1. A simplified radiative-convective model

In this problem we consider a simplified atmosphere which nevertheless has a stratosphere/troposphere structure. It is like the standard grey-gas radiative/convective model, but differs in that all of the tropospheric IR absorptivity is concentrated in a cloud layer located at the tropopause. Other than this layer, the troposphere is transparent to IR. The stratosphere, located above the tropopause, is considered to be a grey gas with constant specific absorption coefficient (i.e. well mixed, with no collisional broadening effect). As usual the stratosphere is assumed to be in local radiative equilibrium. For the purposes of this problem, you may assume that neither the cloud nor the atmosphere absorb any solar radiation. You may also assume, albeit unrealistically, that the cloud does not reflect any solar radiation back to space. Thus, it might be better to think of this as a layer of strong gaseous IR absorber (like CFC) rather than a real cloud.

You may also make the following subsidiary assumptions. (1) The cloud is strongly coupled thermally to the troposphere, so that its temperature is equal to the tropopause temperature. (2) The troposphere is dry adiabatic (i.e. has constant dry potential temperature). (3) Turbulent transfers are efficient enough to keep the ground temperature equal to the overlying air temperature, so $T_s = T(0)$. (4) The cloud is a blackbody in the IR, with emissivity $\epsilon < 1$. (5) The cloud is geometrically thin, so that the pressure at the top of the cloud is essentially the same as the pressure at the bottom. (6) The surface of the planet has zero albedo. (6) The stratosphere is optically thin, and may therefore be assumed isothermal.

You may also make all the assumptions customary in radiative/convective models, namely that the troposphere as a whole must be in balance between heating and cooling, and that radiative fluxes are continuous (i.e. have no jumps) between the troposphere and stratosphere.

(a) Find an expression implicitly determining the tropopause pressure in terms of the emissivity of the cloud, the surface pressure, and the optical thickness parameter $\tau_1 = \kappa p_s / g$. Note that $\tau_1$ in this case is not the optical thickness of the atmosphere as a whole, since
the troposperhe is transparent. You won't be able to invert this equation explicitly for 
tropopause pressure, but you can easily find the answers you want by drawing a suitable 
graph.

(b) Discuss how the surface temperature and tropopause height depend on the emissivity of 
the cloud, and on $\tau_1$.

2. **Runaway icehouse on Mars**

Suppose that Mars has a large CO2 ice-cap at its North pole. By “large” I mean that you 
may assume that it will never all sublimate into the atmosphere. Suppose that the 
atmosphere is pure CO2, so that the surface pressure is $p = a \exp(-L/(RT))$, where $T$ is the 
surface temperature of the ice cap. Suppose that the planet has constant mean albedo $\alpha$, 
and is in equilibrium with incident solar radiation $S$. Now assume that the OLR as a 
function of temperature and surface pressure can be represented as

$$\text{OLR}(T,p) = A + B T - C \ln(p)$$

where $A$, $B$ and $C$ are positive constants. For simplicity, you may assume that the 
atmosphere is so strongly mixed that $T$ is constant over the entire surface of the planet.

Find an expression of OLR($T$), by eliminating $p$. Use this expression to discuss how the 
surface temperature depends on $S$.

3. **Exercises using the insolation formula**

(a) For a planet with obliquity of $\pi/2$ radians, compute the annual average solar flux at the 
pole and at the equator. For such a planet, where would ice tend to form first? Hint: For 
this obliquity, the expression for the latitude of the Sun becomes $\cos(\pi/2 - \delta) = \cos(\kappa)$, 
where $\kappa$ is the season angle.

(b) In the Data Lab Seasonal Cycle Lab folder, you will find an hdf file "solar.hdf" which 
contains the daily mean flux factor $f$ for the Earth as a function of day of year and latitude. 
Transpose the data, and copy the column corresponding to the latitude of Chicago (about 
45N), and paste it into Plot. Make a graph of the daily radiative-equilibrium temperature 
vs. time, assuming that $\text{OLR} = 150 + (T - 240)*(250)/(90)$, and an albedo of 20%.

(c) Using the formula for the seasonal and latitudinal distribution of solar radiation given in 
class, *in the limit of small obliquity*, derive an expression for the annual-average amount of 
solar radiation per unit area received at the pole, as a function of the planet's obliquity.