GEOS 24705 / ENST 24705
Problem set #10
Due: Tuesday May 5th

Background
In class we said that every motor could be a generator, and every generator could be a motor. I gave definitions of a motor and a generator:

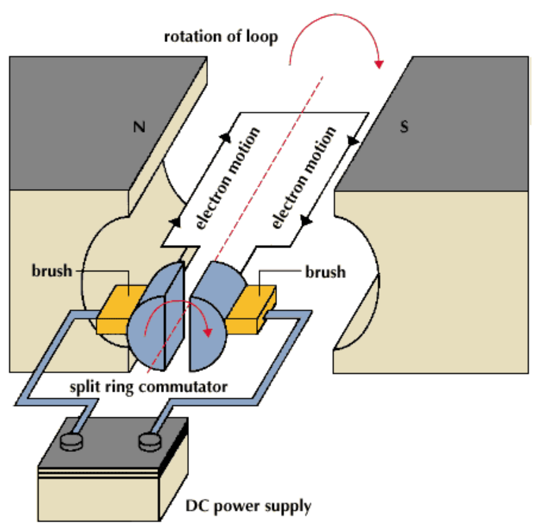
- An electrical motor is a device that, if you flow current through it, will rotate and do work.
- An electrical generator is a device that, if you do work to rotate it, will cause a current to flow.

But these definitions are a little incomplete. First, in both cases, you need interaction with a magnetic field. Second, talking about the *current* as the driver of a motor, or the product of a generator, is not quite right. We should write the definition in terms of *voltage*.

Motors: You should ask yourself, well, ***how*** could you make current flow through a motor? Just as water flows from high to low, and heat flows from hot to cold, current only flows from high to low voltage. To get current to flow through the motor, you have to impose a voltage difference across its wires. The definition of a motor would then be:

- An electrical motor is a device that, if you impose a voltage on it (and allow current to flow), will (in the presence of a magnetic field) rotate and do work.

In the picture to the right, of a simple DC motor like you built in class, the battery imposes a constant voltage difference across the two ends of the loop wire. Current then flows from one battery terminal to the other, through the motor. The voltage is the means by which you drive the motor, and the current that flows just follows from that voltage.



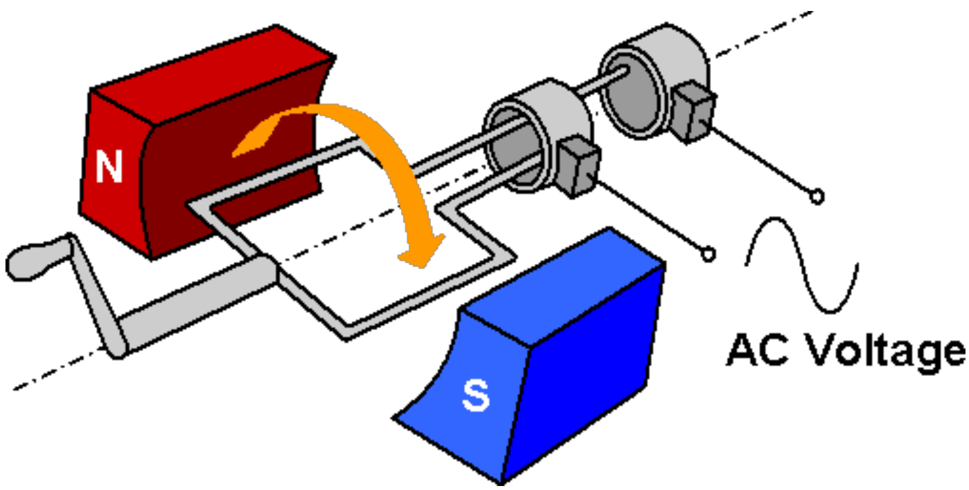
The next reasonable question you should ask is, how *much* current would flow? I'll answer that after talking about generators.

Generators

Just as you decided above that the way you drive a motor is by applying a voltage, you should think of what a generator does as generating a voltage rather than a current. Think: what happens to a generator if you cut its wires so that no current could flow out of it? The generator would still make a voltage – a difference in electrical potential. It's just that no current would flow til you connected the wires. So a better definition might be

- An electrical generator is a device that, if you do work to rotate it (in the presence of a magnetic field), will produce a voltage difference (that can cause a current to flow).

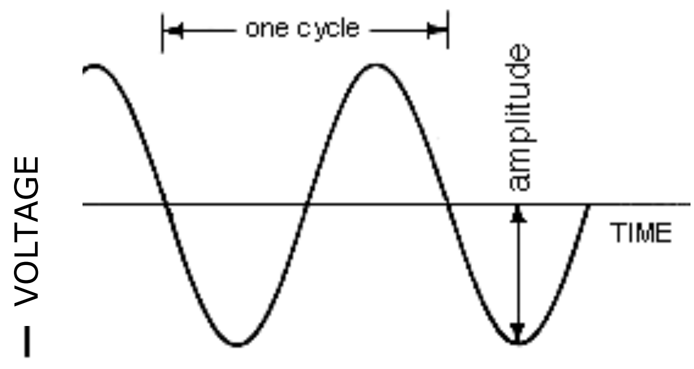
It is easy to make an "alternating current" (AC) generator with a setup similar to the loop motor above. The simplest possible generator is a single loop of wire rotating in the field of a permanent magnet:



The generator produces a voltage because the magnetic field "captured" by the wire loops varies over time. That voltage is given by Faraday's law which is, in our notation:

$$V = N \cdot d\Phi/dt = N \cdot d(A_\perp \cdot B)/dt$$

where V is the voltage produced in the current loop, N is the number of loops, B is the amplitude of the magnetic field, $A_\perp$ is the area of the loop that is perpendicular to the magnetic field, and d( )/dt means a rate of change. Φ is the "magnetic flux" captured by the loop, or $A_\perp \cdot B$. AC generating systems produce voltage that rises and falls in a sine wave, as in the picture below.



If you wanted to make a constant instead of alternating voltage, you'd have to play the same tricks that we discussed for DC motors: switching the wires' connections every half-cycle to prevent the voltage from going negative, and adding more loops to smooth it out.

You can see both AC and DC generation at this nice animation:
http://www.walter-fendt.de/ph14e/generator_e.htm
(Note: to get the animation to play on a Mac, you may have to go into System Preferences, click on Java and then on "Security" and add the website to the "Exception Site list" and then restart your browser).
Set the "without commutator" option to make a simple AC generator. Click "with commutator" to see a kind of bumpy DC generation.

AC generation may seem strange, but it can power devices just fine (and is how most of our electrical system works). If you connect an AC generator to a wire, the alternating voltage in turn drives an alternating current that moves back and forth at the same frequency. Think of the current as sloshing back and forth through whatever device is attached to the generator. Purely resistive devices – like incandescent lightbulbs, or toasters – don't care whether they are powered with alternating or direct current. An incandescent lightbulb will light no matter what direction the current is flowing through it. It will therefore light even if the current changes direction and sloshes back and forth through it.

For motors, you saw that an alternating current might actually allow a simpler motor, since it could solve the problem of the loop that "flopped" back and forth when connected to a battery. Instead of having to switch the connections, you could just switch the direction of the current – exactly as an AC generator already does. Alternating current can therefore have advantages in making simple electric motors. We won't talk about AC motors til next lecture, but I'll cover DC motors now…

DC motors *(See also posted reading on motors)*

I asked earlier, when you connect a DC motor to a battery, how much current would flow? The answer to that question is a little counter-intuitive, but it's deep and interesting and governs why electric motors are particularly useful for certain applications.

First, think about what might happen if the motor loop wasn't allowed to turn at all – if you put some kind of brake on it. Connecting the loop to a battery would then be just like placing a wire across the battery terminals. Because the wire has low resistance, a huge amount of current would flow (set by Ohm's law, $\Delta V_{battery} = I\ R_{wire}$, so $I = \Delta V_{battery}/R_{wire}$). The battery would lose all its energy very quickly, since the power flowing out of the battery must be $P = I * \Delta V_{battery} = \Delta V_{battery}^2/R_{wire}$. And, since the only place for that power to go is into the wire as heat, you'd probably melt your motor.

But you know that motors don't actually melt if you allow them to turn. If you went to lab, you saw that when you connected a wire to a battery with a magnet nearby, and allowed it to turn, the wire turned but didn't heat up. In some way, allowing the motor to turn must *reduce* the current that flows through it! That's odd, and fascinating.

What the DC motor actually does in the process of turning is to create a kind of "back" voltage (call it V_e) proportional to how fast it is turning, and in the opposite direction. (Why does it do this? Because a motor is also a generator, so turning the loop makes a counter-acting voltage.) This "back voltage" means that in a motor that is turning, the total voltage drop that pushes current around the wire is no longer $V_{battery}$ but a smaller number, $V_{battery} - V_e$. (I'm dropping the Δ notation here just for clarity.) The resulting current that flows is then only:

$$I = (V_{battery} - V_e)/R_{wire}$$

And the power lost to resistive heating is also correspondingly smaller:

$$P_{heat} = I * (V_{battery} - V_e)$$

But the total power lost from the battery must still be

$$P_{battery} = I * V_{battery}$$

How can those two quantities ($P_{battery}$ and P_{heat}) be different? Because some of the total power goes to work! The work that the motor does is the *difference* between the total power lost by the battery and the part of that power that goes to resistive heating:

$$P_{work} = P_{battery} - P_{heat.}$$

Or $P_{work} = I * V_e$

When does the motor do maximum work? As the motor spins faster, V_e increases, which would tend to increase the work, but I decreases, which would tend to decrease the work. To answer the question, you have plug in the expression for I:

$$P_{work} = V_e*(V_{battery} - V_e)/R_{wire} = [-V_e^2 + (V_{battery} *V_e)] * 1/R_{wire}$$

Check the logic of this quadratic equation. Remember that the back-voltage V_e is proportional to how fast the motor is turning. If the motor stops (V_e =0), it can (obviously!) do no work. But if the motor is going so fast that $V_e = V_{battery}$, it again can do no work. Somewhere between 0 speed and this top speed, the motor must do its maximum work.

Now, how do you set how fast the motor goes, and how much work it does? The answer is, you don't get to tell the motor what to do. You have only two things you can control:

- what voltage you apply to the motor
- what "load" you connect to the motor

Think of the load as a weight you're asking the motor to lift. It's analogous to the problem of the Newcomen engine and water pump. If you attach too heavy a load to the motor, it can't turn and doesn't do work. If you attach no load to the motor, it has no way to do work and spins uselessly at its maximum rate. You get the maximum work from the motor if you match the load and voltage just right (just as the Newcomen engine designers had to match each engine to its pump.)

What happens if you overload the motor? We said in class that if you overloaded the Newcomen engine, it would just stop. The cylinder would heat and cool but the piston would not move. That's wasteful, but not dangerous. Overloading a DC motor is however dangerous. The overloaded motor also stops, but now you have a bigger problem, because as the motor slows, V_e goes down and the current rises. That is, as the motor slows, the battery delivers MORE power, but LESS of that power goes to doing work. Instead, the extra power is dumped as heat into the wires. If you overload a DC motor, it can melt.

Problem 1: Characteristics of motors and generators

A. In the generator animation of the link above, explain what the "slip rings" do in the case where "without commutator" is set. Why are these necessary?

B. In generator animation, explain what the commutator is and what it is doing when "with commutator" is set. Are the connections here the same or different than you would find on a DC motor?

C. The pictures below show the pieces of a disassembled simple DC motor: the rotating part or "rotor" (left) and the stationary part (right) inside which the "rotor" turns. Annotate the images: describe and mark the different parts of the motor.


D. ***(Optional)*** In the discussion of the DC motor above (the "Background" section), prove that the maximum power occurs when $V_e = V_{battery}/2$. Discuss.

E. Make a power-speed curve for a DC motor: draw a graph showing the variation of the power output P_{work} vs. the rotations per time at which the motor turns (call it ω). The "back voltage" V_e is proportional to that speed, so let $V_e = \omega / k$, where k is some constant determined by the motor design. You can use the result above that the maximum power comes at $V_e = V_{battery}/2$.

F. ***(Optional)*** Make a torque-speed curve for a DC motor: draw a graph showing torque vs. ω. We haven't discussed torque yet; we'll talk about it in the next class. Torque (τ) is a kind of "turning force" that is the force exerted times a lever arm distance; it has the units of energy. The power output is then equal to torque x rotation rate: $P_{work} = \tau * \omega$.

Problem 2: The trouble with DC electrical systems

Edison's vision of the electrical system was easy to understand: his generating station would generate a high constant voltage, and electrical current would flow from that station down wires to houses and businesses. The system would be analogous to a hydropower system: think of Edison's generators as pumping electric charge up to some "height" from which it flows down.

Edison established the first commercial electricity company in the U.S. based on his DC technology. Edison's first customers used his electricity for lighting – remember, Edison also invented the first commercial light bulb – but factories were another potential customer base for him, since DC motors had also been invented by this time.

Given all those advantages, why did Edison not succeed in having his technology become the standard? The only drawback to his system was that Edison had no easy way of transforming voltages.

In the modern electrical system, your household electricity is at 110 V but the wires from the power plant are carrying electricity at much, much higher voltage: over 100,000 V. Between the power plant and your house are various stations or devices that can lower the voltage without altering the total power being transmitted. (Those transformations don't throw power away. Since power $P = I * V$, they just trade off current and voltage, reducing voltage V and correspondingly raising current I). Note that V here is the total voltage drop to ground.

Edison couldn't transform voltage. If 110 V is the voltage that was safe to use in households, then Edison had to transmit electricity at 110 V. And that limits how he could operate his business.

A. Write the expression for resistive losses (P_{loss}) in any power cable as a function of current and resistance R_{wire} in the lines.

B. Now write down an expression for the "line loss" ratio P_{loss}/P, where P is the total power transmitted = I*V, where V is the total voltage difference from generating station to ground. For the same wires, how much more significant are line losses for Edison transmitting at ~100 V than for the power company transmitting at ~ 100,000 V?

C. Read the 1882 article on Edison's first generating company in

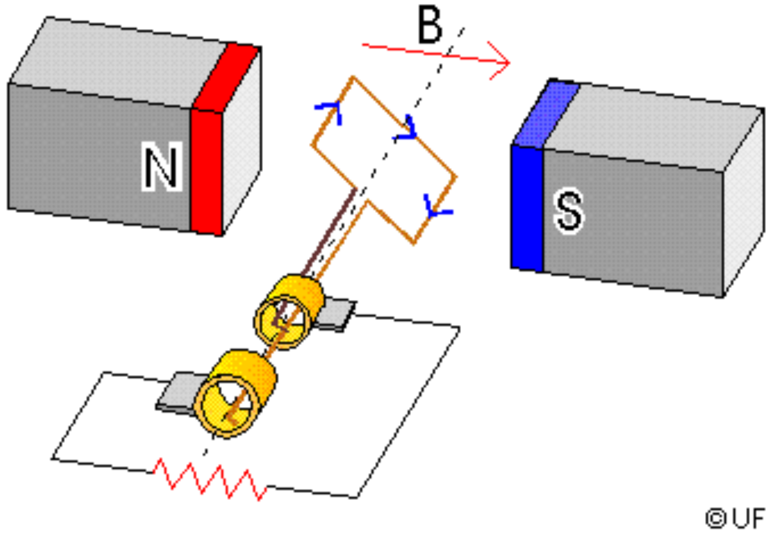
New York that was posted on the website. Print out a map of modern Manhattan (the street plan is the same) and color in Edison's service area. State its dimensions.

D. Now calculate his line losses. You'll need Edison's total power transmitted per line and the resistance of each line. Edison's electrical power output doesn't seem to be recorded, but I've seen statements that his first engines totalled 175 hp, and we can assume that he converted something like 20% of that power into electricity. Then you need to estimate how many lines he was sending power over. The Pearl Street station served 400 lamps when it started up, and from the New York Times article you can see how many of them were in a single building. I'd just assume that each building gets its own transmission line. You can also assume that Edison is using modern metal cables for power transmission with resistance of ~ 0.3 ohm/kilometer.

Then answer: **How far could Edison transmit before he has lost half his power to resistive heating in the wires? How much power would he lose in transmitting to the edge of his service area?**

Problem 3: Generating AC voltages

The modern electricity system follows Tesla's standard of "alternating current" (AC). In this problem you'll plot out the behavior of a simple AC loop generator like the one below. There are two options on this problem, one for those comfortable with calculus/dot products/sines and cosines, and a no-calculus option.



Make sure to read the AC generator description in the “background” section. Then answer one of the following sets of questions.

Option 1 (no calculus):

A. **Draw the position of the loop at various time points over the course of a single oscillation of period T.** (For example, draw it at t=0, t =¼ T, t=1/2 T, t=3/4 T, t=T).

B. **Make a graph of A⊥·B vs time.** That is, identify A⊥·B at each of your time points, plot them on the graph, and link them.

C. **On the same figure, plot the resulting voltage vs. time.**

Then assume that you are now rotating the loop twice as fast, so that you can complete two full revolutions in your original period T. If it helps, draw the system again (and add some more intermediate time points now that you’re going faster).

D. **Add plots of the new $A_\perp \cdot B_{\_faster\ rotation}$ and the resulting voltage to your previous graphs.** Make sure that you draw amplitudes correctly to scale, and that you’re thinking carefully here.

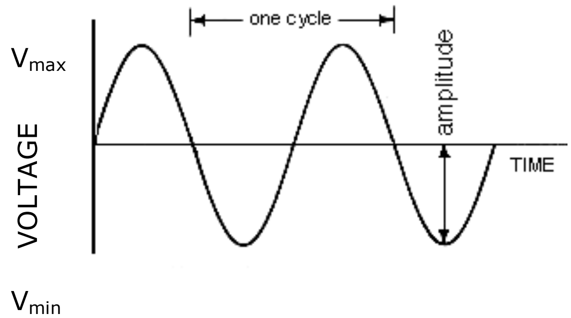
E. **Come up with an expression for the average absolute value of the voltage $|V|_{average}$ in terms of generator characteristics.** (The “absolute value” means ignore the fact that it goes negative.) Write your expression in terms of B, A, and the frequency of rotation ($\omega = 1/T$). It may help to think about only the first ¼ cycle.

Option 2 (with calculus):

A. **Write a formula for A⊥·B as a function of time**
B. **Write the formula for voltage as a function of time**
C. **Write the formula for for A⊥·B at 2ω** (2 x rotation rate).
D. **Write the formula for voltage at 2ω.**
E. **Integrate the absolute value of the voltage to get $|V|_{average}$** (and write with your answer in terms of not the period of the oscillation but the frequency).

Problem 4: Specifying AC voltages

The voltage in the U.S. electrical system is alternating current, switching back and forth 60 times a second, with a voltage specified as about 110 Volts. (It can fall in range of about 110-120 V).



But what does that specified voltage really mean? Voltage in an AC system varies from 0 to some maximum amplitude Vmax back down through 0 and then to a negative –Vmax. The specified voltage is just some single value that represents that whole pattern.

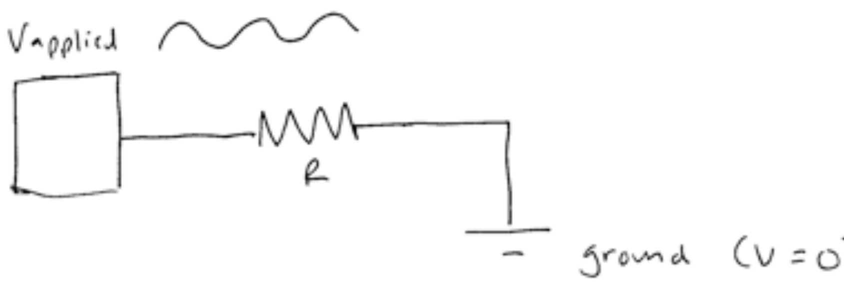
How to pick that representative value? We obviously can't use the average voltage, since that's zero. We could use the maximum voltage V_{max}, or the average absolute voltage $|V|_{average}$ that you just calculated in problem 2. But that's not what is used. Instead, voltage is given as the "root-mean-squared" voltage: the square root of the average of the square of the voltage:

$$V_{rms} = \sqrt{\overline{V^2(t)}}$$

(where the horizontal bar represents a time average). That seems complicated. In this problem you'll understand why that was chosen.

A. **Determine Vmax, the peak voltage you get on your household electrical system, from your known V_{rms}.** *For anyone with calculus, integrate to find Vrms. For others, use the formula $V_{rms} = V_{max} / \sqrt{2}$*

The diagram below describes a simple system, with current flowing from an applied voltage V to ground through a resistive load R. (That resistive load could be a lightbulb, for example).



The power dissipated in the resistive load is the product of current and voltage:

$$P = I \cdot \Delta V$$

where current I has units of charge/time and V has units of energy/charge, so that I·V has units of energy/time, or power. The current that flows in a resistive system is proportional to a voltage drop (Ohm's law: $\Delta V = I \cdot R$, where ΔV is the voltage drop).

B. Write the expression for the power dissipated in the resistive load in terms of both I and R and also in terms of V and R. (You wrote down the expression in terms of I and R already in Problem 2.)

C. Your answer in B should give you insight: **Why do we use V_{rms}? What's so special about squaring the voltage?**

Problem 5: Motor identification.

A. Find something around your house that has an electric motor. If the motor plugs into the wall, it is AC. If it runs off a battery, it is DC. Photograph the motor and describe it. Extra points for doing both AC and DC motors.

B. ***(Optional):*** if the motor is broken (or is used in some broken object you don't care about anymore), bring it in to class and we will dissect it in lab.

Optional thermodynamics problems
For physics students with some calculus and thermo background.
These can be done & turned in anytime, and Andrew can provide hints.

Problem 6 *(Optional)*: Describing the Carnot cycle.

Background:
The gas in an engine cylinder is governed by both its equation of state, the ideal gas law, and by conservation of energy. The ideal gas law is

$$PV = nkT$$

(where n is the number of molecules and k is the Boltzmann constant).

Conservation of energy means that the heat input into (or removed from) the system must be accounted for by changes in the internal energy of the system and/or by work done on or by the system, i.e.

$$dQ = dU + dW$$

(The differential notation, e.g. "dW", is standard in thermodynamics; if it bothers you, you can rewrite this as a differential equation in time). Work dW is given by p*dV, as we've used in class.

A change in internal energy is manifested as a temperature change:

$$dU = cv*dT$$

where cv is the specific heat at constant volume (J/K). (This is in fact the definition of temperature).

The Carnot cycle consists of two isothermal and two adiabatic legs. It is trivial to derive that during an isothermal stage, pressure and volume in the cylinder are related by $P \propto 1/V$. It is less trivial to determine the governing equation for an adiabatic stage.

Questions

A. Derive the fundamental relationship for an adiabatic expansion or compression. This is known as "Poisson's equation":

$$PV^{\gamma} = \text{constant}$$

where the constant $\gamma \equiv cp/cv$, with cv = the specific heat at constant volume and cp the specific heat at constant pressure, in units of J/(kg*K). The two are related via cp = cv + R, where R = k/m and m is the molecular mass.

B. Explain in words why cp must be greater than cv.

Problem 7 *(Optional)*: Integrating around the P-V diagram

Background:

The work done by a single engine stroke is the area of the P-V diagram. The discussion in problem 6 should help you think about how to describe the heat flows into and out of an ideal Carnot engine.

The math for this problem is relatively simple – this is not an exercise in integration - but the problem can still seem confusing. The challenge is in thinking your way through to make it uncomplicated.

Questions:

A. **Draw the Carnot cycle, and mark on which legs heat flows into or out of the system.**

B. **Integrate P*dV on the legs that have net heat flow in or out of the system.**

C. **Either integrate P*dV on the legs that do not have net heat flow in, or construct an argument why you shouldn't need to do this.**

D. **Derive the efficiency of the Carnot cycle** from your answers above.