Geosci 232 Problem Set 5
Fall Quarter 2007

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5 Problem Set:Blackbody radiation, energy balance and bifurcation

Problem 5.1 Compute the total power radiated by a person with a normal body temperature of $37C$. Why is this so much greater than the typical daily energy consumed by a person in the form of food (equivalent to about $100W$)? Next, compute the power radiated by the person in the visible wavelength band (about $.5$ to 1 microns). Approximately how many visible light photons per second are radiated? About how long would you have to wait before the person emitted a *single* ultraviolet photon (about $.1$ micron wavelength)?

For the purposes of estimating the surface area needed in this problem, you may assume that the person is shaped approximately like a rectangular prism, with height $1.5m$, width $.5m$ and depth $.25m$.

Problem 5.2 For Jupiter, the observed OLR is $14.3W/m^2$. Compute the effective radiating temperature. Referring to the Jupiter temperature profile given in Chapter ??, estimate the effective radiating pressure of Jupiter. Is there more than one possible value? Which of these do you consider more likely?

Problem 5.3 You are designing a spherical planet to be placed at the orbit of Mercury. The planet will have a Nitrogen atmosphere that has no effect on the infrared radiated by the planet, but which is dense enough and mixes heat rapidly enough that the entire planetary surface is isothermal. What albedo should the planet have in order for its surface temperature to be a comfortable $300K$?

Problem 5.4 A cylindrical space station with length h and radius r is in an orbit about the Sun at a distance where the solar constant is $L_\odot$. The space station has zero albedo in the visible range and radiates as a perfect blackbody. The flow of air inside keeps the entire station at the same temperature, and the skin is a good conductor of heat, so that its temperature is the same as that of the interior. The orientation of the station is such that the axis of the cylinder is always perpendicular to the line joining the center of the station to the center of the Sun. Find an expression for the temperature of the station. Put in numbers corresponding to the mean Solar constant at Earth's orbit, assuming $r = h$.

Now suppose that the equipment in the interior of the space station consumes 1 megawatt of solar-generated electrical power, which is dissipated as heat. How much warmer would this make the station once the equipment was turned on? To get rid of this excessive heat, you are to design a radiator, which is a large, thin flat plate heated by pumped water from the space station so that its temperature is the same as the interior of the space station. The radiator is perfectly reflective in the visible range, but acts as a perfect blackbody in the infrared range. How large should the radiator plate be in order to get rid of the excess heat? For the purposes of this part of the problem you may assume $r = h = 50m$.

Problem 5.5 Taking into account its observed albedo, Titan absorbs $2.94W/m^2$ of solar radiation (averaged over its entire surface). The observed surface temperature is 95K. If you assume that the temperature profile of the atmosphere is given by the dry adiabat for pure N_2 having a surface pressure of $1.5bar$, what would the radiating pressure for Titan have to be in order to account for the observed surface temperature?

Problem 5.6 *Computing the Ice-Albedo Hysteresis Diagram*

Calculations with a complete real-gas radiation simulation indicate that, for a CO_2 concentration of $300ppmv$, and with an atmosphere on the moist adiabat, a reasonable fit to the actual OLR curve in the range of $220K$ to $310K$ is the linear fit $OLR(T) = a + b \cdot (T - 220)$ where $a = 113W/m^2$ and $b = 2.177W/m^2K$. Compute the ice-albedo hysteresis diagram giving the set of equilibrium temperatures as a function of the solar constant $L_\odot$. Note that there is a simple trick for getting the bifurcation plot. The equation determining the equilibrium is $\frac{1}{4}L_\odot(1 - \alpha(T)) = OLR(T)$ Instead of specifying $L_\odot$ and finding the T that satisfy the equation, we can re-write the equation as

$$L_\odot = 4\frac{OLR(T)}{1-\alpha(T)} \tag{1}$$

Now, if we call the right hand side $G(T)$, then $G(T)$ gives the unique value of $L_\odot$ which supports the temperature T. Hence, to get the bifurcation diagram, you can just plot $G(T)$ and then turn it sideways.

Use the same albedo-temperature function defined in Chapter 3. Assume that the albedo for an ice-free Earth is .2 and for an ice-covered Earth is .6.

Based on your calculations, if CO_2 were held constant how much would $L_\odot$ have to be reduced from its modern value before Earth was forced to fall into an inevitable snowball state? Using the inverse-square law and assuming a circular orbit, compute how far out from the Sun the Earth would have to be displaced (relative to its present orbit) to achieve this solar constant. Conversely, how close to the Sun would you have to place the Earth before a Snowball state became impossible? *Note:* The assumption of fixed CO_2 is very unrealistic, since tectonically active planets with water have a way of adjusting CO_2 in response to changes in the solar constant. This will be discussed in Chapter ??.

Python Tips: The script `IceAlbedoZeroD.py`, found in the Chapter Scripts collection for this chapter, calculates and plots hysteresis diagrams using this method, and also makes various other plots useful in understanding bifurcations due to ice-albedo feedback. You can easily modify this script to solve this and similar problems, by redefining the `OLR(T)` function, and (in other cases) also the albedo function.