# 3 Problem Set: Clausius Clapeyron and Moist Adiabats

**Problem 3.1** Titan has a surface temperature of about $95K$. Suppose that there is a lake of liquid methane ($CH4$) on the surface, and that the air just above the lake is in equilibrium with the lake, and therefore saturated with respect to methane. Use the simplified form of Clausius-Clapeyron to determine the partial pressure of methane in the air near the surface. What would the partial pressure be if the temperature rose to 120K? Based on a surface pressure of $1.5bar$, what would the molar mixing ratio of methane in the atmosphere be? What would the mass mixing ratio, the molar concentration and the mass specific concentration be? What would the partial pressure and molar mixing ratio be if the temperature rose to 120K? What would the mass mixing ratio, the molar concentration and mass specific concentration be?

**Problem 3.2** Something is about to happen. Something wonderful. To promote life on Jupiter's moon Europa, which currently is composed of a liquid water ocean covered by a very thick water ice crust, the alien race which built Tycho Magnetic Anomaly 1 ignites thermonuclear fusion on Jupiter, heating Europa to the point that its icy crust melts, leaving it with a globally ocean covered surface having a temperature of $280K$. Water vapor is the only source of atmosphere for this planet. Describe what the atmosphere would be like, and calculate $T(p)$ for this atmosphere. Give a rough estimate of the depth (in km) of the layer containing most of the mass of the atmosphere. (The gravitational acceleration at Europa's surface is $1.3m/s^2$).

**Problem 3.3** *Temperature-dependent specific heat*

This is a computer problem involving temperature-dependent specific heats. To do the problem, you will need to know the following programming techniques: (1) Defining and evaluating a function; (2) Tabulating a set of values and either plotting them or writing them out for plotting with a separate program; (3) Summing up a set of values in a loop.

The *Shomate equation* is an empirical formula for the dependence of specific heat on temperature. It works well for a broad range of gases. The formula reads:

$$c_p = A + B * (T/1000.) + C * (T/1000.)^2 + D * (T/1000.)^3 + E * (T/1000.)^{-2} \quad (1)$$

where $T$ is the temperature in Kelvins and $A, ...E$ are gas-dependent constants. Some coefficient sets are given in Table 1.

Write a function to compute `cp(T)`. Explore the behavior for the three gases, taking note of the degree to which the specific heat varies. Make a plot of the temperature dependence. From a physical standpoint, why do the specific heats increase with temperature? Measurements show that the specific heat of $He$ is very nearly temperature independent. Why?

| | A | B | C | D | E |
|---|---|---|---|---|---|
| $N_2$ | 931.857 | 293.529 | -70.576 | 5.688 | 1.587 |
| $CO_2$ | 568.122 | 1254.249 | -765.713 | 180.645 | -3.105 |
| $NH_3$ | 1176.213 | 2927.717 | -904.470 | 113.009 | 11.128 |

Table 1: Shomate coefficients for selected gases, valid from $300K$ to $1300K$ for $CO_2$ and $NH_3$, and $300K$ to $6000K$ for $N_2$

Compute the energy it would take to raise the temperature of 1kg of $N_2$ from 300K to 700K, in steps of .1K (assuming $c_p$ to be constant within each step). Compare this to the value you would have gotten if $c_p$ were assumed constant at its value in the middle of the temperature range. *Hint:* Write a loop to sum the energies in each step.

*Python Tips:* It is convenient to store the Shomate coefficients for each gas in a list,e.g.

```
N2_coeffs = [931.857,293.529,-70.576,5.688,1.587]
```

If you do this, you can set all the coefficients at once with a statement like `A,B,C,D,E = N2_coeffs`. A good trick is to define the lists as globals outside the function definition for `cp(T)`, and to have the function refer to the list of coefficients as, e.g. `ShomateCoeffs`. Then, to switch from one gas to another, all you have to do is set `ShomateCoeffs` to the list you want before you call `cp(T)`, as in `ShomateCoeffs = N2_coeffs`.

**Problem 3.4** In this computer problem, you will compute the dry adiabat $T(p)$ for an ideal gas whose specific heat depends on temperature, in accord with the Shomate equation (See Problem 3.3). In addition to basic skills such as defining functions and loops, you will need to know how to write programs that find approximate solutions to an ordinary differential equation of the form $dY/dx = f(x, Y)$

First, use the First Law of Thermodynamics to derive a differential equation for $d\ln T/d\ln p$ assuming $\delta Q = 0$. This defines the dry adiabat. Note that since $c_p$ is a function of $T$, you can no longer treat it as a constant in doing the integral.

Write a program that tabulates approximate solutions to the differential equation. Note that your dependent variable is $Y \equiv \ln T$ whereas the right hand side of the differential equation involves $T$. This is not a problem, since you can write $T = \exp(\ln T)$. In writing your program, assume that $c_p(T)$ is defined by the Shomate equation.

Apply your program to obtain an improved approximation to the dry adiabat in a pure $CO_2$ Venusian atmosphere, which you originally computed in Problem **??**. Start your computation at the ground ($p_s = 92bars$) with the observed mean surface temperature of Venus ($737K$). Integrate up to the $100mb$ level, and compare the temperatures you get with those in the Magellan observations shown in Figure 2.2 of the text. Make a plot comparing your calculations with the dry adiabat obtained by keeping $c_p$ constant at $820 J/kg$.

**Problem 3.5** *Water on the Hadean Earth*

Ancient zircon crystals tell us that liquid water existed somewhere on the surface of the Earth as early as 4.2 billion years ago. What does this tell us about the surface temperature? Does it imply that the temperatures had to be below $100C$ (the "boiling point")? To answer this question, compute the saturation partial pressure of water vapor at an ocean surface as a function of surface temperature. Do this three ways. First, use the exponential form of the Clausius-Clapeyron relation with constant latent heat, based on triple point data. Next, use the exponential form based on "boiling point" data (i.e. that the vapor pressure is $1bar$ at $373.15K$. Finally, use the empirical Antoine equation:

$$p_{sat}(T) = 10^{A-B/(T+C)} \tag{2}$$

where $A$,$B$ and $C$ are empirical coefficients determined by fits to experimental data. In the range $379-573K$, the coefficients are $(A, B, C) = (3.55959, 643.748, -198.043)$ if the pressure is given in $bars$. For the purposes of this problem you may assume that it is safe to push the equation $250K$ or so above the range of the fit.

Using your results, answer the following. At $100C$ ($373.15K$) what is the partial pressure of water vapor at the surface? What fraction of the molecules of the atmosphere are water at the surface (assuming that the non-condensing part of the atmosphere has partial pressure of $1bar$)? Does the ocean boil under these conditions? What if the surface temperature is $200C$ instead? At this temperature, what proportion of the mass of water on the planet is in vapor form in the atmosphere, assuming the total water mass to be the same as that of the present ocean ($1.4 \cdot 10^{21} kg$)? How hot would the surface have to be in order for all the inventory of water to go into the atmosphere?

*Hint:* At temperatures of $400K$ and above, it is a reasonable approximation that water vapor dominates the mass of the atmosphere, so that by the Hydrostatic Law, the mass of water vapor in the atmosphere can be approximated by $A \cdot p_{sat}/g$, where $p_{sat}$ is the saturation water vapor pressure at the surface (in $Pa$) and $A$ is the surface area of the Earth.