

# Geosci 232 Problem Set 4

## Fall Quarter 2007

November 6, 2007

### 4 Problem Set: Moist adiabats and Black Body Radiation Basics

**Problem 4.1** Using the formula for the slope of the saturated moist adiabat for a condensible/noncondensible mixture, compute  $d\ln(T)/d\ln(p_a)$  for a mixture of water vapor and Earth air at a pressure of 1000mb and (a) a temperature of 300K, (b) a temperature of 250K. List the values of the individual terms in the numerator and denominator, so that you can get a feel for which are big and which are small. Compare the values with the slope of the dry adiabat.

The slope is also called the "lapse rate". The term is most commonly used for  $dT/dz$ , but can by extension be used for other characterizations of the rate of change of temperature with vertical coordinate.

**Problem 4.2** The atmosphere of Titan consists of a mixture of  $N_2$  and  $CH_4$  (Methane). The methane is condensible at Titan temperatures but for the purposes of this problem the nitrogen can be regarded as non-condensable. (In reality, it does condense a little bit in Titan's upper troposphere). Assume that there is sufficient methane ice or liquid at the surface to maintain methane saturation.

Compute and plot the moist adiabatic  $T(p)$  for Titan assuming 1.5bar of  $N_2$  at the ground. You may also assume that methane is saturated throughout the atmosphere (requiring a reservoir of methane at the ground). Plot and discuss results for the mixing ratio as a function of  $p$  as well. Discuss results for three different cases: (a)  $T = 95K$  at the surface, as for Titan at present, (b) a hot Titan, with  $T = 120K$ , and (c) a cold Titan, with  $T = 80K$

*Python Tips:* You can modify the script `MoistAdiabat.py` for use with a mixture as defined above. Gas data for  $N_2$  and  $CH_4$  are both in the `phys` module, under the names `N2` and `CH4`.

**Problem 4.3** Consider an atmosphere consisting of a saturated mixture of condensible water vapor with noncondensable Earth air. The way we have been setting up the calculation, the partial pressure of noncondensable air,  $p_a$  is specified at the ground, and the total pressure  $p$  (including water vapor) is computed. From the hydrostatic relation, the total mass of the atmosphere per square meter is  $p/g$ , but the mass of air is not  $p_a/g$  and the mass of water is not  $p_w/g$  because water vapor is not well-mixed, i.e.  $p_w/(p_w + p_a)$  is not constant.

Write a function to compute the total mass of water vapor and of air in Earth's atmosphere as a function of  $p_a$  at the surface and the surface temperature. Explore the behavior with  $p_a$  fixed at `1bar`, as the temperature ranges from `270K` to `400K`. How much does the mass of noncondensing air vary? Why does it vary? How does the mass of water in the atmosphere compare with the estimate provided by  $p_w/g$ , which is accurate when water vapor dominates the atmosphere?

*Hint:* If  $q(p)$  is the mass concentration of a substance, then the mass of the substance per unit area is  $\int_{p_s}^0 q(p)dp/g$ . The integral can be approximated by a sum over layers.

*Python Tips:* You can do this problem by modifying the chapter script `MoistAdiabat.py`, or by using the `phys.MoistAdiabat` class to make a function to compute the moist adiabat and mass concentration, then summing up the output.

**Problem 4.4** Consider a patch of the photosphere of the Sun with an area of  $10 \text{ m}^2$ . The patch is small enough that it can be considered flat. Assume that the temperature of the photosphere is `6000K` and that it radiates like an ideal blackbody. Now consider all the light with wavenumbers between `10000` and `10100 cm-1`, which leaves the patch in directions making angles between `45` degrees and `50` degrees with the normal to the patch. How much power (in  $W$ ) is contained in the light meeting these requirements? What is the rate (in  $W$ ) at which energy is carried *through* the patch by such light? For the purposes of this problem, you may assume the Planck function to be constant within the specified wavenumber band.

**Problem 4.5** What is the total power radiated by a blackbody sphere of radius  $1m$  having uniform temperature  $300K$ , in the wavenumber range between  $500cm^{-1}$  and  $750cm^{-1}$ . You may assume the Planck density to be approximately constant over this range. Compare this to the total power radiated over all wavenumbers. *Note:* Blackbody radiation is emitted with equal intensity in all directions, so you must remember to take this into account in computing the total energy flux coming from the surface.

**Problem 4.6** Write down the Planck density corresponding to the case in which position in the electromagnetic spectrum is measured by  $\ln \lambda$ , where  $\lambda$  is the wavelength. Estimate the value of  $u = h\nu/(kT)$  for which this density has its peak. (You will probably need to do a small numerical calculation to do this). How does your result change if you use the log of the wavenumber or frequency as the coordinate instead?