Problem 0.1 This is a computer problem involving temperature-dependent specific heats. To do the problem, you will need to know the following programming techniques: (1) Defining and evaluating a function; (2) Tabulating a set of values and either plotting them or writing them out for plotting with a separate program; (3) Summing up a set of values in a loop.

The Shomate equation is an empirical formula for the dependence of specific heat on temperature. It works well for a broad range of gases. The formula reads:

\[ c_p = A + B \times (T/1000.) + C \times (T/1000.)^2 + D \times (T/1000.)^3 + E \times (T/1000.)^{-2} \]  

where \( T \) is the temperature in Kelvins and \( A, \ldots E \) are gas-dependent constants. Some coefficient sets are given in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_2 )</td>
<td>1863.714</td>
<td>587.0</td>
<td>-141.153</td>
<td>11.377</td>
<td>3.167</td>
</tr>
<tr>
<td>( CO_2 )</td>
<td>568.122</td>
<td>1254.249</td>
<td>-765.713</td>
<td>180.645</td>
<td>-3.105</td>
</tr>
<tr>
<td>( NH_3 )</td>
<td>1176.213</td>
<td>2927.717</td>
<td>-904.470</td>
<td>113.009</td>
<td>11.128</td>
</tr>
</tbody>
</table>

Table 1: Shomate coefficients for selected gases, valid from 300K to 1300K

Write a function to compute \( c_p(T) \). Explore the behavior for the three gases, taking note of the degree to which the specific heat varies. Make a plot of the temperature dependence. From a physical standpoint, why do the specific heats increase with temperature?

Compute the energy it would take to raise the temperature of 1kg of \( N_2 \) from 300K to 700K, in steps of .1K (assuming \( c_p \) to be constant within each
step). Compare this to the value you would have gotten if $c_p$ were assumed constant at its mean value over the temperature range under consideration. 

*Hint:* Write a loop to sum the energies in each step. You can use a similar loop to compute the mean specific heat.

*Python Tips:* It is convenient to store the Shomate coefficients for each gas in a list, e.g.

```plaintext
N2_coeffs = [1863.7142857142858, 587.0, -141.15292857142856, 11.376714285714286, 3.1671428571428568]
```

If you do this, you can set all the coefficients at once with a statement like `A,B,C,D,E = N2_coeffs`. A good trick is to define the lists as globals outside the function definition for $c_p(T)$, and to have the function refer to the list of coefficients as, e.g. `ShomateCoeffs`. Then, to switch from one gas to another, all you have to do is set `ShomateCoeffs` to the list you want before you call $c_p(T)$, as in `ShomateCoeffs = N2_coeffs`.

**Problem 0.2** In this computer problem, you will compute the dry adiabat $T(p)$ for an ideal gas whose specific heat depends on temperature, in accord with the Shomate equation (See Problem 0.1). In addition to basic skills such as defining functions and loops, you will need to know how to write programs that find approximate solutions to an ordinary differential equation of the form $dY/dx = f(x, Y)$.

First, use the First Law of Thermodynamics to derive a differential equation for $d\ln T/d\ln p$ assuming $\delta Q = 0$. This defines the dry adiabat. Note that since $c_p$ is a function of $T$, you can no longer treat it as a constant in doing the integral.

Write a program that tabulates approximate solutions to the differential equation. Note that your dependent variable is $Y \equiv \ln T$ whereas the right hand side of the differential equation involves $T$. This is not a problem, since you can write $T = \exp(\ln T)$. In writing your program, assume that $c_p(T)$ is defined by the Shomate equation.

Apply your program to obtain an improved approximation to the dry adiabat in a pure CO$_2$ Venusian atmosphere, which you originally computed using constant $c_p$ in an earlier problem. Start your computation at the ground ($p_s = 92$ bars) with the observed mean surface temperature of Venus (737K). Integrate up to the 100mb level, and compare the temperatures you get with those in the Magellan observations shown in Figure 2.2 of the text. Make a
plot comparing your calculations with the dry adiabat obtained by keeping $c_p$ constant at 820 J/kg.

**Problem 0.3** Use the script `MoistAdiabat.py` to compute and plot the water-air moist adiabat for Earth, corresponding to a surface temperature of 310 K. Compare your results to the dry adiabat and the single-component condensible adiabat for pure saturated water vapor. Also plot and discuss the profile of the molar concentration of water vapor.

**Problem 0.4** The atmosphere of Titan consists of a mixture of $N_2$ and $CH_4$ (Methane). The methane is condensible at Titan temperatures but for the purposes of this problem the nitrogen can be regarded as non-condensible. (In reality, it does condense a little bit in Titan’s upper troposphere). Assume that there is sufficient methane ice or liquid at the surface to maintain methane saturation.

Compute and plot the moist adiabatic $T(p)$ for Titan assuming 1.5 bar of $N_2$ at the ground. You may also assume that methane is saturated throughout the atmosphere (requiring a reservoir of methane at the ground). Plot and discuss results for the mixing ratio as a function of $p$ as well. Discuss results for three different cases: (a) $T = 95 K$ at the surface, as for Titan at present, (b) a hot Titan, with $T = 120 K$, and (c) a cold titan, with $T = 80 K$

*Python Tips:* You can modify the script `MoistAdiabat.py` for use with a mixture as defined above. Gas data for for $N_2$ and $CH_4$ are both in the `phys` module, under the names `N2` and `CH4`. 