(\kappa e_\infty/g)}$$

The second logarithm in the denominator is so enormous that the first can essentially be neglected. In the optically thick limit we then have (approximately)

$$OLR = \sigma(\frac{L}{R})^4 \frac{1}{[\ln(\kappa e_\infty/g)]^4} = \sigma\, T_{rad}^4$$

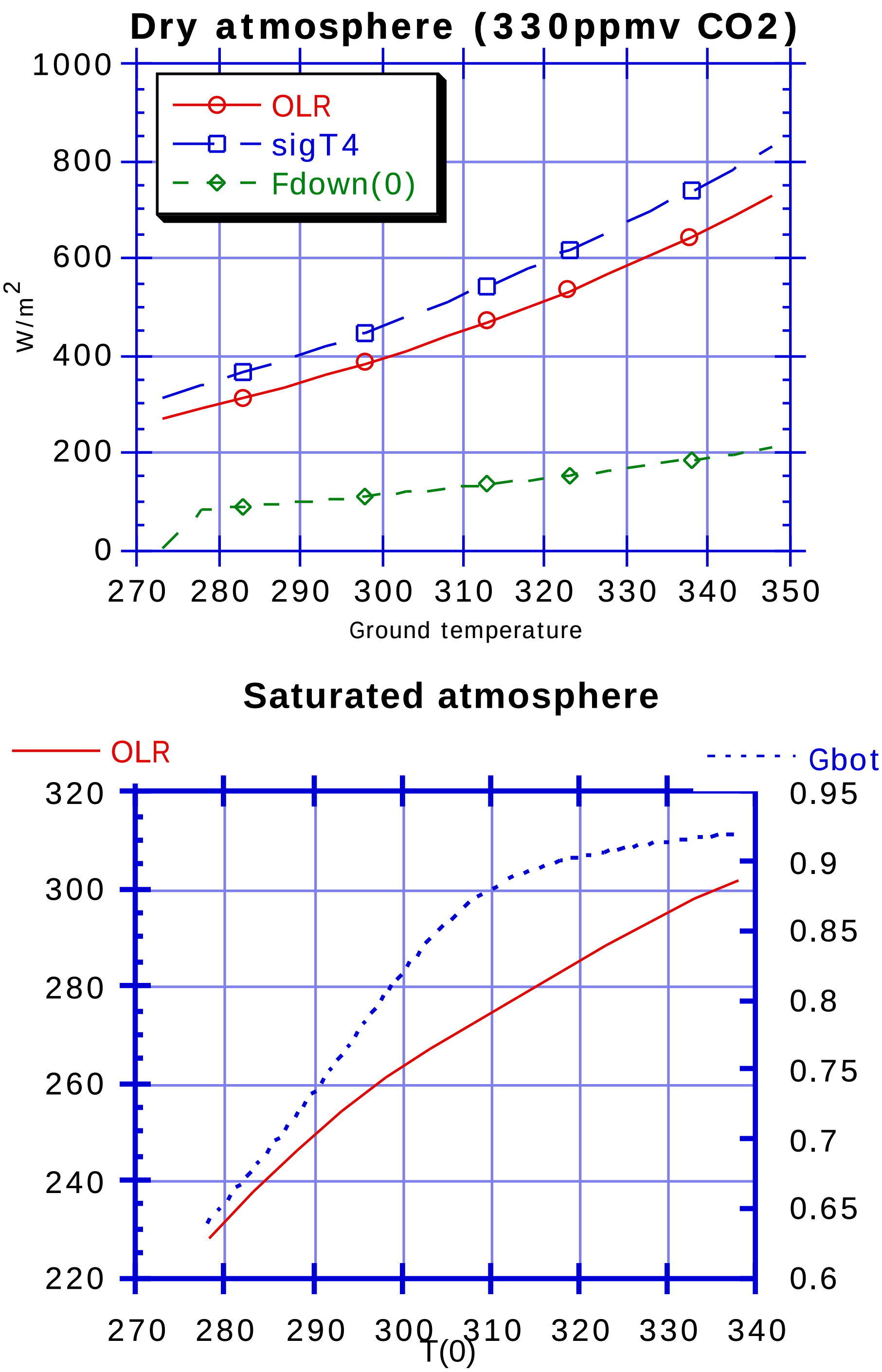
where T_{rad} is the temperature of the saturated atmosphere corresponding to the pressure where $\kappa p/g = 1$. [**In other words, the OLR has an upper bound because from space we only see the radiation from the first optical depth of the atmosphere, but for a saturated atmosphere this cap always has the same temperature. Limiting OLR depends on L/R, on absorption coefficient, and on g. Give some crude estimates for a steam atmosphere on Earth and Venus.]

[**Alternate way of looking at the problem: If we know the mass of the atmosphere, as represented by p_s, we can compute the surface temperature by radiative-convective equilibrium. On the other hand, if there is a repository of condensate which can evaporate to form more atmosphere, then the surface pressure p_s is determined as a function of T_s by the Clausius/Clapeyron relation, since at equilibrium the surface is at saturation. A solution for the atmosphere is obtained when the two curves cross.



[**Estimates of maximum OLR based on a non-grey radiation code.]

Less extreme example of water vapor feedback. The present-day terrestrial regime. Effect of doubling CO_2. Importance of water vapor feedback. Warmer atmosphere *can* hold more water, but *will* it? Disruption of tropical dry air pools? Cumulus drying? Note also large effects expected due to cloud feedback. Both of following curves computed using an idealized tropical temperature profile with fixed lapse rate, tropopause at 100mb, and constant temperature above the tropopause.



4.8 The surface energy budget

Here, we discuss what determines the surface temperature T_s once the atmospheric temperature profile T(z) is known. [**For an optically thick atmosphere, the OLR is nearly independent of T_s, so the T_s problem decouples from the T(z) problem. However, for an optically thin atmosphere T_s affects OLR, and the two problems must be solved together so as to obtain a self-consistent solution.]

[**Observational background. Observations of T_s - T(2meters). Note differences for Sahara vs. tropical ocean, and tropics vs. extratropics.]

Surface energy budget (assuming no horizontal heat transport in the oceans):

$$F_{turb} = S + I^-(0) - \sigma T_s^4$$

where $I^-(0)$ is the IR back radiation from the atmosphere into the surface, S is the solar radiation absorbed at the surface (net of albedo and atmospheric absorption effects), and F_{turb} represents all non-radiative heat exchanges between the surface and the overlying atmosphere (i.e. those due to turbulent fluid motions in the air). The right hand side of this expression is the net radiative heating of the surface, which must balance the net cooling of the surface due to turbulent heat fluxes.

First look at what happens if the balance were purely radiative ($F_{turb} = 0$). In warm regions $I^-(0)$ is quite close to $\sigma T(0)^4$, owing to the optically thick moist lower atmosphere. Thus T_s has to get much greater than T(0) to close the budget. [**Insert figure illustrating this. Fixed $I^-(0)$ corresponding to 80% saturated boundary layer and 300K T(0). T_s - T(0) approaches 30^oK]. These values are greatly in excess of what is observed in the tropical oceans. [**N.B. Sahara is a special case, though, because the boundary layer can be quite dry, whence the evaporation is limited.] In this situation, turbulent heat transport is crucial to closing the budget. Approximate drag law formulation:

$$F_{turb} = F_{sens} + F_{latent}$$

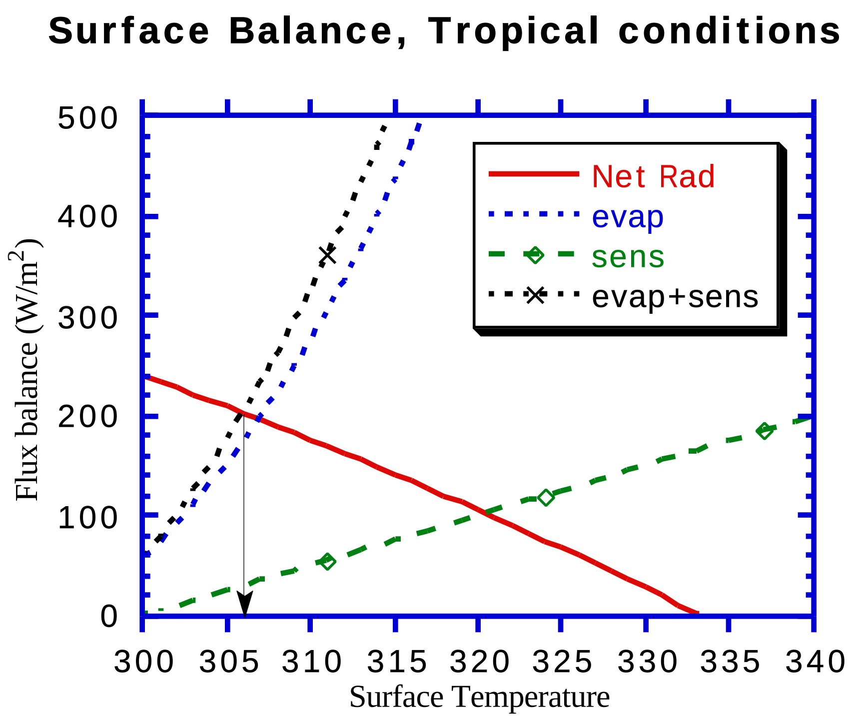
$$F_{latent} = \rho_{air} C_D U L (q_{sat}(T_s) - r\, q_{sat}(T_s))$$

$$F_{sens} = \rho_{air} C_D U C_p (T_s - T(0))$$

[**Define terms. q_{sat} is saturation mixing ratio, evaluated at surface pressure.] The first term is the latent heat transport and the second is the sensible heat transport. C_D is around .001-.002 over the oceans or other smooth surfaces. ρ_{air} is the air density at the surface. Note that the latent heat transport increases *exponentially* with T_s , while the sensible heat transport only increases linearly, all other things being equal. [**Discuss evaporative/radiative balance in figure. Steep evaporation curve for warm temperatures tightly couples T_s to T(0).] [**Over a desert, sensible heat term has to do the work instead.

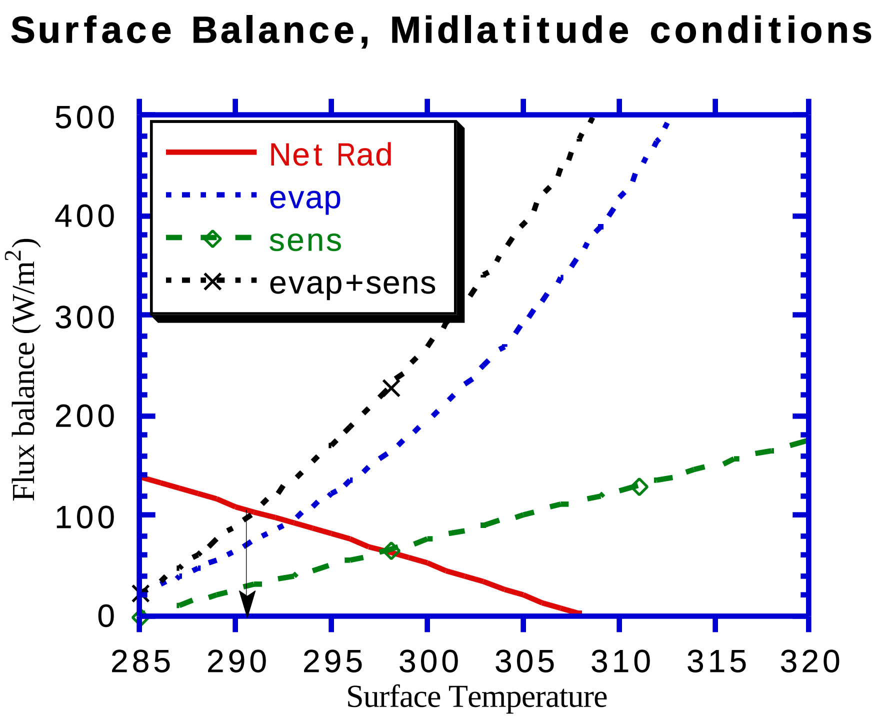
Good possibility for exercise on the Sahara here, figuring out what range of bdd. layer humidities are consistent with the observed T_s.]

The following three graphs illustrate the way in which the surface energy balance operates. All were computed with C_D = .001, U = 5 m/s, and a boundary layer relative humidity of 80%. The first graph is for tropical conditions. We took the surface absorbed solar radiation to be 300 W/m^2, allowing crudely for clear sky albedo and atmospheric absorption. The IR back radiation was taken to be 87% of $\sigma T(0)^4$, with T(0) = 300K. Note that by far, evaporation is the dominant term in the budget, and that the steepness of the evaporation curve keeps the surface temperature from much exceeding the overlying air temperature. The curve labeled "Net Rad" is the aggregate of absorbed solar heating plus net infrared cooling.

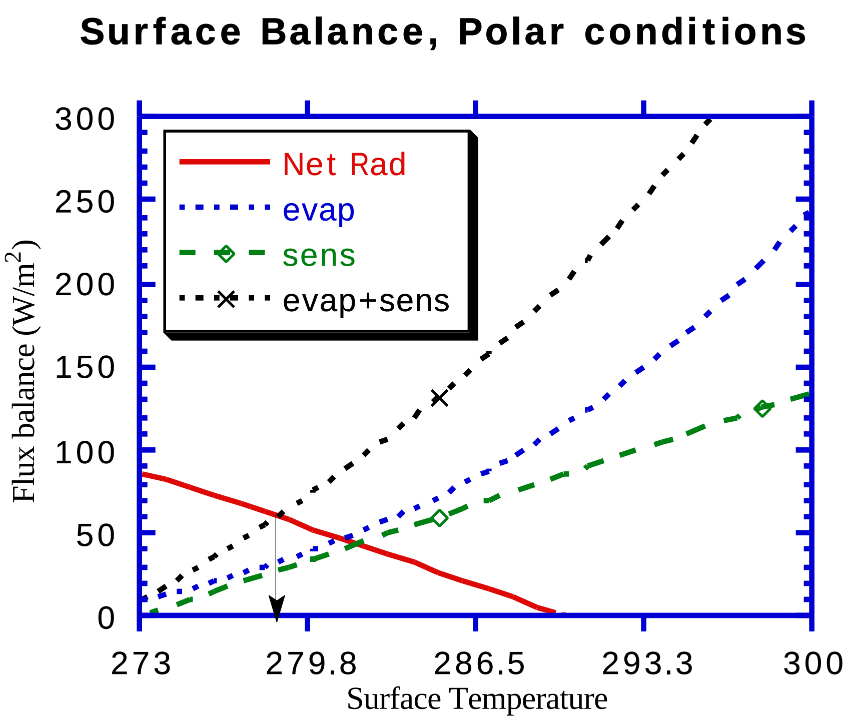


In the absence of turbulent fluxes, the surface would be more than 30K in excess of the overlying air temperature, owing to the weak net IR cooling. Sensible heat flux by itself would bring this down to about 20K, whereas the combination of all fluxes brings the difference to 6K.

Next we consider midlatitude conditions, with T(0) = 285K, IR back radiation set to 75% of $\sigma T(0)^4$, and absorbed solar set to 230 W/m^2. Note that evaporation is still dominant, but that both net radiative cooling and sensible heat fluxes play a more significant role. This is largely because the lower moisture content of the air means both less effective evaporation, and more efficient radiative cooling owing to the lower optical thickness of the lower atmosphere.



Finally, we show the balance for polar conditions. Now, the absorbed solar is 160 W/m^2, T(0) is 273K, and the IR back radiation is still 75% of σ T(0)^4. For these conditions, sensible heat flux and evaporation play nearly equal roles.



Note that in all three cases, the fluxes conspire to keep the surface temperature to within about 5K of the overlying air temperature. Compared to oceanic observations, this value is generally too large. [**Give a map showing typical values. Main culprit in overestimate is probably clouds. Clouds reduce the solar radiation absorbed at the surface, but don't affect the IR back radiation much. Could also increase turbulent fluxes with a larger C_D or U.]

[**Analysis of "stiffness" of surface temperature, by linearized exchange coefficients. Table of exchange coefficients at two temperatures.] At $T=T_o$,

$$k_{rad} = \frac{d}{dT}\ \sigma T^4 = 4\ \sigma\ T_o^3$$

$$k_{sens} = \frac{d}{dT}\ F_{sens} = \rho_a C_p C_D U$$

Before we do the latent heat, note the approximate expression:

$$q_{sat} = q_{sat}(T_o)\ \exp\{\frac{L}{R_w T_o^2}(T-T_o)\}$$

so

$$k_{latent} = \frac{d}{dT}\ F_{latent} = k_{sens}\ \frac{L^2}{R_w C_p T_o^2}\ q_{sat}(T_o)$$

In the exchange coefficient for latent heat, the ratio appearing second has the value 156 at T_o = 300K (when the condensible substance is water). It is not very sensitive to T_o. The main temperature dependence comes through the exponential dependence of q_{sat} on temperature. q_{sat} = 3.5 g/Kg at 273K, 6.2 g/Kg at 280K, 22.3 g/Kg at 300K, and 68.2 g/Kg at 320K.

The following table gives values of the exchange coefficients, in W/(m^2K). Based on C_d = .002, U=10m/s, and r = 100%.

T_o	k_{rad}	k_{sens}	k_{latent}
280	5.	20.	20.
300	6.	20.	70.

[**When can the low layer air become decoupled from the air aloft? Tropical boundary layer, and the trade inversion.]

4.7 Models of convective heat and moisture transport

"Just enough convection." A fascinating and wide-ranging subject. Refer to Emanuel book for those wishing to pursue the matter in depth.

Dry incompressible convection. Scaling laws for heat flux in an infinitely thick layer. Dynamics of ascending thermals. Summary of modern laboratory results.

Dry compressible convection.

Convection for a thermal penetrating a statically stable environment.

Moist convection. Classical entraining-plume models and stochastic mixing models. Moist convective adjustment.

5. Seasonal, diurnal and latitudinal variations of temperature

5.1 Introduction

Geometry and astronomy. Geometric effects on sphere with no axis tilt, i.e. effect of orientation of surface upon which radiation is incident.

Effect of axis tilt. Geometric effects vs. length-of-day effects. Have been doing global average radiation balance -- assumes that response of system averages out inputs over days, latitudes, etc. Some planets can do this, some can't. Calculation of equatorial,noontime radiative equilibrium temperature on Earth. (Based on 1380 W/m^2, even without Greenhouse effect this would be 395^{o}K; at night, of course, it would fall to absolute zero) Relative unimportance of eccentricity of Earth's orbit in seasonality (not quite so with Mars, though!). Time scales for equilibration?. Compare to diurnal cycle on Moon Mercury, which have no oceans or atmospheres. In lunar tropics, surface temperature at local noon reaches 403^{o}K. At midnight and dawn, lunar surface temperature is -173^{o}C. Temp changes penetrate only to about 50cm; below temp is constant at around -30^{o}C. Mercury: Mean surface temperature 350^{o}C by day, -170^{o}C by night.

5.2 Thermal inertia

Issue of thermal inertia. Thermal inertia for a mixed layer of fluid.

$$\frac{d}{dt}\rho H A C_p T = (F - \sigma T^4)A \qquad (2.3.1)$$

where H is the depth of the well-mixed layer, A is the area of the surface at the top of the layer, ρ is the density of the substance making up the mixed layer (e.g. 1000 Kg/m^3 for water) and F is the incident radiation absorbed (or more generally the heat flux into the surface from whatever sources. The factors of A cancel out, leaving the equation

$$\frac{dT}{dt} = \frac{1}{\rho H C_p}(F - \sigma T^4) \qquad (2.3.1a)$$

which is now a closed first-order ordinary differential equation for T(t). In general, F could be a function of time, but if F is constant it has the equilibrium solution T_e, defined by $\sigma T_e^4 = F$. [**Linearization around T_e. Radiative damping time, $\tau = \rho HCp/(4\sigma T_e^3)$. Typical numbers.]

[**Simple mixed layer model of polar night cooling. Set F=0 in (2.3.1a) and let the initial temperature be T_o. This somewhat exaggerates cooling because the actual heat loss to space is less than σT^4, owing to the greenhouse effect. In the dry polar regions, this is not as serious a flaw as it would be in the moist tropical regions. Solution:]

$$\frac{T}{T_o} = (\tau/(t+\tau))^{1/3}$$

where now $\tau = \rho HCp/(3\sigma T_o^3)$, which is similar in form to the radiative damping time we computed earlier, except it is now based on the initial temperature. For $t \ll \tau$, the temperature drops linearly with time, but at large times, the temperature decays much more slowly — namely, like $(\tau/t)^{1/3}$.

[**Review of diffusion equation. Characteristic scale $\sqrt{Dt}$. Self-similar Gaussian solution for evolution of initial spike.]

The diffusion coefficients for solids are typically very small. For ice near 0^{o}C, D is approximately .0114 cm^2/s; the diffusive length scale corresponding to 1day is only 31 cm., growing to 3.1 meters at 100 days; even after 100000 years, heat diffuses only to a depth of 1.8 km (neglecting the effect of the slow creeping flow of the ice). Other solids have similar or even lower diffusivities (Quartz, .044 cm^2/sec, clay .015, organic matter .001). The shallow penetration depth implies a rapid initial drop in surface temperature in response to radiative cooling, with the rate of drop gradually decreasing as the thermal influence taps the reservoir of heat at progressively greater depth.

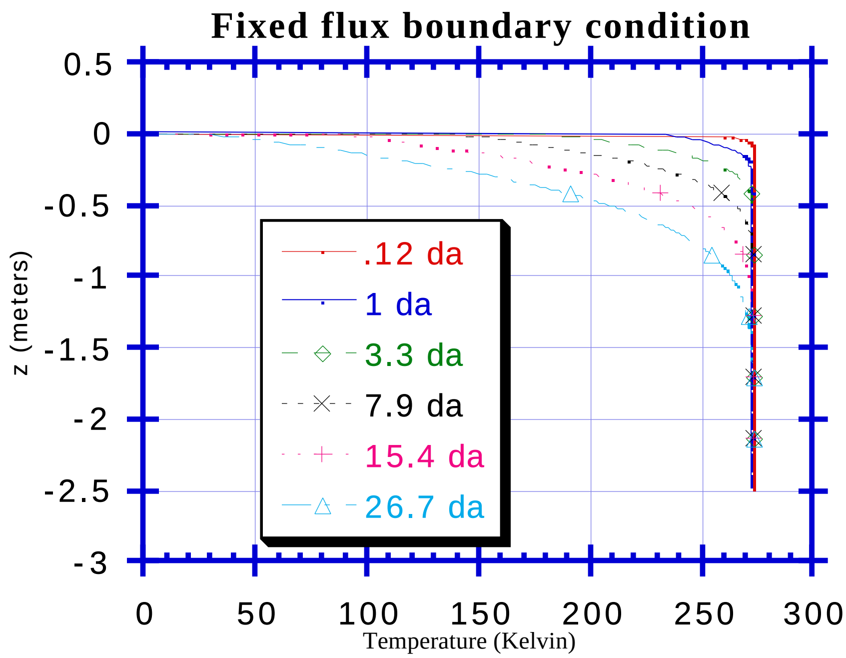
[**Handwaving solution for surface temperature in terms of equivalent mixed layer.] Qualitatively, we might expect the thermal behavior of a solid to behave like a mixed-layer model with a time-dependent mixed layer depth given by $H=\sqrt{Dt}$. Let's substitute this in 2.3.1 and ignore the radiative cooling for the moment, assuming instead that the flux F represents a constant cooling. Then the solution for the surface temperature T(t) is

$$T = T_0 + \frac{F}{\rho\sqrt{D}C_p}\sqrt{t}$$

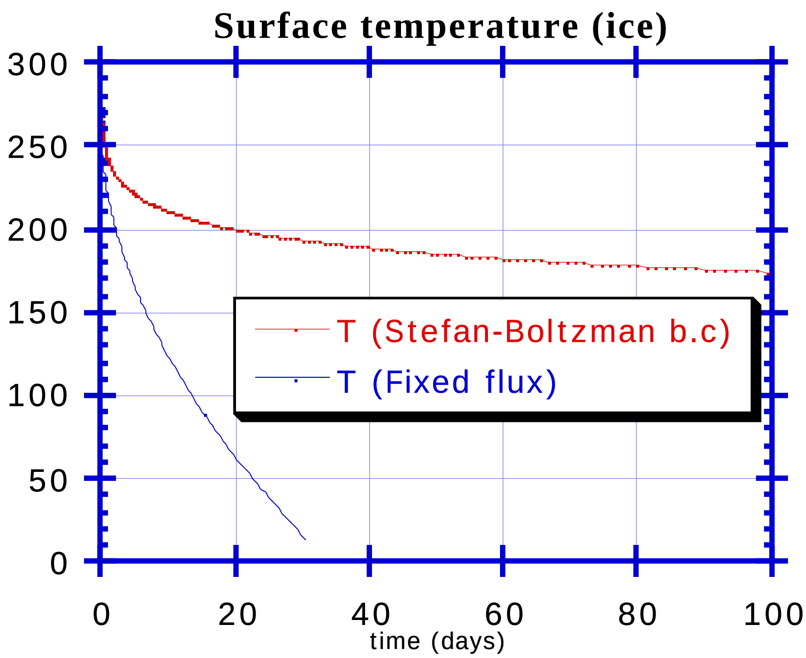
where T_o is the initial temperature (which comes in as an initial condition. Since F is negative, this yields a steady reduction in temperature, which starts off infinitely steeply, but is slower than linear over long times.

This solution may be obtained somewhat more rigorously in terms of a "broken stick" approximation to the temperature profile. [**Discuss "broken stick solution. In fact, it gives the rigorously correct asymptotic solution at long times.]

In the following we show numerical solutions for the actual diffusion equation cooled from the surface, for the diffusivity of ice. The first figure gives the depth profile of the cooling.



The next shows the surface temperature, for both the fixed-flux case and the case where surface cooling satisfies the Stefan-Boltzman law. Note the implication of rapid cooling of the Polar night.



Note that cooling of the polar surface is fundamentally different from the surface heating that leads to a radiative-equilbrium ground temperature higher than the overlying atmosphere. The latter favors mixing by creating an unstable temperature gradient (convection). For the polar cooling case, one expects a thin layer of cold, dense air to form over the surface. This is very stable, and *inhibits* vertical transport. Atmosphere slows down cooling somewhat by re-radiating IR back to the surface, but this can't be maintained for long without the polar atmosphere cooling down. Importance of horizontal atmospheric heat transport to the poles.

In view of the low thermal conductivity of solids, the *first* role of a planet's fluid envelope (ocean or atmosphere) is *heat transport* , which allows tapping a large reservoir of stored heat to smooth out day/night and North/South temperature differences.

[**Thermal inertia of the Atmosphere. Importance of downwelling IR in moderating diurnal cycle, since IR doesn't have much of a diurnal variation (owing to time constants for the atmosphere. At night, surface cools toward IR radiative equilibrium with the atmosphere, because absorbed solar is shut off. Thus, diurnal cycle gives valuable information on downwelling IR. Surface cooling leads to stably stratified nocturnal boundary layers. Note that this (and many things we have been relying on) can't be discussed without the stuff on atmospheric thermodynamics from Section 3.1-3.2. How should material be re-organized?]

[**The above calculations were done incorporating only radiative coupling between the surface and the atmosphere. Give some discussion of thermal inertia for atmosphere/surface system, and for surface alone with atm. fixed.]

[**Important of different time scales for the "effective" depth of the ocean. If heat is only going into a 50m mixed layer, then time scale is on the order of a year or so. However, in the long run, ocean currents distribute heat over the entire depth of the ocean. Can increase response time to centuries.]

[**Note there can also be thermal inertia due to phase changes. Melting of ice cap is a good example, both on Earth and Mars.]

[**Importance of sea-ice feedback on climate calculations. Given diffusivity of ice, formation changes surface from a high-inertia ocean which doesn't cool down much in the winter, to a low-inertia solid, which does. Also cuts off latent heat flux to atmosphere. Warming melts ice, and this inhibits cold winters.]

5.3 Orbital parameters and the seasonal cycle

Astronomical effects on climate. Obliquity, eccentricity, and precession of the equinoxes. First look at "normal" latitudinal and seasonal fluctuation. Then take up subject of astronomical control of the ice ages.

[**Effects of obliquity on seasonal and latitudinal distribution of insolation.] Let the point P be the center of the Earth, and S be the center of the sun. If we draw a line from P to S, it will intersect the surface of the earth at a latitude δ, which is called "the latitude of the sun." It is a function of the orientation of the Earth's axis alone, and serves as a characterization of where we are in the march of the seasons. If the obliquity of the Earth is γ, then δ ranges from γ at the Northern Hemisphere summer solstice to $-\gamma$ at the Southern Hemisphere summer solstice. Let Q be a point on the Earth's surface, characterized by its latitude ϕ and its "hour angle" h, which is the longitude relative to the longitude at which local noon (the highest sun position) is occurring throughout the globe. For radiative purposes, what we want is the "zenith angle" α, which is the angle between the local vertical and the line-of-sight from point Q to the sun. To get the zenith angle, we only need to take the vector dot product of the vector QS and the vector PQ. To do this, it is convenient to introduce a local Cartesian coordinate system centered at P, with the z-axis coincident with the axis of rotation, the x-axis lying in the plane containing the rotation axis and PS, and the y-axis orthogonal to the other two, chosen to complete a right-handed coordinate system.

First note that, by the definition of the dot product

$$\cos(\alpha) = \frac{PQ \cdot QS}{|PQ|\,|QS|}$$

Further, PQ + QS = PS, whence

$$\cos(\alpha) = -\frac{PQ \cdot PQ}{|PQ|\,|QS|} + \frac{PQ \cdot PS}{|PQ|\,|QS|} \approx \frac{PQ \cdot PS}{|PQ|\,|QS|}$$

where we could drop the first term because the radius of the Earth is a small fraction of its distance from the Sun. For the same reason, |QS| in the denominator can with good approximation be replaced by |PS|, leaving the expression in the form of a dot product between two unit vectors. Letting $\mathbf{n}_1$ = PQ/|PQ| and $\mathbf{n}_2$ = PS/|PS|, the unit vectors have the following components in the local Cartesian coordinate system.

$$\mathbf{n}_1 = (\cos(\varphi)\cos(h), \cos(\varphi)\sin(h), \sin(\varphi)) \quad \mathbf{n}_2 = (\cos(\delta),0,\sin(\delta))$$

whence

$$\cos(\alpha) = \cos(\varphi)\cos(\delta)\cos(h) + \sin(\varphi)\sin(\delta)$$

It only remains to express δ as a function of the time of year. The time of year may be conveniently expressed in terms of the "season angle" κ, which is the angle made between PS and the projection of the Northern half of the Earth's rotation axis on the plane of the ecliptic (see Fig **). [**κ is basically the same as "the longitude of the sun"] This angle acts like a clock, going from zero at the Northern summer solstice to π at the Northern winter solstice. Let **n** be the unit normal vector to the plane of the ecliptic, and $\mathbf{n}_a$ be the unit vector in the direction of the rotation axis. Introduce a new cartesian coordinate system with x pointing along PS, z pointing along **n**, and y perpendicular to the two in a right-handed way. Then

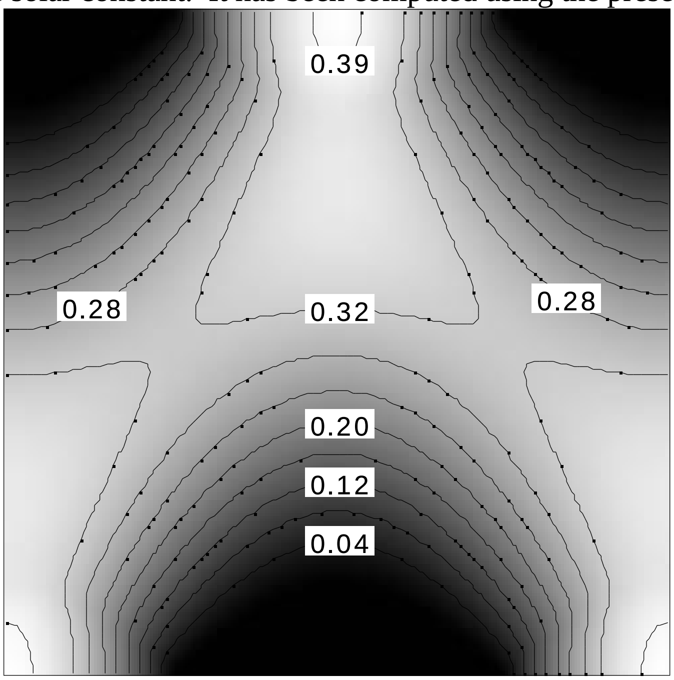
$$\mathbf{n}_a = (\cos(\kappa)\sin(\gamma), \sin(\kappa)\sin(\gamma),\cos(\gamma))$$

and the latitude of the sun is the complement of the angle between $\mathbf{n}_a$ and the x-axis, whence

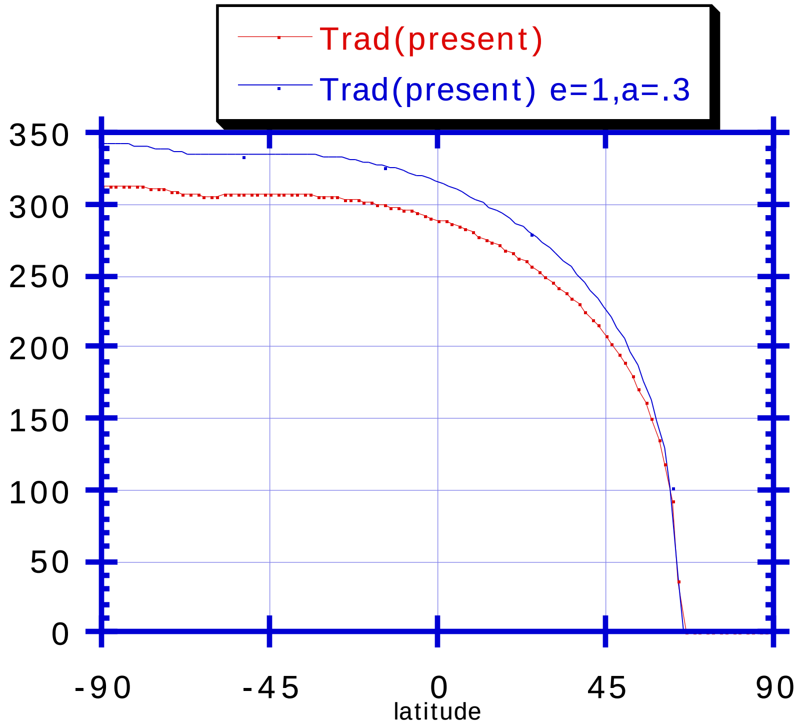
$$\sin(\delta) = \cos(\pi/2 - \delta) = \cos(\kappa)\sin(\gamma)$$

[**Position of terminator. Discuss diurnal average and annual average insolation. Note that to get the diurnal *average*, one needs to integrate $\cos(\alpha)$ with respect to h (over the daylight hours, then divide by 2π. Need to give a few examples of working with these formulae, and to show how they reduce to obvious limits (equinox, zero obliquity, 90^o obliquity, etc.)]

The following contour plot shows the flux factor *f* as a function of latitude and season, with the latitude being the vertical axis and the season being horizontal. *f* is defined such that the diurnal mean flux received at the specified latitude circle is L*f* watts/m^2, where L is the solar constant. It has been computed using the present value of the obliquity.

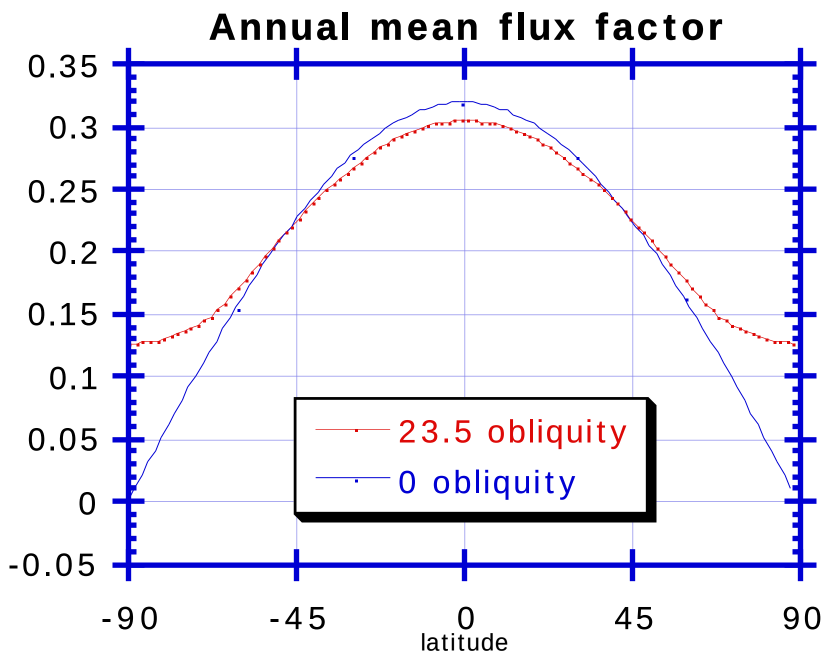


[**Discussion. Large polar fluctuations. Polar hotspot in summer, owing to long polar day. No radiation at all in polar night. Importance of thermal inertia in midlatitude and polar seasonal cycle.] The following graph illustrates this point dramatically. Here, we have computed the radiative equilibrium temperature for the solar flux computed at the solstice. Results are plotted for the Norther Hemisphere Summer solstice, but the Northern Winter results may be obtained by simply flipping the plot about the equator. We show results for a case with no atmosphere and zero albedo, and also for a case with an atmosphere of bulk emissivity 1, and an albedo of 30%.



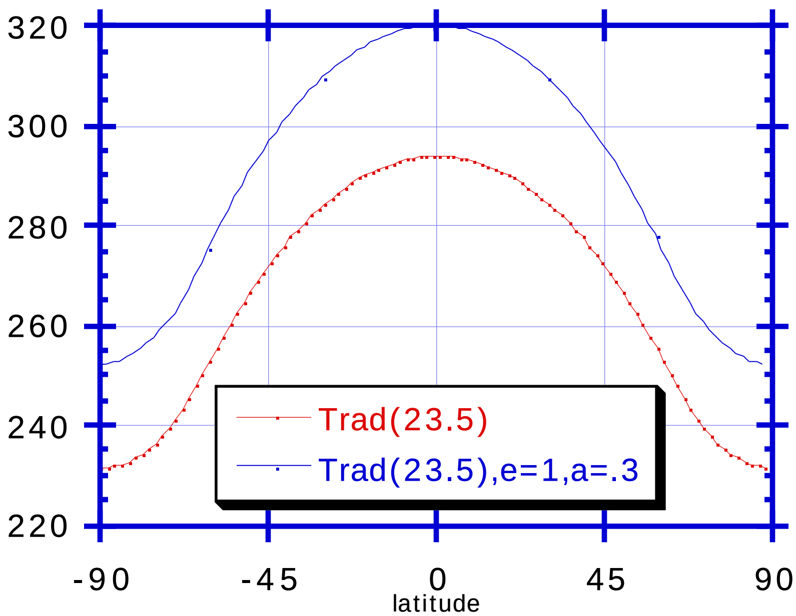
First, we note that, since the winter pole is receiving no energy at all (in equilibrium), it cools down to absolute zero. Even at 45º in the winter hemisphere, the temperature is between 200ºK and 230ºK, which is postively polar! Moreover, the summer pole reaches temperatures that are more than tropical (in fact, at the extreme, not really habitable).

Now let's look at the opposite extreme, in which the thermal inertia is everywhere so large that the surface temperature responds only to the annual mean radiation. The following graph gives the annual mean flux factor *f(*latitude*)*, for two different obliquities. *f* is defined such that the annual mean flux received at the specified latitude circle is L*f* watts/m^2, where L is the solar constant.



[**Discuss graph. Polar hotspot averages out. Importance of obliquity effects especially at polar regions. In annual mean, equator does indeed get more solar energy.]

The following graph gives the local radiative equilibrium obtained by solving the equation $(1\text{-}a)Lf + eT_a^4 = \sigma T^4$, where e is the bulk emissivity of the atmosphere (which is in radiative equilibrium) and a is the albedo. Two cases are shown: (a) no atmosphere, albedo = 0, and (b) 30% albedo, and atmosphere with e=1.



[**Discuss graph].

[**Discuss seasonal cycle. Main problem with local radiative equilibrium model of the seasonal cycle is the extreme cooling of the polar night. Why not hot polar regions at solstice? Importance of heat storage in surface and meridional heat transport. Illustrative example for role of energy exchange with surface: seasonal cycle in stratosphere vs. troposphere. Importance of ozone heating in stratosphere. Another illustration: Seasonal cycle of Mars.]

[**Milankovich theory. Effects of changing obliquity. Range of variation and time scale. Main effect is in high latitude regions. Importance of mild high-latitude summers *and* winters for glaciation.. Symmetry of effect between hemispheres. Why the Northern Hemisphere is the pacemaker for global glaciation. Discuss observed seasonal cycle of global mean temperatueres (which are highest in the Northern summer, owing to the greater land coverage).]

Orbital parameters for earth

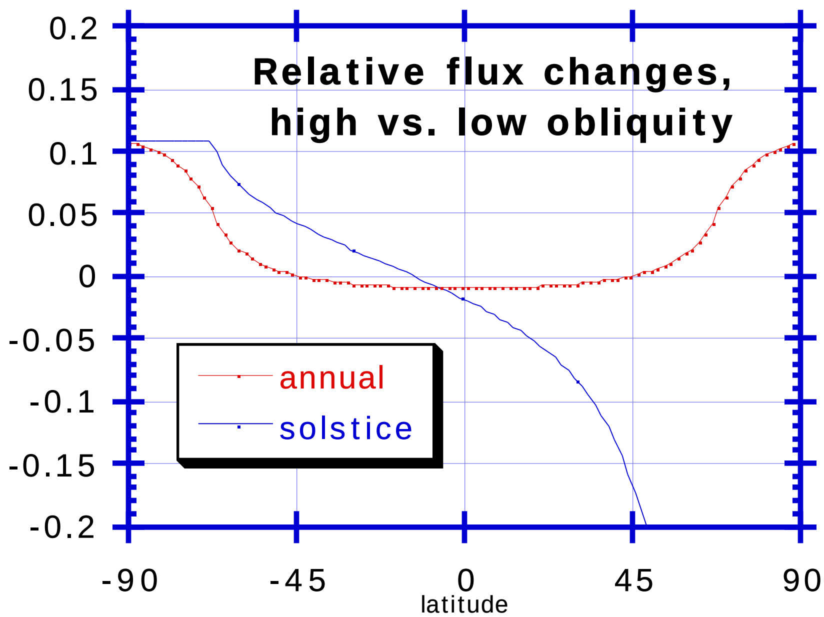
Parameter	Range	period
obliquity	23.1±1.3 deg	41000 yr
eccentricity	0 to .06	97000 yr
precession of equinox		22000 yr

[**N.B. from definition of eccentricity, axis ratio is actually sqrt(1-e**2), which is about 1-e**2/2 for small e]

Some current values

Planet	Obliquity	eccentricity
Earth	23,27'	.0167
Mars	23, 59'	.0934
Jupiter	3, 5'	.0485

The following graph shows the fractional change in flux factor (i.e. 2 (f(high) - f(low))/(f(high) + f(low))) between the high obliquity and low obliquity cases, for both the annual mean values and the solstice values.



In the annual mean, the higher obliquity state has slightly reduced fluxes in the low latitudes, but sharply increased (over 10%) fluxes in the polar regions. The obliquity effects are similar in the summer hemisphere for the solstice fluxes, except that the warming effect of high obliquity extend further toward the equator. Very large relative changes in the solstice flux are seen in the high latitudes of the winter hemisphere, but these represent large relative changes in a quantity that is rather small to begin with. [**Discuss what temperature increases this yields in radiative equilibrium.] The high obliquity states favor warm polar summers and generally warm poles. The winters in polar regions can't get much colder, because polar night already has zero incident solar flux, which therefore can't be reduced further. One expects, then, that the low obliquity states will be favorable for glaciation.

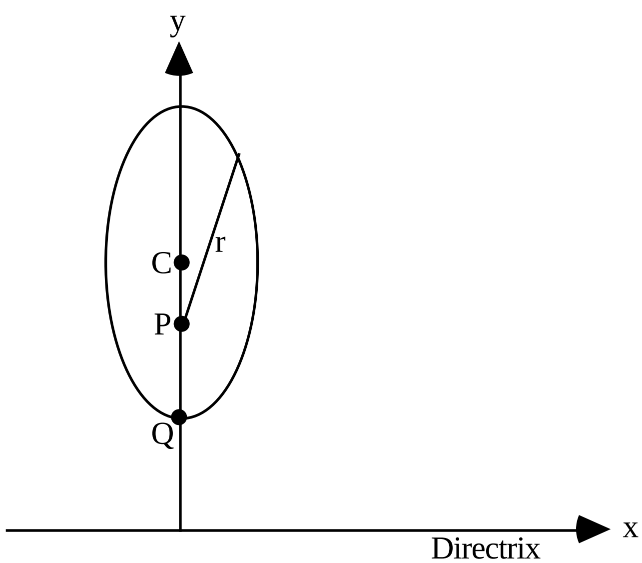
[**Eccentricity effects. Review of orbital mechanics. Definition of eccentricity (refer to figure below). Let major axis a = CQ, P be the focus of the ellipse, and h be the height of the focus above the directrix. Δy is the shift of the center (C) relative to the focus. The ellipse is defined by the relation:

$$\frac{r^2}{y^2} = e^2$$

Multiply out and complete the square to yield:

$$\frac{x^2}{1-e^2} + \left((y-h) - \frac{he^2}{1-e^2}\right)^2 = \frac{h^2e^2}{(1-e^2)^2}$$

so that $(\Delta y)/a = e$.



Thus, if e is small, the orbit is approximately circular, but the Sun is shifted by Δy relative to the center of the circle. If L_o is the solar luminosity, then to lowest order, the fluctuation in the solar constant between perihelion and aphelion is

$$F = \frac{L_0}{4\pi r^2} = (1 \pm 2e)F_o$$

where F_o is the mean solar constant.

[**Has weak effect on annual average insolation, at the next order. Velocity effect also; spends less time near perihelion than aphelion, but this is a weak effect. Main effect is on summer-winter contrast. "tilt seasons" vs. "distance seasons." Antisymmetry between hemispheres in eccentricity effect. Note also, eccentricity signal is the strongest periodicity in the ice record, but is a relatively weak radiative effect. Conflict with historical data: In Pleistocene, Northern Hemisphere is the pacemaker for ice ages *globally*. Present evidence for climatic effect of eccentricity: N-S asymmetry of seasonal cycle on Mars.]

[**Effect of precession of perihelion (important only in conjunction with seasonal cycle.). Split precessional cycles, and effects on the tropics. Bond cycles, Dansgaard-Oeschgar events. Important example of the precession signal: the Holocene optimum.]

5.4 Ice albedo feedback

6. Afterword to Volume I

[**Philosophical remarks on climate. CO_2, water, and life on earth.]

[**Dynamics. Role of rotation of planets.]

7. Fluid mechanics: Fundamentals

[**Compressible PE equations on a sphere, in pressure coordinates. Rotating reference frame. Isentropic coordinates. Ertel PV conservation. One layer (shallow-water) and two-layer variants.]

8. The Tropics, part I: Hadley circulations

8.1 Diabatic heating and vertical velocity

Role of water in energy exchange between surface and atmosphere: Solar energy absorbed in sea evaporates water, which goes into atmosphere and releases energy at higher levels and other latitudes when it condenses.

Steady state balance. Heating and ascent

Role of radiative cooling in limiting ascent

8.2 Mass continuity relation.

Mean meridional circulation

ITCZ

8.3 Angular momentum conservation. Trade winds

8.4 Momentum budget (Bernoulli equation)

Pressure forces determine why the tropics cannot persist in a state of local radiative-convective equilibrium. If it did, tropical low would lead to low pressure, and air would rush in, creating the Hadley circulation. Because the upper level outflow must fight an adverse pressure gradient, the pressure forces also act to limit the meridional extent of the Hadley cell (but not enough to account for the observations, evidently).

Observations show asymmetry of Hadley cell descending branch between the hemispheres. The strong descent is always in the winter hemisphere (and is very sharply defined, contradicting the picture of gentle descent due to radiative cooling). Why?

8.5 A two-box model of the tropical circulation

8.6 The Held/Hou model.

8. The Tropics, part II: Zonally asymmetric motions

8.1 Basic wave concepts

[**Dispersion relation, phase velocity, group velocity, ray paths.]

8.2 Equatorial beta-plane waves

8.3 The Walker circulation

9. Fundamentals of midlatitude dynamics

9.1 Geostrophic balance

9.2 Quasigeostrophic theory: Shallow water case

9.3 Quasigeostrophic theory: General case

Geostrophy. Basic concepts QG theory.

9.4 Fundamental conservation laws

[**Pseudomomentum and pseudo energy. Things that will be of importance both for wave theory and for stability theory.]

9.5 The Ekman layer

10. Rossby waves

10.1 Free wave dispersion relation

10.2 Free waves in vertical shear

10.3 Forced Stationary waves

10.4 Ray tracing theory. Wave trains

11. Synoptic scale transient eddies

[**Basically the stuff in my Ann. Rev. review article.]

[**Perhaps we can also include some numerical methods for stability theory here.]

12. General circulation models

[**This section introduces numerical concepts needed to do modeling on a sphere. It leads up to discussions of the behavior of the models, and their general use in addressing climate problems.]

[**It might actually be better to have this part earlier (before Chapter 7). This would permit us to use the GCM's to illustrate basic phenomena (like synoptic eddies) as they come along. The main question is how to arrange presentation of the basic physics (e.g. the Rossby wave chapter) vs. the presentation of nonlinear transient models, which can be used to illustrate the concepts in a more realistic context.]

12.1 Shallow water model on a sphere

12.2 A two layer primitive equation model

12.3 A dry multi-level GCM

12.4 Multi-level GCM with a hydrological cycle

A1 A brief handbook of atmospheric physics

Thermodynamics

1mb = 100 Pascals = 100 $Kg/(m\ sec^2)$

T_{abs} = -273.15 ºC

$p = \rho R T$, where T is in ºK and R is the gas constant for the gas in question. R is related to the *universal gas constant* R^* by the relation $R = R^*/M$, where M is the molecular weight of the gas.

R^* = 8314.3 Joule/(ºK kmol)

M_{water} = 18.016, so R_{vapor} = 461.

$M_{dry\ air}$ = 28.97, so $R_{dry\ air}$ = 287.

M_{CO2} = 44, so R_{CO2} = 188.96

Specific heats $c_v = (dq/dT)|_{\rho=const} = (du/dt)|_{\rho=const}$, where u=u(T) is the internal energy of the gas. Let specific volume $\alpha = 1/\rho$. Since $dq = du + p d\alpha = c_v dT + p d\alpha = (c_v+R)dT - \alpha dp$ we have $c_p = c_v + R = (dq/dT)|_{p=const.}$

Dry air: c_v = 717 J/(º Kg), c_p = 1004 J/(º Kg) , R/c_p = 2/7

CO_2: c_p = 820 at 0 ºC, about 1000 at 400 ºC.

Water vapor: c_v = 1463, c_p = 1952

Liquid water: c_v = 4218 (approximately incompressible, i.e. $c_v = c_p$)

Water ice: c_v = 2106 (approximately incompressible, i.e. $c_v = c_p$)

Water: Latent heat of melting L = 3.34 10^5 J/Kg at standard temperature and pressure. Latent heat of vaporization L = 2.5 10^6 J/Kg at 0ºC (2.25 10^6 at 100ºC). Critical point (p,T,ρ) = (22.064 MegaPascals, 647.15 ºK, 3110 Kg/m^3)

Clausius-Clapeyron relation: $de_s/dT = L/((\alpha_2 - \alpha_1)T)$, where e_s is saturation vapor pressure and α_2 is the specific volume of the vapor phase. If L = const. and $\alpha_2 >> \alpha_1$, then

$$e_s = e_\infty \exp(-L/RT)$$

where R is the gas constant for the pure vapor in question (e.g. the gas constant for pure water vapor, in the case of water). For water not too far from STP, e_∞ = 2.6 10^{11} Pa and L/R = 5423 ºK .

For water, e_s = 6.108mb at 0ºC, and .189 mb at -40ºC.

If the water mixing ratio in air is small, then the mass mixing ratio is given approximately by $w_s = e_s/p$. e.g. w_s = 3.5 g/Kg at 0ºC for p = 1000mb.

1 Megaton TNT-equivalent = 4.2 10^{22} ergs = 4.2 10^{12} Joules.

1 Joule = .000238662 Kg-cal

1 g-cal = 4.1860 Joule

Radiation

σ(Stefan-Boltzman constant) = 5.6696 10^{-8} Watt/(m^2 ºK^4)

Solar flux L at mean radius of Earth's orbit = 1370 W/m^2

Fossil Fuel Facts

1 Joule = .000948608 BTU

Petroleum facts:

1 Barrel Petroleum = 158.98 Liter

Gasoline combustion releases 5.253 10^6 BTU/barrel

Motor gasoline: 19.5 Million Metric Tons Carbon per Quadrillion BTU

[**Carbon emission per liter.]

n-octane is C_8H_{18}; heat of combustion = 1303 Kg-cal/gram-mole

n-Hexane is C_6H_{24}; heat of combustion = 995 Kg-cal/gram-mole

Coal facts:

Natural gas facts:

Earth constants

Radius of Earth = 6.37 10^6m

g = acceleration of gravity = 9.8 m/sec^2

ω_{day} = 7.292 10^{-5} radians/sec

mean distance from Sun = 1.5 10^{11} m

Mars constants

Radius = 3.394 10^6 m

g = acceleration of gravity = 3.72 m/s^2

ω_{day} = 7.078 10^{-5} radians/sec

mean distance from Sun =

References

Bloggs. 19xx: Title. *J. Fluid Mech* **nnn**,xx-xx.

Notes for book with Isaac

With all the basic climate stuff normally included in 232, the project begins to become unwieldy. Also, there may be too much overlap of the introductory material with Hartmann's book. I suggest that the book with Isaac concentrate on the more dynamical end (Sections 6 and onward), though keeping a good emphasis on application to real data and providing the readers with working source code for models. Perhaps at some later point I can write a "climate primer" companion volume with consistent notation.

Notes for revision

[**Following can be used as outline for introductory GFD course, GFD I.]

1. Basic principals of fluid dynamics

1.1. What is a fluid and how is it characterized?

1.2. Deriving the equations of motion

Material (comoving) derivatives

Pressure Force

2D Incompressible flow

2D equations of motion. Incompressibility constraint $\rho=$ const., $\cdot \mathbf{u} = 0$. Diagnostic equation for pressure. Vorticity equation. Interpretation of vorticity (circulation). Streamfunctions. Poisson equation for vorticity in terms of streamfunction.

1.3. Fluid dynamics in a rotating reference frame

Review of particle dynamics in rotating frame

Hockey puck on carousel. Centrifugal "force". Coriolis "force". Inertial circles.

3D momentum equations in a rotating frame

Rapidly rotating limit: Rossby numbers and the Geostrophic approximation

2D special case. Unstratified special case (Taylor-Proudman).

Geostrophic flow in a spherical shell