

# Geosci 232 Problem Set 6

## Fall Quarter 2007

November 27, 2007

### 6 Problem Set: Blackbody radiation review problems, and thin atmospheres

**Problem 6.1** Show that if both  $\nu_1$  and  $\nu_2$  are in the frequency range where  $h\nu/kT \ll 1$ , then the total flux per steradian emitted between  $\nu_1$  and  $\nu_2$  is  $\frac{2}{3}(kT/c^2)(\nu_2^3 - \nu_1^3)$ .

**Problem 6.2** *Microwave brightness of Venus*

The thick  $CO_2$  atmosphere of Venus is nearly opaque in the infrared, so observations of Venus in the infrared spectrum provide no direct information about the temperature of the ground. However, the atmosphere is nearly transparent to microwave radiation, so the microwave emission of the planet can be used to infer its surface temperature. Indeed, this is how it was inferred in the 1960's that the surface of the planet was hotter than a pizza oven, rather than the steamy jungle world earlier science fiction authors had envisioned.

Assuming the ground to radiate like a blackbody in the microwave, compute the net power (in  $W$ ) radiated by Venus in the wavelength band from 1 millimeter to 100 millimeters, if the surface temperature is uniform at  $737K$ . What would the radiated power be if the surface temperature were  $300K$  instead? *Hint:* You can use the result from Problem 6.1. Why is this valid?

At its closest approach, Venus is about 41 million  $km$  distant from Earth. Using the  $1/r^2$  law, what microwave energy flux (in  $W/m^2$ ) would be seen from Earth orbit by a microwave antenna directed toward Venus? How much microwave power would be collected by an antenna with area  $100m^2$ ?

**Problem 6.3** Compute the equilibrium temperature at the subsolar point of Europa, which is in orbit around Jupiter and therefore has the same solar constant as that planet. The greenhouse effect of Europa's tenuous atmosphere can be neglected. For the purposes of this problem, you may assume that the albedo of Europa is .67 . Assuming Europa to have a water-ice surface, what would be the saturation vapor pressure of the water vapor atmosphere immediately above the subsolar point? Suppose that there is some methane, carbon dioxide and ammonia mixed in with the ice. What would be the partial pressures of these gases? *Hint: Recall that the subsolar point is the point at which the Sun is directly overhead. In this problem you are to assume that the temperature at the subsolar point instantaneously comes into equilibrium with the absorbed solar radiation*

**Problem 6.4** Consider a planet covered in water ice with a uniform albedo of .7 . The planet is tide-locked, so that the same face always points to the Sun; the other side is in perpetual night. The atmosphere has negligible greenhouse effect. Compute the solar constant needed to begin melting the ice under the following three alternate scenarios: (a) The atmosphere is so efficient at transporting heat that the entire surface of the planet (dayside and nightside) has the same temperature, (b) The atmosphere is only moderately efficient, so that the dayside temperature is uniform but essentially no energy is carried away to the nightside, (c) There is no atmosphere, so that each bit of the planet's surface is in equilibrium with the solar radiation it absorbs.

**Problem 6.5** Consider a sandwich made of three slabs of material. The two outer slabs are blackbodies, with temperatures  $T_1$  and  $T_2$  respectively. Between these two is a slab which is a grey body having emissivity  $e$ . What is the temperature of the grey-body slab?

**Problem 6.6** Suppose that the ground has temperature  $T_s$ , which is a given. The ground has unit infrared emissivity. There is one isothermal layer above the ground with emissivity  $e_1$ , and another one above that with emissivity  $e_2$ . Neither layer absorbs any Solar radiation, and neither layer is affected by any heat transfer mechanism other than radiation.

Find an expression for the temperatures of the two layers. Do not assume that the emissivities are small.

**Problem 6.7** Consider an optically thin atmosphere which has a nonzero emissivity  $e$  within a very narrow frequency band of width  $\Delta\nu$  centered on frequency  $\nu_o$ . This atmosphere is above a surface having a known temperature

$T_g$ , which radiates like a blackbody. What is the temperature of the atmosphere if no convection is allowed to take place? Explore how the temperature varies as a function of  $\nu_o$ . Does the result depend on  $\Delta\nu$ ? On the emissivity? Put in numbers corresponding to Martian afternoon conditions:  $T_g = 230K$ , and  $\nu_o$  corresponding to a wavenumber of  $650 \text{ cm}^{-1}$  (approximately the center of the principle absorption band of  $CO_2$ ).