1 Simple models of ice-albedo feedback

In class we considered a model of the ice-albedo feedback in which the albedo decreased linearly with temperature. One could argue that when the earth is nearly ice-free and the ice is largely in the polar regions, the change in albedo with temperature should be weak, because of the small area near the poles. This suggests the following quadratic representation of albedo.

\[ \alpha(T) = \alpha_o + (\alpha_i - \alpha_o) \frac{(T - T_o)^2}{(T_i - T_o)^2} \]  

which has zero slope near \( T = T_o \). As usual we assume that \( \alpha = \alpha_i \) for \( T < T_i \) and \( \alpha = \alpha_o \) for \( T > T_o \).

(a) Discuss the possible equilibrium states of the climate in this system, and how the equilibria change as the incident solar flux \( S_o \) is varied. Keep \( \alpha_o \) fixed at .2 and discuss how the picture changes as \( \alpha_i \) is varied between .2 (very dirty ice) and 1. (very clean snow).

(b) The above analysis identifies which states are equilibria of the system but says nothing about their stability. If the system is started exactly at one of the equilibria, it will stay there forever, as long as we don’t change any of the parameters (e.g \( S_o \)). However, if the system is displaced a little bit from an equilibrium, in this case by slightly increasing or decreasing the temperature, the system may either return to the equilibrium (the stable case) or run off to another state (the unstable case).

To determine stability, you have to look at the time dependant problem. Suppose \( c_p M \) is the effective thermal inertia of the system, where \( M \) is the mass per square meter of the relevant layer of the atmosphere/ocean system. Then, the equation for the time evolution of temperature is:
\[ c_p M \frac{dT}{dt} = (1 - \alpha(T)) S_o - \sigma T^4 \]  

(2)

Take a case from Part (a) in which two of the equilibria have partial ice cover, and integrate this equation numerically to explore the stability. Assume a value of thermal inertia equivalent to a 50m mixed layer water ocean. How does the thermal inertia affect your results? Does it alter the stability properties or just the time scale? Can you understand the stability in terms of the slope of the right hand side of the above equation, expressed as a function of \( T \)?

2 Effect of temperature profile on the greenhouse effect

Consider an atmosphere with no condensible component, which is completely transparent to infrared, and which is stirred strongly enough that the temperature structure is on the dry adiabat for some specified value of \( R/c_p \).

Suppose that the surface pressure is \( p_s \) and the surface temperature is \( T_s \). Suppose \( p_s \) is 1000mb.

Now suppose a layer of a very effective greenhouse gas is injected at a pressure \( p_1 \). The greenhouse gas makes the atmosphere act like a perfect blackbody with temperature equal to the local temperature; it is also perfectly transparent to solar radiation. Plot the curve of OLR vs surface temperature for \( p_1 \) equal to 1000mb, 500mb and 100mb and \( R/c_p = 2/7 \). Do the same for \( R/c_p = 4/7 \). Discuss your results.

Suppose that in all cases the absorbed solar radiation heating the planet is 300 W/m\(^2\). Find the surface temperature of the planet in each of the six cases described above.

3 Calculation of optical depth

The optical depth \( \tau \) is determined by the number of infrared absorbers per square meter encountered by the beam of radiation, and by the efficiency of the infrared absorbers (water vapor and carbon dioxide are very efficient, whereas oxygen and nitrogen are very inefficient). Suppose the gas consists of a mixture of transparent gases with a single greenhouse gas whose mass
mixing ratio (i.e. kg $CO_2$ per kg air) given by a constant $q$. Suppose that the parameter $\kappa$ describes the absorptivity per unit mass of the greenhouse gas. Then the optical thickness is given by the equation

$$\frac{d\tau}{dp} = -\frac{q}{g} \kappa$$

(3)

because $-dp/g$ is the mass per unit area of the atmosphere contained in a thickness $dp$.

Suppose that $q = 300 \times 10^{-6}$ and $\kappa = .5$. Compute the optical depth vs pressure from 1000 mb to 10 mb.

In reality $\kappa$ isn’t a constant, because molecules are more excited at high pressure and absorb better. This is called “collisional broadening.” Compute the optical depth vs. pressure if $\kappa$ is pressure dependant and given by

$$\kappa(p) = \kappa_0 \frac{p}{p_s}$$

(4)

with $p_s = 1000mb$.

Note that all pressures need to be converted to Pascals before you do the calculation.