Geosci 232 Problem Set 1
Fall Quarter 2009

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1 Problem Set:Energy and radiation basics

Problem 1.1 An impactor with the mass of the Moon hits the Earth with a speed of 15 km/s. The object is made of silicate rock. Is the energy released enough to vaporize the object? Does your answer depend on the mass of the object? *Note:* It takes about $2 \cdot 10^7 J$ to vaporize 1 kg of silicate rock.

Problem 1.2 Luminosity is the net power output of a star, which we will measure in W. The proton process fuses four protons (hydrogen nucleii) into helium, yielding $4.17 \cdot 10^{-12} J$ of energy. Assuming constant luminosity, find a relation between the mass of the star and its lifetime on the main sequence, assuming the star to be initially hydrogen-dominated. How does your answer change if you use Eq. 1.1 to take into account the increase in luminosity over time? In this application of the formula, the parameter $t_\odot$ should assumed to represent half the star's lifetime on the main sequence, and the formula should be considered valid only out to times 2 $t_\odot$.

If T is the surface temperature of the star, then the radiation flux in W/m^2 out of the surface is $5.67 \cdot 10^{-8} T^4$. You will learn more about this formula in Chapter ??. Use this to estimate the surface temperature of the star in terms of its radius. This also gives you the spectral class of the star, since cooler stars are redder. Then, if you have a relation between the mass of the star and its radius, you have a relation between lifetime and spectral class. Use this to show that the bluer stars are more short-lived, assuming that the density is independent of mass. A more accurate calculation would need to take into account the compressibility of the star's substance.

Problem 1.3 The Sun puts out energy at a rate of $3.84 \cdot 10^{26} W$. A resting human consumes (and puts out) energy at a rate of about $100W$. Look up the mass of the Sun, and determine which of these two bodies requires more energy production per unit of mass. Using the data given in Problem 1.2, determine how much mass of hydrogen would constitute a lifetime supply of food if humans could eat hydrogen and fuse it into helium?

Problem 1.4 Show that if both ν_1 and ν_2 are in the frequency range where $h\nu/kT \ll 1$, then the total flux per steradian emitted between ν_1 and ν_2 is $\frac{2}{3}(kT/c^2)(\nu_2^3 - \nu_1^3)$.

Problem 1.5 *Microwave brightness of Venus*

The thick CO_2 atmosphere of Venus is nearly opaque in the infrared, so observations of Venus in the infrared spectrum provide no direct information about the temperature of the ground. However, the atmosphere is nearly transparent to microwave radiation, so the microwave emission of the planet can be used to infer its surface temperature. Indeed, this is how it was inferred in the 1960's that the surface of the planet was hotter than a pizza oven, rather than the steamy jungle world earlier science fiction authors had envisioned.

Assuming the ground to radiate like a blackbody in the microwave, compute the net power (in W) radiated by Venus in the wavelength band from 1 millimeter to 100 millimeters, if the surface temperature is uniform at $737K$. What would the radiated power be if the surface temperature were $300K$ instead? *Hint:* You can use the result from Problem 1.4. Why is this valid?

At its closest approach, Venus is about 41 million km distant from Earth. Using the $1/r^2$ law, what microwave energy flux (in W/m^2) would be seen from Earth orbit by a microwave antenna directed toward Venus? How much microwave power would be collected by an antenna with area $100m^2$?

Problem 1.6 Compute the total power radiated by a person with a normal body temperature of $37C$. Why is this so much greater than the typical daily energy consumed by a person in the form of food (equivalent to about $100W$)? Next, compute the power radiated by the person in the visible wavelength band (about $.5$ to 1 microns). Approximately how many visible light photons per second are radiated? About how long would you have to wait before the person emitted a *single* ultraviolet photon (about $.1$ micron wavelength)?

For the purposes of estimating the surface area needed in this problem, you may assume that the person is shaped approximately like a rectangular prism, with height $1.5m$, width $.5m$ and depth $.25m$.

Problem 1.7 You are designing a spherical planet to be placed at the orbit of Mercury. The planet will have a nitrogen atmosphere that has no effect on the infrared radiated by the planet, but which is dense enough and mixes heat rapidly enough that the entire planetary surface is isothermal. What albedo should the planet have in order for its surface temperature to be a comfortable $300K$?