1 Radiative transfer in a simple 1-D single-slab model atmosphere

In class you saw that some, but not all, of terrestrial IR radiation from the surface of the Earth reaches space. The atmosphere is partially transparent to the IR, and therefore is not a perfect absorber. By Kirchoff’s law, that absorptivity is also the emissivity, that means that the atmosphere isn’t a perfect emitter either. Its emissivity $e_a$ must be less than 1. In the real world, emissivity is frequency-specific: some frequencies of light are absorbed more strongly than others. But it’s possible to get insight from a simple model that assumes that the atmosphere is “grey”, i.e. that the atmosphere is equally absorptive at all wavelengths, and is characterized by a single value for $e_a$.

Consider the simple model below (Fig. 1.1). The ground acts like a near-perfect blackbody ($e_s = 0.95$), emitting radiation (a flux $F_s$) according to its surface temperature $T_s = 287$ K, the global mean temperature. Some part of that radiation is transmitted through the atmosphere to space and some part is absorbed by the atmosphere. Meanwhile the atmosphere itself (at temperature $T_a$) emits IR radiation (flux $F_a$) both upwards and downwards. Assume that the system is in equilibrium, i.e. the atmosphere is not warming or cooling. In this case the upwelling radiation at the ground must equal the upwelling radiation at the top of the atmosphere. (If it didn’t, heat would be gained in or lost from the atmosphere). Assume that the outgoing longwave radiation is that of the average Earth, i.e. that the total power emitted to space is that of a blackbody with $T_E = 255$ K.
(A) Write down the values for all the energy fluxes in terms of $T_s$, $T_a$, and $e_a$, the atmospheric emissivity.

(B) Use the assumption of energy balance to solve for the atmospheric temperature: write a formula and state a value.

(C) Solve for the atmospheric emissivity $e_a$: again write a formula in terms of other variables and state a value.

(D) Comment on your values for $T_a$ and $e_a$. Are these reasonable? Is $T_a$ larger or smaller than $T_E$, and does that make sense?

(E) What fraction of IR radiation from the ground is emitted to space?

(F) What fraction of net outgoing longwave radiation is derived from the ground, and what fraction from the atmosphere?

2 Radiative transfer in the real atmosphere

Instead of asking you (at this point) to program a more realistic model of radiative transfer in the atmosphere, this problem asks you to just look at output from an existing standard radiative transfer model (MODTRAN), that has been implemented in an online version by David Archer. You can access the model here: [http://climatemodes.uchicago.edu/modtran/modtran.html](http://climatemodes.uchicago.edu/modtran/modtran.html)
The model calculates the net outgoing longwave radiation (OLR) at the top of the atmosphere, for a prescribed surface temperature and atmospheric temperature structure and for whatever combination of atmospheric gases are specified by the user. The surface temperature and temperature profile in each case (tropics, midlatitudes summer and winter, etc.) is chosen such that the atmosphere in the real Earth would be in balance: that is, it’s chosen so that the model-calculated OLR would balance real-world incoming solar radiation.

Gas concentrations can be changed on the right-hand panel. (Just click next to the graphs to recalculate them.) Default specifications for the well-mixed gases (those that exist in constant proportions throughout at least the troposphere, the region from the surface to \(~10-17\text{ km}\) ), are 400 parts per million for CO\(_2\), 1.7 parts per million for methane, and 28 parts per billion for O\(_3\). (That is, out of every million atmospheric molecules, 400 are CO\(_2\) and 1.7 are CH\(_4\); out of every billion molecules 28 are O\(_3\).) The gases that have strongly altitude-dependent concentrations (water vapor and stratospheric ozone) are specified just by adjusting a relative scale: scale “1” means the present-day profile; scale “2” doubles concentration at every altitude, etc. The graph on the far right lets you see the assumed profiles. Unless otherwise specified, use the default case of “no clouds or rain.” The resulting spectrum of outgoing longwave radiation (OLR) is shown in the center graph. The x axis units are essentially frequency: the “wavenumber” of light is \(\nu/c\), with units of 1/cm. Using wavenumber lets you use the more “natural” frequency form of the Planck function while not having to suffer the awfulness of writing frequencies in Terahertz.

**IR absorption by atmospheric gases.**

First, play qualitatively with reducing and increasing the concentration of individual IR-absorbing gases, and observe how the OLR spectrum changes.

Note: the MODTRAN model in this version keeps a fixed surface temperature and temperature profile, and simply recalculates OLR for a different atmospheric composition. If you add certain gases you will produce a decrease in OLR. In a full model, that change means the Earth would gradually warm. If OLR is suddenly reduced, the planet would be emitting less energy to space than it is taking in, and so would warm. That warming would cause OLR to increase, and would progress until the system came back into energy balance.

Write down the bands (regions specified in both wavenumber and wavelength) where individual gases absorb strongly:

(A) CO\(_2\)
(B) CH\(_4\)
(C) H\(_2\)O
(D) O\(_3\)
(E) Based on what you see, rank the gases in terms of their total IR-absorbing ability. (That is, rank the various species according to what role each
actually plays in the Earth’s atmosphere, not by their absorbance per molecule).

(F) Now, clear all absorbers out of the atmosphere except CO\textsubscript{2} and CH\textsubscript{4}, and put in equal amounts of each gas. Compare their absorption per molecule (i.e. the fraction of radiation absorbed from the surface).

For the default atmosphere, the OLR at the frequencies of individual absorption bands looks something like blackbody emission but from a temperature cooler than the Earth’s surface. For each of the gases listed, state the apparent emission temperature within its deepest absorption band. Then look at the figure showing the assumed temperature profile, and translate that emission temperature into an altitude. This is the altitude from which it appears that radiation originates.

(A) CO\textsubscript{2}
(B) CH\textsubscript{4}
(C) H\textsubscript{2}O
(D) O\textsubscript{3}

(E) Try playing with gradually increasing the concentration of a single gas from zero to very large. (CH\textsubscript{4} is a good one to try). Describe what happens to the apparent emission temperature in the absorption band center as you increase the concentration of the IR absorber, and discuss.

**Atmospheric emissivity** For the following cases, calculate the emission temperature \( T_E \) (the temperature of a perfect blackbody that would produce the observed OLR) and estimate the atmospheric emissivity \( e_a \) (using your thinking from Problem 1). Note that for the cases when you remove all IR-absorbing gases, the solid Earth is presumably no longer in energy balance, but the atmosphere itself is assumed to be in balance, so your framework from Problem 1 still holds.

(F) tropics, default gases
(G) midlatitudes summer, default gases
(H) midlatitudes winter, default gases
(I) subarctic winter, default gases
(J) subarctic winter, remove all IR-absorbing gases
(K) tropics, remove all IR-absorbing gases

**Climate sensitivity** You can use the MODTRAN model to get a rough estimate of the “climate sensitivity” \( \Lambda \). \( \Lambda \) measures the amount by which the Earth would warm if you changed “radiative forcing”, i.e. produced some change that altered outgoing longwave radiation. The units of \( \Lambda \) are therefore K/W/m\textsuperscript{2}.
so higher climate sensitivity produces greater warming: a given W/m$^2$ change results in a larger change in temperature. In the popular press (and even in many science papers), climate sensitivity is often given instead as $T_{\text{doubling}}$, the temperature change that would result if you doubled atmospheric CO$_2$. Note that because MODTRAN doesn’t include any feedbacks due to clouds, loss of sea ice, or other factors, it can provide only the direct radiative effects of IR-absorbing gases. The climate sensitivity in a full model with more complexity might be different.

(L) First, progressively alter CO$_2$ – alter it from 100 → 200 → 400 → 800 → 1600 ppm. Write down the change in OLR in W/m$^2$ for each stepwise change in CO$_2$. Use the “midlatitudes summer” setting, just so that the whole class is consistent.

(M) Now, repeat that sequence but with each step, after increasing CO$_2$, manually adjust the surface temperature until you produce the original OLR. That is, adjust temperature until you have warmed to the new equilibrium state. Write down the changes in temperature. At each step, the incremental temperature increase is your “doubling temperature” $T_{\text{doubling}}$. In this proble, you should leave the default the “hold fixed: water vapor pressure” setting. That means that water vapor does not increase as you increase temperature. You are measuring the warming properties of CO$_2$ alone, without any feedback from simultaneous changes in water vapor.

(N) Divide your two results above to get the climate sensitivity in K/W/m$^2$ at each step.

(O) Does that sensitivity appear to be a fundamental climate property, i.e. is it approximately constant at each step?

(P) Repeat the temperature adjustment with the setting “hold fixed: relative humidity”. Now you’re allowing water vapor to change as you increase temperature. But water vapor is also an IR absorber, so you are providing a positive feedback. You should need to adjust temperature more to achieve the same OLR, because the very process of increasing temperature also allows the atmosphere to hold more water vapor, which reduces OLR somewhat. Write down the temperature increment you need for each stage.

(Q) Again, divide your temperature adjustment needed to come back to balance by your OLR change to get climate sensitivity in K/W/m$^2$.

(R) By what factor does changing water vapor amplify the warming effect of CO$_2$ alone?

(S) The 2013 5th Assessment Report of the Intergovernmental Panel on Climate Change (IPCC) reported the results of state-of-the-art numerical
simulations of the Earth system, and stated a range for the doubling temperature of 1.5–4.5 degrees K. In these models, is the Earth system characterized by positive feedbacks (which amplify the warming due to the radiative effects of CO$_2$ and water vapor) or negative feedbacks (which reduce that warming)?

(T) Now look at the total effect of all IR absorbing gases. For the cases of tropics, midlatitudes summer, and subarctic winter, write down the OLR at default gas concentration values, then remove all IR absorbers from the atmosphere, then write down: how low do you have to drop surface temperature to bring OLR back to its original value? If the “greenhouse” effect in the different locations is different, discuss why. Is the magnitude of the surface warming produced by IR absorbers roughly consistent with the offset between the Earth’s $T_{\text{mean}}$ and $T_E$ that we have discussed repeatedly in class?

Graduate students or for extra credit also do:

(U) Repeat some of the above exercise with CH$_4$ (with water vapor pressure held fixed). (For simplicity you might want to start with 1 ppm CH$_4$). Progressively double CH$_4$ and describe the OLR change from altering CH$_4$ concentrations. Is the OLR change the same at each doubling step? If not, describe the pattern it follows.

(V) Now repeat the exercise with CH$_4$ at the concentrations you used for CO$_2$. Describe the response of OLR to doubling CH$_4$ in this regime.

(W) Discuss / explain the above results. Refer to your result from question F, where you looked at the absorption per molecule of CH$_4$ and CO$_2$ when each had only low concentrations.