Robust elements of Snowball Earth atmospheric circulation and oases for life

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[1] Atmospheric circulation in a Snowball Earth is critical for determining cloud behavior, heat export from the tropics, regions of bare ice, and sea glacier flow. These processes strongly affect Snowball Earth deglaciation and the ability of oases to support photosynthetic marine life throughout a Snowball Earth. Here we establish robust aspects of the Snowball Earth atmospheric circulation by running six general circulation models with consistent Snowball Earth boundary conditions. The models produce qualitatively similar patterns of atmospheric circulation and precipitation minus evaporation. The strength of the Snowball Hadley circulation is roughly double modern at low CO2 and greatly increases as CO2 is increased. We force a 1-D axisymmetric sea glacier model with general circulation model (GCM) output and show that, neglecting zonal asymmetry, sea glaciers would limit ice thickness variations to $O(10\%)$. Global mean ice thickness in the 1-D sea glacier model is well-approximated by a 0-D ice thickness model with global mean surface temperature as the upper boundary condition. We then show that a thin-ice Snowball solution is possible in the axisymmetric sea glacier model when forced by output from all the GCMs if we use ice optical properties that favor the thin-ice solution. Finally, we examine Snowball oases for life using analytical models forced by the GCM output and find that conditions become more favorable for oases as the Snowball warms, so that the most critical time for the survival of life would be near the beginning of a Snowball Earth episode.


1. Introduction

[2] At least three global-scale glaciations occurred during the Neoproterozoic at $\approx 715$ Ma (Sturtian), $\approx 635$ Ma (Marinoan), and $\approx 580$ Ma (Edicaran) [Hoffman and Li, 2009]. Both the Sturtian and Marinoan glaciation show strong evidence for ice flowing from land into the ocean in tropical latitudes with carbonate sequences overlying glacial sequences. In addition, numerous banded iron formations, which represent possible evidence for global glaciation, are found in the Sturtian glaciation, but not generally in the others. U-Pb zircon radiometric dating suggests that the Sturtian glaciation may have lasted for as much as 70 Myr, the Marinoan for up to 20 Myr, and the Edicaran for a few million years [Hoffman and Li, 2009]. In addition, other global or nearly global glaciations, for which less data are available, occurred in the Paleoproterozoic (at $\approx 2.2$ Ga [Hoffman and Schrag, 2002]).

[3] The Snowball Earth hypothesis attempts to explain these global-scale glaciations, or at least the more severe ones. According to the Snowball Earth hypothesis, the entire ocean was covered with ice and there was extensive low-latitude continental glaciation for a few million years, during which time CO2 from volcanic outgassing built up to high enough values to melt tropical ice and reverse the global glaciation [Kirschvink, 1992; Hoffman et al., 1998]. Alternative hypotheses have been suggested to explain Neoproterozoic glaciations, or at least the less severe ones, in which low-latitude continental glaciation occurred without the ocean completely freezing over [Hyde et al., 2000; Chandler and Sohl, 2000; Peltier et al., 2007; Micheels and Montenari, 2008; Liu and Peltier, 2010; Abbot et al., 2011; Yang et al., 2012a, 2012b], but we will consider the traditional (sometimes called “hard”) Snowball Earth hypothesis here. The main outstanding theoretical questions for the Snowball Earth hypothesis are as follows: (1) What caused
Snowball Earth events? (2) Can a Snowball Earth deglaciate at a reasonable CO₂ level? and (3) How did life survive Snowball Earth events? The first question is not considered here (see Tziperman et al. [2011], Yang et al. [2012a, 2012b, 2012c], Voigt and Marotzke [2010], Voigt et al. [2011], and Voigt and Abbot [2012] for recent work on this problem), as we assume Snowball Earth conditions in our calculations.

[4] The question of whether a Snowball Earth could deglaciate at a realistic CO₂ level was raised by early general circulation model (GCM) simulations [Pierrehumbert, 2004, 2005], which led to theories implicating albedo reduction through volcanic or aeolian dust [Le Hir et al., 2010; Abbot and Pierrehumbert, 2010; Abbot and Halyev, 2010]. In a companion paper [Abbot et al., 2012], we showed that clouds could warm a Snowball climate by ≈10 K, which would reduce the CO₂ needed for deglaciation by at least an order of magnitude. In the current cloud paper, we focus on atmospheric circulation and the hydrological cycle in a Snowball. The mechanism for optically thick cloud formation in a Snowball identified by Abbot et al. [2012] was lifting of air by a vigorous Hadley circulation, which makes understanding the Snowball atmospheric circulation important. Moreover, atmospheric circulation in the Snowball determines heat export from the tropics to higher latitudes, which is crucial for the Snowball deglaciation threshold. Additionally, atmospheric circulation produces the precipitation minus evaporation pattern, which augments the flow of thick, floating “sea glaciers” on the Snowball ocean [Goodman and Pierrehumbert, 2003; Pollard and Kasting, 2005; Goodman, 2006; Li and Pierrehumbert, 2011; Tziperman et al., 2012]. Sea glaciers flow strongly toward the equator because annual mean evaporation exceeds precipitation in the tropics in a Snowball as a result of annual mean descent in the tropics [Pierrehumbert, 2005; Abbot and Pierrehumbert, 2010]. Over hundreds of thousands of years, this would tend to concentrate surface dust in the tropics where it would be most important for deglaciation [Abbot and Pierrehumbert, 2010; Li and Pierrehumbert, 2011].

[5] Assuming global ice coverage, sea glaciers are expected to grow to a thickness of hundreds of meters, which would prevent photosynthetic activity in underlying oceans. Given this, two alternatives have been proposed to explain the survival of photosynthetic marine life through Snowball Earth events. The first proposal is that life survived in some small oases, possibly associated with volcanic activity [e.g., Hoffman and Schrag, 2000]. Campbell et al. [2011] considered whether oases could exist in narrow channels, into which sea glacier flow would be reduced by friction with the side walls. Similarly, Tziperman et al. [2012] considered the potential for oases in seaways constricted by a narrow entrance. These oases become more viable at lower ice temperature and higher evaporation minus precipitation. The second proposal for the survival of life is that the tropical net ablation zone, which is driven by atmospheric circulation, could lead to a Snowball solution with tropical ice thin enough that photosynthesis could occur beneath it [McKay, 2000; Pollard and Kasting, 2005]. We will consider both types of oases here. This paper therefore addresses how the atmospheric circulation affects both the CO₂ threshold for Snowball deglaciation and the survival of life through Snowball Earth episodes.

[6] GCMs are the main tools we have to make detailed predictions about atmospheric temperature, circulation, precipitation, and evaporation in a Snowball Earth, and they have been applied to the problem of global glaciation at least since Wetherald and Manabe [1975]. Because GCMs have not always yielded consistent results in Snowball conditions, detailed comparisons between GCMs are required to determine robust predictions. Initial efforts to compare GCMs run in Snowball situations have been performed by Le Hir et al. [2007, 2010], Abbot and Pierrehumbert [2010], Hu et al. [2011], Pierrehumbert et al. [2011], and Abbot et al. [2011]. Preliminary results suggest that models simulate atmospheric circulation of consistent pattern, but widely varying strength, under Snowball conditions [Abbot and Pierrehumbert, 2010; Hu et al., 2011]. In this paper and its companion [Abbot et al., 2012], we perform a rigorous comparison of GCMs (Table 1) run in a consistent Snowball Earth configuration. In the Snowball Earth hypothesis CO₂ accumulates throughout the event, so we run the models at CO₂ = 10⁴ (10² ppm), as a proxy for early Snowball conditions, and at CO₂ = 0.1 (10⁵ ppm), as a proxy for a nearly deglaciating Snowball.

[7] This paper is organized as follows. In section 2 we describe the models and simulations. We highlight the important results from these simulations in section 3. In particular, we find that the pattern of atmospheric circulation and precipitation minus evaporation are consistent across the models, although the magnitude of the stream function maximum differs by up to 100% among models. This confirms that the shape of the hydrological forcing of sea glacier flow is a robust feature of Snowball Earth simulation. We also find...
that differences in model cloud simulation are much more important than differences in heat export from the tropics for determining the Snowball deglaciation threshold. We consider sea glacier flow and oasis solutions in section 4. We find that sea glacier flow is efficient enough when forced by output from all GCMs to make sea glacier thickness fairly uniform and that oases become increasingly favorable as the Snowball ages and warms, mostly due to the reduction in sea glacier thickness outside of the oasis region. We summarize our conclusions in section 5.

2. Brief Description of Models and Simulation Specifications

We provide a brief description of the simulations here. Readers interested in more detail should consult the supplementary material of Abbot et al. [2012]. We run General Circulation Models (GCMs) FOAM, CAM, LMDz, ECHAM, and GENESIS (Table 1) for 10 years and average results over the final 5 years. SP-CAM contains a numerically expensive embedded cloud resolving scheme, so we initialize SP-CAM simulations from converged CAM simulations, run them for 2 years, and average variables over the final year. We run each model with CO$_2$ = 10$^{-4}$ and CO$_2$ = 0.1 (10$^6$ ppm), and we have validated the radiation schemes to within 2 W m$^{-2}$ at CO$_2$ = 0.1 [Abbot et al., 2012]. ECHAM only contributes to the CO$_2$ = 10$^{-4}$ simulation because it becomes unstable when CO$_2$ is increased to 0.1.

To avoid ice vertical resolution issues [Abbot et al., 2010], we set the land surface to “glacial ice,” like Greenland and Antarctica in modern simulations, everywhere. This is roughly equivalent to assuming that all continents are covered with low-elevation land ice. We initialize snow cover at zero everywhere. We set the surface albedo to 0.6 everywhere, for both snow and glacial ice, regardless of temperature and age. This allows comparison of atmospheric behavior among models without introducing surface albedo differences due to different simulation of snow cover. We set the solar constant to 1285 W m$^{-2}$ (94% of its present value) and set the radiative effect of all aerosols to zero, thereby neglecting potential dust aerosol radiative forcing [Abbot and Halevy, 2010]. To minimize simulation differences among models, we set ozone and all greenhouse gases other than CO$_2$ and H$_2$O to zero. For the warmer simulations with tropopause heights of $\approx$15 km, setting ozone to zero could increase the tropopause height by a few kilometers [Thuburn and Craig, 1997]. We set the obliquity to 23.5° and the eccentricity to 0.5.

3. Simulation Results

3.1. Review of Temperature

The models show a similar meridional surface temperature distribution (Figure 1). The surface temperatures of all models except FOAM cluster within about 5 K, and FOAM is about 10 K colder than the other models. This is due to extremely low cloud condensate simulation in FOAM, such that clouds have very little radiative effect [Abbot et al., 2012]. Since snow has a higher albedo than 0.6, the temperature of a Snowball Earth would likely be lower than that depicted in Figure 1 in snow-covered regions. Similarly, the temperature would be lower in regions made darker by the mixture of dust with snow and ice [Le Hir et al., 2010; Abbot and Pierrehumbert, 2010]. At both CO$_2$ = 10$^{-4}$ and 0.1, all models produce strong inversions in the winter hemispheres (Figure 2), and in the annual mean, the atmosphere in all models is highly stable poleward of about 30° (not shown). This results from an annual cycle in which a region of convection tracks the insolation maximum and the atmosphere is stable elsewhere. The temperature inversion in the winter extratropical profile is a basic feature of the Snowball climate that was identified in FOAM [Pierrehumbert, 2004, 2005] and is now seen to be robust across other models. This inversion is important because the greenhouse effect is greatly reduced if the atmospheric emission temperature approaches the surface temperature.

3.2. Hadley Cell Strength and Heat Export From Tropics

In this paper we are primarily interested in tropical cloud formation, heat export out of the tropics, and the tropical net ablation zone, because of their effects on deglaciation and sea glacier flow. Because all of these factors are to a large extent governed by the tropical mean meridional circulation, we will focus on the Hadley cell here.
The primary purpose of this section is to compare circulation results from the GCMs we have run and to outline potential explanations. Further research with a hierarchy of models would be required to fully resolve the relative contribution of the effects we discuss here.

The pattern of zonal-mean Snowball atmospheric general circulation (Figures 3 and 4) is similar among the models. In the annual mean, the tropical circulation is thermally indirect (Figure 3), with descent where heating is highest. As described by Abbot and Pierrehumbert [2010], the cause of this thermally indirect net circulation is the low surface heat capacity of ice, which allows the region of maximum upwelling to deviate far from the equator (Figure 4) so that the two solstitial circulation patterns dominate the annual mean. We will therefore focus on understanding the solstitial circulation in what follows. Increasing CO$_2$ from $10^{-4}$ to 0.1 does not substantially change the width of the circulation, but does increase the circulation depth.

The models produce much stronger circulation with Snowball boundary conditions than in the modern climate. For example, the maximum January stream function is 2–4 times the modern value of $160 \times 10^9$ kg s$^{-1}$ [Dima and Wallace, 2003] at CO$_2$ = $10^{-4}$ and 4–6 times stronger at CO$_2$ = 0.1 (Table 2). The main reason for this is the monsoon-like behavior of the Snowball Hadley circulation. Because of the low heat capacity, the maximum surface temperature in January is at $30–35^\circ$S (Figure 1). This means that the effective heating of the atmosphere is delivered far off the equator, which significantly enhances the strength of the Hadley circulation [Lindzen and Hou, 1988;
**Figure 4.** January mean mass Eulerian stream function for the models. Clockwise circulation is depicted by thin solid lines, counterclockwise circulation is depicted by thin dashed lines, and the zero stream function contour is thick and solid. Contour spacing is $50 \times 10^9$ kg s$^{-1}$. Maximum stream function values for each model are given in Table 2. The region where the local Rossby number is greater than 0.5 is shaded red. The local Rossby number is defined as $Ro = \frac{u}{\Omega a \sin \phi \cos \phi}$, where $a$ is the Earth’s radius, $\Omega$ is the Earth’s rotation rate, $\phi$ is the latitude, and $u$ is the zonal mean zonal velocity.

A robust feature of all the GCMs in this study is an increase in circulation strength with increasing CO$_2$ (Table 2), which is consistent with previous work \cite{Pierrehumbert2005, Hu2011}. As a first step to qualitatively understand this, we apply the axisymmetric angular-momentum-conserving model of Held and Hou \cite{Held1980}, which yields qualitatively similar parameter scalings as eddy-dominated theories \cite{Walker2006, Schneider2006}, although with different power laws. Consideration of the local Rossby number (the local ratio of relative to planetary vorticity) suggests that the angular-momentum-conserving limit is a reasonable starting point when considering the Snowball circulation. The angular-momentum-conserving limit corresponds to a local Rossby number in the upper branch of the Hadley cell near one \cite{Walker2006}, and it is greater than 0.5 in our simulations (Figure 4) and does not change much as CO$_2$ is increased.

Following Schneider \cite{Schneider2006} and Held and Hou \cite{Held1980} theory yields a stream function maximum ($\Psi_{\text{max}}$) that scales as

$$\Psi_{\text{max}} \sim \rho_0 \frac{a^2H}{r} \left( \frac{gH}{\Omega^2 a^2 \tau_0} \right)^{\frac{3}{2}} \Delta h \Delta_\theta,$$

where $\rho_0$ is the reference density, $g$ is the gravitational acceleration, $\Omega$ is the planetary rotation rate, $H$ is the height of the tropopause, $r$ is the radiative timescale (relaxation time to radiative-convective equilibrium), $\tau_0$ is the reference surface temperature, $\Delta h$ is the difference between the surface

<table>
<thead>
<tr>
<th>Model</th>
<th>Annual, CO$_2 = 10^{-4}$</th>
<th>Annual, CO$_2 = 0.1$</th>
<th>January, CO$_2 = 10^{-4}$</th>
<th>January, CO$_2 = 0.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOAM</td>
<td>39.3</td>
<td>41.8</td>
<td>307.8</td>
<td>651.7</td>
</tr>
<tr>
<td>CAM</td>
<td>132.1</td>
<td>153.4</td>
<td>548.1</td>
<td>900.2</td>
</tr>
<tr>
<td>SP-CAM</td>
<td>158.2</td>
<td>139.0</td>
<td>611.4</td>
<td>928.7</td>
</tr>
<tr>
<td>LMDz</td>
<td>105.1</td>
<td>129.5</td>
<td>547.0</td>
<td>803.6</td>
</tr>
<tr>
<td>ECHAM</td>
<td>86.6</td>
<td>—</td>
<td>475.5</td>
<td>—</td>
</tr>
<tr>
<td>GENESIS</td>
<td>59.1</td>
<td>65.0</td>
<td>413.0</td>
<td>640.4</td>
</tr>
</tbody>
</table>

\(^a\)Note that the tropical circulation is thermally indirect in the annual mean (Figure 3).
temperature at the equator and the pole of a forcing temperature field, and $\Delta S$ is the difference between the potential temperature at the surface and the upper branch of the Hadley cell (gross stability). Fang and Tung [1996] find a similar scaling as Held and Hou [1980] with maximum heating off the equator rather than symmetric forcing. As $CO_2$ is increased, tropopause height, which increases by 20–100%, is the only variable in equation (1) that changes in the right direction to increase circulation when $CO_2$ is increased from 10^{-4} to 0.1 (Table 3). Based on equation (1), this increase in tropopause height is enough to explain the increase in circulation strength with $CO_2$. Large increases in tropopause height are reasonable given the increase in surface temperature and emission height that the models experience when $CO_2$ is increased [Schneider et al., 2010], even if water vapor changes are minimal [Thuburn and Craig, 1997].

Although Held and Hou [1980] theory provides a reasonable explanation for the increase in circulation strength with $CO_2$ (via tropopause height), it would also predict an increase in circulation extent, which we do not observe (Figure 4). This suggests that other effects may also be important. For example, idealized GCM simulations suggest that for global mean temperatures below freezing, moist dynamical effects lead to evaporation exceeding precipitation and mean ascent driven by atmospheric circulation. Mean descent tends to lead to evaporation exceeding precipitation and mean ascent leads to the opposite. The pattern of precipitation minus evaporation is robust across models, with a broad region of net ablation in the tropics (Figure 5). The amount by which evaporation exceeds precipitation and mean ascent tends to lead to evaporation exceeding precipitation and mean ascent leads to the opposite. The pattern of precipitation minus evaporation is robust across models, with a broad region of net ablation in the tropics (Figure 5). The amount by which evaporation exceeds precipitation in the tropics increases with $CO_2$, due to increased circulation strength (see below) and warmer air, which can hold more moisture. Differences in the magnitude of the precipitation minus evaporation

### Table 3. Held and Hou Model Parameters

<table>
<thead>
<tr>
<th>Model</th>
<th>$H_0$ [km]</th>
<th>$\delta H$ [km]</th>
<th>$\Delta S$ [K]</th>
<th>$\delta \Delta S$ [K]</th>
<th>$\Delta \delta S$ [K]</th>
<th>$\delta \Delta \delta S$ [K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOAM</td>
<td>8</td>
<td>+6.3</td>
<td>19</td>
<td>+24.0</td>
<td>62</td>
<td>+1.5</td>
</tr>
<tr>
<td>CAM</td>
<td>11</td>
<td>+4.7</td>
<td>42</td>
<td>+18.6</td>
<td>66</td>
<td>-8.3</td>
</tr>
<tr>
<td>SP-CAM</td>
<td>12</td>
<td>+3.9</td>
<td>45</td>
<td>+6.6</td>
<td>61</td>
<td>-0.9</td>
</tr>
<tr>
<td>ECHAM</td>
<td>12</td>
<td>—</td>
<td>48</td>
<td>—</td>
<td>62</td>
<td>—</td>
</tr>
<tr>
<td>LMDz</td>
<td>10</td>
<td>+2.1</td>
<td>30</td>
<td>+1.6</td>
<td>57</td>
<td>-2.0</td>
</tr>
<tr>
<td>GENESIS</td>
<td>10</td>
<td>+8.1</td>
<td>38</td>
<td>+53.6</td>
<td>58</td>
<td>-3.7</td>
</tr>
</tbody>
</table>

*The tropopause height at the equator in January defined using a 4 K km^{-1} lapse rate criterion [Voigt et al., 2012] at $CO_2 = 10^{-4}$ ($H_0$), the change in tropopause height when $CO_2$ is increased from 10^{-4} to 0.1 ($\delta H$), the change in stability when $CO_2$ is increased from 10^{-4} to 0.1 ($\Delta S$), the change in surface temperature between a latitude of 20°S and 50°N in January at $CO_2 = 10^{-4}$ ($\Delta \delta S$), and the change in $\Delta S$ when $CO_2$ is increased from 10^{-4} to 0.1 ($\delta \Delta S$). We choose the latitude range for $\Delta S$ to extend roughly from the January surface temperature maximum to where differences in simulation of the winter temperature inversion cause large divergences in surface temperature simulation among the models. Note that we calculate the values of $\Delta S$ and $\Delta \delta S$ after equilibration with atmospheric dynamics, because this is what we have, rather than the values of an imagined forcing temperature field, which is what would be required for strict consistency with Held and Hou [1980]. The tropopause height calculated here corresponds roughly to where the mass stream function approaches zero. The calculated tropopause height is quantitatively similar if a lapse rate criterion between 3–5 K km^{-1} is used.

### Table 4. Tropical (20°S to 20°N) Surface Temperature (TS), Top-of-Atmosphere Cloud Radiative Forcing (CRF), and Heat Export to Higher Latitudes (Absorbed Shortwave Radiation Minus Outgoing Longwave Radiation in the Tropics) for Each Model

<table>
<thead>
<tr>
<th>Model</th>
<th>$CO_2$</th>
<th>TS [K]</th>
<th>CRF [W m^{-2}]</th>
<th>Heat Export [W m^{-2}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOAM</td>
<td>10^{-4}</td>
<td>238.9</td>
<td>1.2</td>
<td>6.7</td>
</tr>
<tr>
<td>CAM</td>
<td>10^{-4}</td>
<td>244.9</td>
<td>14.5</td>
<td>10.8</td>
</tr>
<tr>
<td>SP-CAM</td>
<td>10^{-4}</td>
<td>246.5</td>
<td>18.8</td>
<td>11.3</td>
</tr>
<tr>
<td>LMDz</td>
<td>10^{-4}</td>
<td>244.6</td>
<td>11.3</td>
<td>10.3</td>
</tr>
<tr>
<td>ECHAM</td>
<td>10^{-4}</td>
<td>247.1</td>
<td>20.5</td>
<td>12.1</td>
</tr>
<tr>
<td>GENESIS</td>
<td>10^{-4}</td>
<td>248.1</td>
<td>35.1</td>
<td>23.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model</th>
<th>$CO_2$</th>
<th>TS [K]</th>
<th>CRF [W m^{-2}]</th>
<th>Heat Export [W m^{-2}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOAM</td>
<td>0.1</td>
<td>254.8</td>
<td>1.9</td>
<td>11.9</td>
</tr>
<tr>
<td>CAM</td>
<td>0.1</td>
<td>263.5</td>
<td>11.4</td>
<td>18.0</td>
</tr>
<tr>
<td>SP-CAM</td>
<td>0.1</td>
<td>266.0</td>
<td>17.1</td>
<td>18.4</td>
</tr>
<tr>
<td>LMDz</td>
<td>0.1</td>
<td>261.7</td>
<td>14.6</td>
<td>16.7</td>
</tr>
<tr>
<td>GENESIS</td>
<td>0.1</td>
<td>265.3</td>
<td>39.7</td>
<td>29.4</td>
</tr>
</tbody>
</table>
pattern are largely driven by differences in model temperature. Additionally, all of the models show a small region in the subtropics where evaporation exceeds precipitation, which we will return to when discussing the thin-ice Snowball model (section 4.3). The subtropical net ablation zone is due to moisture flux out of the subtropics by midlatitude eddies and is stronger and larger in GCMs that produce warmer and moister conditions.

[20] Based on simulations using an ice sheet model driven by output from the LMDz GCM, Donnadieu et al. [2003] argued that the hydrological cycle during a global glacia-
tion would be sufficiently strong to produce the observed evidence of flowing continental glaciers. Measured by precipitation minus evaporation extrema, all models except FOAM simulate a hydrological cycle at least as strong as LMDz (Figure 5). Measured by total global evaporation (latent heat), all models simulate a hydrological cycle stronger than LMDz (not shown). This suggests that the conclusions of Donnadieu et al. [2003] should be robust to changing the GCM driving the glacial model.

4. Sea Glacier Flow

4.1. Axisymmetric Flow Makes Sea Glacier Thickness Uniform

[21] In a Snowball Earth, thick “sea glaciers” that could flow viscously would be expected to form on the surface of the ocean, rather than the thin sea ice that forms in polar regions of the modern climate [Goodman and Pierrehumbert, 2003]. The flow of these glaciers would be driven by surface temperature, precipitation, and evaporation patterns. As a preliminary means of investigating the effect of the climate simulated by the different GCMs on sea glacier flow, we run the one-dimensional (latitude) axisymmetric model of Goodman and Pierrehumbert [2003] with a uniform geothermal heat flux of 0.08 W m$^{-2}$. We use the code of Li and Pierrehumbert [2011], which fixes a programming error contained in Goodman and Pierrehumbert [2003]. A zonally symmetric model is appropriate for the zonally symmetric boundary conditions we apply to the GCMs, but will not capture ice thickness variations due to heterogeneity introduced by continents [Campbell et al., 2011; Tziperman et al., 2012].

Additionally, in this section, we neglect solar radiation absorption by transmission through the ice and, therefore, do not consider the tropical thin ice Snowball solutions proposed by McKay [2000] and Pollard and Kasting [2005].

[22] Consistent with the results of Li and Pierrehumbert [2011] and Tziperman et al. [2012], zonally symmetric sea glacier flow is efficient and leads to variation in ice thickness of only $\mathcal{O}(10\%)$ or less when forced by output from all of the GCMs (Figure 6). In steady state, the global mean of the equation for mass conservation of ice [Li and Pierrehumbert, 2011; Tziperman et al., 2012] simplifies to

$$F_g = \frac{k(T_f - T_s(\theta))}{h(\theta)} h(\theta),$$

where $\theta$ is the latitude, $h(\theta)$ is the ice thickness, $k$ is the thermal conductivity of ice, $T_f$ is the freezing point of water, $T_s(\theta)$ is the surface temperature at the top of the ice, and $h(\theta)$...
is the global mean of a variable \( x \). If variations in \( h(\theta) \) are small, then

\[
\langle k(T_f - T_s(\theta)) \rangle \approx k(T_f - \langle T_s(\theta) \rangle) \langle h(\theta) \rangle
\]

which yields

\[
\langle h(\theta) \rangle \approx \frac{k(T_f - \langle T_s(\theta) \rangle)}{F_g}, \tag{2}
\]

i.e., the average ice thickness can be estimated by the thickness that would result from a 0-D ice thickness model with the surface temperature set to the hemispheric mean surface temperature. We find that this is a very good approximation for the mean ice thickness in the simulations (Figure 7).

[25] The picture that emerges from these 1-D sea glacier flow simulations, therefore, is that, for all GCMs and at both CO2 values, ice flow is efficient enough that sea glacier thickness is effectively uniform. This thickness is then simply determined by the global mean surface temperature. Even in the warmest models at CO2 = 0.1, the ice thickness is still 2 orders of magnitude larger than 5–10 m (Figure 7), which is what would be required for photosynthesis underneath ice [McKay, 2000; Pollard and Kasting, 2005].

### 4.2. “Thin Ice” Solution Possible for Certain Optical Properties

[24] A potential explanation for the apparent survival of photosynthetic life through Snowball Earth episodes is regions of ice thin enough to allow sunlight to pass through it [McKay, 2000; Pollard and Kasting, 2005]. This might be possible in snow-free regions (e.g., the tropics) if the ice can transmit and absorb enough energy to warm and thin it sufficiently. There is, however, significant debate about whether ice with properties that would allow it to be sufficiently thin is possible [Warren et al., 2002; Warren and Brandt, 2006; Pollard and Kasting, 2006]. Moreover, meteoric ice formed by compaction of snow at higher latitudes, which is much more reflective and less transmissive of sunlight than marine ice formed by freezing seawater, could be advected by sea glacier flow into net ablation zones [Goodman, 2006], preventing the formation of thin ice there. In this section, we will assume ice properties that favor the thin-ice Snowball and neglect differences between meteoric and marine ice, and repeat the axisymmetric sea glacier calculations of section 4.1. This will allow us to determine whether a thin-ice solution is possible for all the GCMs and at both low and high CO2 using these optimistic assumptions.

[25] To calculate solar absorption in the ice, we assume exponential decay of solar flux with a penetration depth scale, \( h \), in all ice that is not covered with snow [McKay, 2000]. McKay [2000] used \( h = 0.8 \) m, but Warren et al. [2002] argued that doing a proper spectral integral leads to a mean value closer to \( h = 0.05 \) m, which is the value Goodman and Pierrehumbert [2003] used. Pollard and Kasting [2005] used a two-stream model of solar penetration, but based on their Figure A2, the effective penetration depth is \( h \approx 1.5 \) m. We will use \( h = 1 \) m here, but still specify a surface albedo of 0.6 even for this highly transparent ice.

[26] Using these assumptions, we find that all GCMs produce ice thinner than 1 m in the tropics at both CO2 = 10−4 and 0.1 (Figure 8). The ice is thicker in colder GCMs and at lower CO2 for a given GCM, but in all cases, it would be thin enough for photosynthesis to occur beneath the ice [McKay, 2000; Pollard and Kasting, 2005].

![Figure 7](image1.png)

**Figure 7.** Comparison of the approximate mean ice thickness calculated using the mean surface temperature (equation (2)) with the mean ice thickness calculated using the 1-D ice flow model.

![Figure 8](image2.png)

**Figure 8.** (top) Ice thickness and (bottom) ice meridional velocity calculated using the one-dimensional (latitude) sea glacier model of Li and Pierrehumbert [2011] for the GCMs including solar absorption in snow-free regions, assuming an exponential decay of solar flux with a penetration depth of 1 m. All models can produce a thin ice region using these solar absorption assumptions.
4.3. Oases Expand in a Warming Snowball

[27] In addition to the thin ice region in the tropics, all GCMs also produce a thin ice region in the subtropics (Figure 8), corresponding to the net ablation region caused by eddy moisture export to higher latitudes (Figure 5). The magnitude of evaporation minus precipitation is much smaller in this region than in the deep tropics, which would make this region of thin ice particularly susceptible to destruction by advection of meteoric ice. For example, if we assume a subtropical ice velocity scale of 100 m yr\(^{-1}\), an evaporation minus precipitation of 0.01 m yr\(^{-1}\), and a meteoric ice thickness of 100 m, then the meteoric ice can penetrate into the subtropical ablation zone on the order of 1000 km. Since this is on the order of the size of the subtropical net ablation zone, it might not be stable in a model including advection of meteoric ice. A full investigation of this effect would require a model that separates meteoric from marine ice, such as that used by Goodman [2006].

### Table 5. Calculations Relevant for Potential Oases for Life During a Snowball Based on Evaporation Minus Precipitation and Surface Temperature in the GCMs

<table>
<thead>
<tr>
<th>Model</th>
<th>CO(_2) = 10(^{-4}), (H_0 = 1200) m</th>
<th>(H_0 = 600) m</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOAM</td>
<td>(\frac{L}{W}) = 16.5  (\Delta H) = 700</td>
<td>(\Delta H) = 9.8</td>
</tr>
<tr>
<td>CAM</td>
<td>(\frac{L}{W}) = 10.6  (\Delta H) = 1200</td>
<td>(\Delta H) = 4.6</td>
</tr>
<tr>
<td>SPCAM</td>
<td>(\frac{L}{W}) = 10.1  (\Delta H) = 1300</td>
<td>(\Delta H) = 4.7</td>
</tr>
<tr>
<td>ECHAM</td>
<td>(\frac{L}{W}) = 12.0  (\Delta H) = 1000</td>
<td>(\Delta H) = —</td>
</tr>
<tr>
<td>LMDz</td>
<td>(\frac{L}{W}) = 12.7  (\Delta H) = 900</td>
<td>(\Delta H) = 4.9</td>
</tr>
<tr>
<td>GENESIS</td>
<td>(\frac{L}{W}) = 11.8  (\Delta H) = 1000</td>
<td>(\Delta H) = 5.3</td>
</tr>
</tbody>
</table>

\(^{a}\)GCM output at the equator is used for the calculations shown here, as this is the most favorable location for oases. \(H_0\) is the sea glacier thickness outside of the oasis, and is set to a constant, which depends on \(CO_2\) for all models. \(\frac{L}{W}\) is the ratio of the penetration length to the channel width (for the channel oasis described by Campbell et al. [2011], and is rounded to the nearest tenth. For reference, \(\frac{L}{W}\) ≈ 6.5 for the Red Sea. \(\Delta H\) is the difference between the sea glacier thickness outside and inside the constricted seaway described by Tziperman et al. [2012], and is rounded to the nearest hundred. The scaling constant for the calculation of \(\Delta H\) is chosen so that the sea glacier thickness is near zero at \(CO_2 = 10^{-4}\). Both types of oases become more favorable at high \(CO_2\), mostly because the ice thickness outside the oasis is reduced.

\[\Sigma \propto \tau^n,\] where \(\Sigma\) is the strain and \(\tau\) is the stress. For a Newtonian fluid, \(n = 1\). Ice is shear thinning, with \(n \approx 3\), which is the value we will adopt. \(\Gamma\) is defined by

\[\Gamma = \rho_i g \left(1 - \frac{\rho_i}{\rho_w}\right),\]

where \(\rho_i\) is the density of ice, \(\rho_w\) is the density of water, and \(g\) is gravitational acceleration. The slight difference between equations (4) and (14) in Campbell et al. [2011] is due to an error in that manuscript (A.J. Campbell, personal communication, 2012).

[30] We calculate \(\frac{L}{W}\) using evaporation minus precipitation and surface temperature for all models at both \(CO_2\) values. We find that conditions for oases are most favorable at the equator, which is a nontrivial result since E-P and surface temperature are in competition, and therefore only display \(\frac{L}{W}\) values calculated there. For easier comparison among models, we set \(H_0 = 1200\) m at \(CO_2 = 10^{-4}\) and \(H_0 = 600\) m at \(CO_2 = 0.1\), which is roughly consistent with the equatorial sea glacier thicknesses in most models (Figure 6). Campbell et al. [2011] use the Red Sea, for which \(\frac{L}{W}\) ≈ 6.5, as a reference for a potential oasis. They argue that if the penetration depth is less than about 6–7 times the channel width, there is a reasonable chance that such an oasis might have existed during a Snowball because we have a large example of such a channel on modern Earth. We find that for most of the models \(\frac{L}{W}\) = 10 – 12 at \(CO_2 = 10^{-4}\) (Table 5), so that a channel much more elongated than the Red Sea would be required for this type of oasis. The situation is significantly worse for a channel oasis when forced with FOAM output, as a result of a much lower evaporation minus precipitation at the equator (Figure 5), which overwhelms the ice stiffening due to colder temperatures. When \(CO_2\) is increased to 0.1, we find that conditions become much more favorable for a channel oasis (\(\frac{L}{W}\) is reduced, Table 5). This is mostly due to the reduction in \(H_0\) for warmer temperatures and means that the most dangerous time for a channel oasis is early
in the Snowball life cycle. The length scale $D$ (equation 4) stays roughly constant as the climate warms, which can be explained by the similar value of the Clausius-Clapeyron exponent and the activation energy for ice creep [Campbell et al., 2011].

[31] Tziperman et al. [2012] find that the thickness difference between the inside and outside of a constricted seaway ($\Delta H$) should scale like

$$\Delta H \sim (E - P)^{\frac{1}{2}} (A(T)^{-\frac{1}{2}}).$$  \hspace{1cm} (5)

Following Tziperman et al. [2012], we use $A(T)$ as given in Goodman and Pierrehumbert [2003], but find similar results when we use the formulae in Campbell et al. [2011], Pollard and Kasting [2005], and Barnes et al. [1971]. The proportionality constant inequation (5) is affected by factors such as the geometry of the seaway. We choose this constant so that $\Delta H \approx H_0$ at $CO_2 = 10^{-4}$ for most models, which is equivalent to assuming that a constricted seaway oasis is possible. We then find that $\Delta H$ stays roughly constant when we repeat the calculation using GCM output at $CO_2 = 0.1$, which is similar to the behavior of $D$ in the channel oasis case. Since $H_0$ is significantly reduced as $CO_2$ is increased, a constricted seaway oasis therefore becomes much more favorable as the Snowball ages. As with the channel oasis, we find that the constricted seaway oasis is significantly less likely in FOAM output than the other models. This implies that considerations of oasis solutions in the past using FOAM output are likely to be conservative.

5. Conclusions

[32] The main conclusions of this work are as follows:

[33] 1. All tested GCMs produce a similar pattern of atmospheric circulation. This causes a precipitation minus evaporation pattern that is consistent across models, with net evaporation in the tropics, which confirms that sea glaciers should flow equatorward in a Snowball Earth.

[34] 2. The Hadley cell is much stronger in all GCMs than in the modern and increases with increasing $CO_2$ in all GCMs. This is important because of the thick clouds that significantly warm the Snowball form in the Hadley ascent region. The increase in Hadley circulation strength with increasing $CO_2$ is consistent with large increases in tropopause height.

[35] 3. Warmer GCMs export more heat from the tropics to higher latitudes, as do all GCMs individually as the $CO_2$ is increased. Heat export from the tropics is therefore a negative feedback on Snowball deglaciation.

[36] 4. We find that in the axisymmetric approximation, sea glacier flow is efficient enough to cause roughly uniform sea glacier thickness regardless of the temperature and strength of precipitation minus evaporation that a GCM produces. This thickness can be predicted from the GCM’s mean surface temperature.

[37] 5. The axisymmetric sea glacier model produces a thin-ice Snowball solution when forced by output from all GCMs if the penetration depth of solar radiation into ice is large enough (and an albedo of 0.6 is used). Future research into whether a thin-ice Snowball solution is possible should therefore focus on ice properties, rather than meteorological forcing.

[38] 6. We find that both the channel and constricted seaway oases become more favorable as the $CO_2$ and temperature increase, implying that the time right after a Snowball Earth is entered is the most dangerous for the survival of photosynthetic marine life. This is due to the decrease in sea glacier thickness outside the oasis as the Snowball warms.

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