6 Problem Set: Radiation balance, Greenhouse Effect and Ice-Albedo feedback

Problem 6.1 Compute the equilibrium temperature at the subsolar point of Europa, which is in orbit around Jupiter and therefore has the same solar constant as that planet. The greenhouse effect of Europa’s tenuous atmosphere can be neglected. For the purposes of this problem, you may assume that the albedo of Europa is .67. Assuming Europa to have a water-ice surface, what would be the saturation vapor pressure of the water vapor atmosphere immediately above the subsolar point? Suppose that there is some methane and carbon dioxide mixed in with the ice. What would be the partial pressures of these gases?

Problem 6.2 Consider a planet covered in water ice with a uniform albedo of .7. The planet is tide-locked, so that the same face always points to the Sun; the other side is in perpetual night. The atmosphere has negligible greenhouse effect. Compute the solar constant needed to begin melting the ice under the following three alternate scenarios: (a) The atmosphere is so efficient at transporting heat that the entire surface of the planet (dayside and nightside) has the same temperature, (b) The atmosphere is only moderately efficient, so that the dayside temperature is uniform but essentially no energy is carried away to the nightside, (c) There is no atmosphere, so that each bit of the planet’s surface is in equilibrium with the solar radiation it absorbs.

Problem 6.3 Early Mars Climate

The surface of Mars has river-like channel networks that suggest that at sometime in the past Mars was warm and wet. Some of these features date back to a time several billion years ago when the Solar luminosity (and hence the Solar constant at the orbit of Mars) was about 25% less than its current value. One school of thought states that the mean surface temperature of Mars would have to have reached values at the freezing point (273K) or above to account for such features.
Assume that Early Mars had a dense atmosphere that kept its surface temperature approximately uniform, and that the planet had an albedo similar to the present one (about .25). Compute the greenhouse effect – as measured by \( p_{rad}/p_s \) – needed to bring the surface temperature of Early Mars up to the freezing point under the following three alternate assumptions regarding the vertical profile of temperature:

- (a) A pure \( CO_2 \) atmosphere on the dry adiabat.
- (b) A mixture of equal parts (molar), of \( CO_2 \) and \( N_2 \) on the dry adiabat.
- (c) A pure \( CO_2 \) atmosphere with 2 bar surface pressure, which is on the dry adiabat where \( CO_2 \) is subsaturated but on the \( CO_2 \) condensing adiabat at altitudes where the temperature falls sufficiently that \( CO_2 \) becomes saturated (as in Figure 2.6.)

For the purposes of this problem, you may assume that the ground temperature is equal to the low-level air temperature.

**Python Tips:** The Chapter Script OneComponentCond.py in the collection of scripts for Chapter 2 will be useful in doing Part (c).

**Problem 6.4** Consider an atmosphere in which, for some reason, the temperature increases with height, according to the formula \( T(p) = T_s + a \cdot (p_s - p) \) with \( a \) a positive constant. Suppose that the addition of a greenhouse gas elevates the radiating level to \( p_{rad} < p_s \). Find an expression for the dependence of the surface temperature on \( p_s \), assuming that the atmosphere is in equilibrium with an absorbed solar flux \( S \cdot (1 - \alpha) \) per unit surface area of the planet.

**Problem 6.5** *Computing the Ice-Albedo Hysteresis Diagram*

Calculations with a complete real-gas radiation simulation indicate that, for a \( CO_2 \) concentration of 300 ppmv, and with an atmosphere on the moist adiabat, a reasonable fit to the actual \( OLR \) curve in the range of 220K to 310K is the linear fit \( OLR(T) = a + b \cdot (T - 220) \) where \( a = 113 W/m^2 \) and \( b = 2.177 W/m^2K \). Compute the ice-albedo hysteresis diagram giving the set of equilibrium temperatures as a function of the solar constant \( L_{\odot} \). Note that there is a simple trick for getting the bifurcation plot. The equation determining the equilibrium is \( \frac{1}{4} L_{\odot} (1 - \alpha(T)) = OLR(T) \) instead of specifying \( L_{\odot} \) and finding the \( T \) that satisfy the equation, we can re-write the equation as

\[
L_{\odot} = 4 \frac{OLR(T)}{1 - \alpha(T)}
\]

Now, if we call the right hand side \( G(T) \), then \( G(T) \) gives the unique value of \( L_{\odot} \) which supports the temperature \( T \). Hence, to get the bifurcation diagram, you can just plot \( G(T) \) and then turn it sideways.

Use the same albedo-temperature function defined in Chapter 3. Assume that the albedo for an ice-free Earth is .2 and for an ice-covered Earth is .6.
Based on your calculations, if CO$_2$ were held constant how much would L$_\odot$ have to be reduced from its modern value before Earth was forced to fall into an inevitable snowball state? Using the inverse-square law and assuming a circular orbit, compute how far out from the Sun the Earth would have to be displaced (relative to its present orbit) to achieve this solar constant. Conversely, how close to the Sun would you have to place the Earth before a Snowball state became impossible? Note: The assumption of fixed CO$_2$ is very unrealistic, since tectonically active planets with water have a way of adjusting CO$_2$ in response to changes in the solar constant. This will be discussed in Chapter ??.

Python Tips: The script IceAlbedoZeroD.py, found in the Chapter Scripts collection for this chapter, calculates and plots hysteresis diagrams using this method, and also makes various other plots useful in understanding bifurcations due to ice-albedo feedback. You can easily modify this script to solve this and similar problems, by redefining the OLR(T) function, and (in other cases) also the albedo function.