1 Planck function

Convince yourself of certain properties of the Planck function.

**Stefan-Boltzmann law.** Convince yourself that if you integrate the Planck function over the entire spectrum, you get back out the Stefan-Boltzmann law.

(A) Quick and easy demonstration of the $T^4$ dependence. In the last problem set you wrote $B(\nu, T)$ instead as $B(u, T)$ where $u$ is the dimensionless parameter $u = \frac{h\nu}{k_B T}$. Now just write the integral and show that it consists of constants, a $T^4$ term, and a dimensionless integral over $du$. First, don’t solve the integral; just show that

$$\int_{u_1}^{u_2} B(u) du = (\text{constants}) \cdot T^4 \cdot \int_{u_1}^{u_2} f(u) du$$

(B) Do that integral numerically (or use the value that was stated in class) and show that the integral times the constants is the Stefan-Boltzmann constant $\sigma$.

**Equivalence of forms** Convince yourself that integrating either form of the Planck function (in terms of intervals in $\nu$ or $\lambda$) produces the same integrated flux, i.e. that

$$\int_{\nu_1}^{\nu_2} B_\nu d\nu = \int_{\lambda_1}^{\lambda_2} B_\lambda d\lambda$$

(C) Pick a test case and describe it: what temperature blackbody, and what wavelength or frequency interval. Make it something that you find interesting and are curious about. (For example, how many W/m$^2$ of microwave radiation ($1 \text{ mm} < \lambda < 1 \text{ m}$) does a human body emit?)
(D) Integrate \( \int_{\nu_1}^{\nu_2} B_\nu \, d\nu \) over your interval

(E) Integrate \( \int_{\lambda_1}^{\lambda_2} B_\lambda \, d\lambda \) over your interval

Below problems are optional extra credit for 232 students, required for 332 students.

**Choosing the “natural” form of the Planck function** Determine in which form the apparent peak lies closest to the midpoint of the distribution (i.e. 1/2 power on either side of the peak). It’s easiest to do this using \( B_\nu \) and \( B_\lambda \) written in terms of \( u \) (especially as you know the peak locations in terms of \( u \)).

(F) What is the split of power on either side of the peak of \( B_\nu \)? You don’t need to give absolute values for a given blackbody. Just give the fraction of power that lies on either side of the peak. That is, you need to integrate \( \int_0^{u_{\text{peak}}} u^2 / (e^u - 1) \, du \) and compare it to the integral from 0 \( \rightarrow \) \( \infty \)

(G) What is the split for \( B_\lambda \)?

(H) Which form seems more “natural”? 
