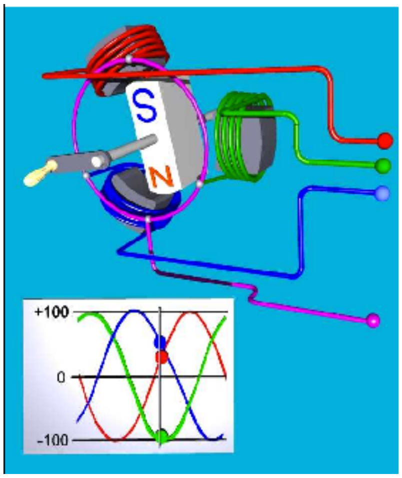
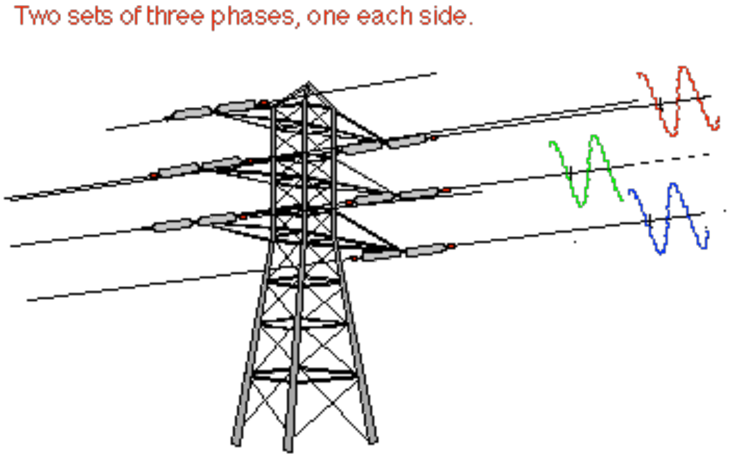
GEOS 24705 / ENST 24705
Problem set #13
Due: Thursday May 14 (but extension is possible)

## Problem 1: Electricity transmission

The standard means by which electricity is generated and transmitted in the U.S. is as 3-phase AC power: three different alternating currents and voltages, each with the same frequency but out of phase with each other. Each generator carries separate windings that are not connected, and the rotating magnetic field in the rotor generates a different voltage in each one as the magnetic field changes in each set of coils successively. See the animation previously shown in class if this is not clear: [http://www.launc.tased.edu.au/online/sciences/physics/3phase/threeph.htm](http://www.launc.tased.edu.au/online/sciences/physics/3phase/threeph.htm)
Each industrial power generator is essentially three generators in one.



3-phase generation.
*Image from koehler.me.uk*



3-phase transmission.
*Image from Launceton College*

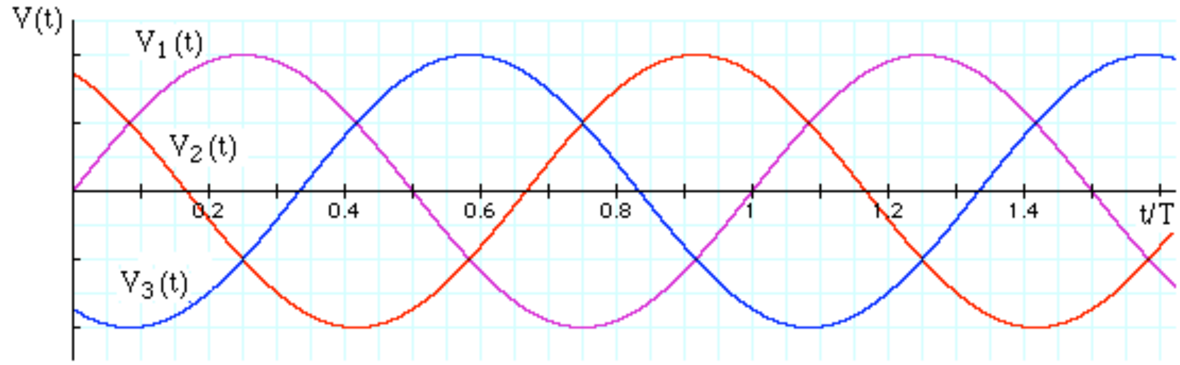
The electrical grid is therefore also essentially three grids in one, with three totally electrically separate sets of wires carrying the three phases. All high-voltage transmission (except the occasional DC line) carry wires in multiples of three. The three-phase transmission is the reason that a bird whose wings span two lines can cause a blackout. If the lines are connected, a current will flow because the voltages are different on the different lines at each moment. The bird will experience an oscillating current flowing back and forth, you will quickly have a dead bird and a blackout. If the three lines were all in phase, their voltages would be identical, and the bird could touch two at once without harm.

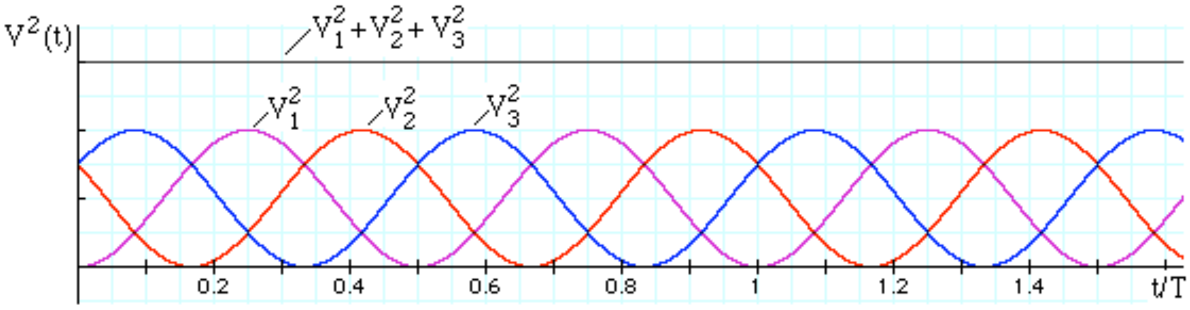
All electrical devices you use in daily life use only a single phase, and each plug a residential wall carries only a single phase. However, some industrial customers receive all three phases and wire them into special outlets, for some industrial equipment that uses all 3 phases at once.

L: Single-phase outlet (a "hot" wire, a "neutral" or "return", and a grounding wire. *(Image source unknown)*.

R: Three-phase outlet (three "hot" wires, one "return", one ground)
*Image from Demand Energy*

The 3-phase AC power system has two tremendous advantages for transmission. 1) First, the voltages always sum to zero, so there is no net movement of charge. In each circuit the AC current is constantly "sloshing" back and forth, but at every moment the sum of all that current is exactly zero. That means that there is no need for return wire in the transmission system – a huge cost savings. 2) Second, the total power (which is proportional to $V^2$) carried by all three phases is constant at all times, so that power transmission is smooth, with no jerkiness – the same power is delivered at all times. The diagram below shows V and $V^2$ for the 3 phases. In each electrical line, current will be proportional to voltage and power is proportional to $V^2$.





A. **Demonstrate that the voltages sum to zero** (and therefore there is no net current in the transmission lines).

   For those who are comfortable with sines & cosines, consider the sum of $V_1=\sin(\omega t)$, $V_2=\sin(\omega t + \phi_2)$, and $V_3=\sin(\omega t + \phi_3)$ and prove that it equals zero. (You need standard trig identities but nothing fancier than that).

   For those who are not comfortable, pick four locations on the top figure above, and at each location measure $V_1$, $V_2$, and $V_3$ by hand and add them to show the sum is constant at all times.

B. **Demonstrate that the sum of the $V^2$s is constant** (and therefore that power transmission is constant). By the same means as you did

   As before, for those comfortable with sine and cosines, show analytically that $V_1^2 + V_2^2 + V_3^2$ = constant. For those not comfortable, pick four locations along the figure and measure and add by hand.

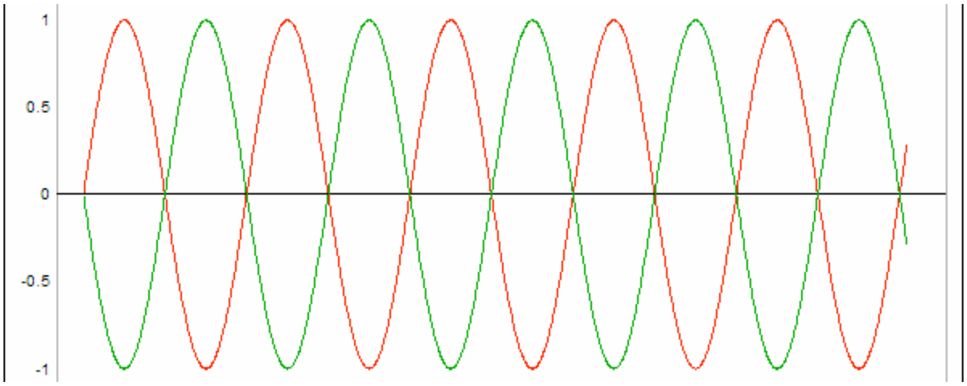
C. **Can you get the same properties if you use only two phases?** Discuss.

**Problem 2: Electricity distribution – a very local field trip.**
In the questions below, answer what you can from you local electrical system. If you can't answer a few, don't worry. These are look-see questions, not time-consuming. It can help to refer to the Chapter 6 from Hayes ("The Grid", posted after Lecture 11 on the class website).

The electrical power in your house is at a relatively low 110-120 V. You know from class discussions that resistive losses would be severe if you tried to transmit power at this low voltage for any significant distance, and that all transmission and distribution systems minimize resistive losses by moving electrical power at much higher voltages (and therefore lower currents). That means there must be a transformer quite close to your house. How close? Edison transmitted DC power at 110 V and couldn't extend his lines longer than 2 km, so you are sure to find a transformer within 2 km. In modern systems it is much closer than that. In the U.S., transformers are usually located on power poles very near each residence.

In the U.S., the transformer not only steps down voltage but it also duplicates and inverts each phase that is fed into it. A single conductor that goes into the transformer is matched with three wires leading out: a "hot" wire that carries the original signal, only now stepped down in voltage to 120V, a "hot" wire that carries that same 120V signal only inverted (180 degree phase shift, for those comfortable with that language), and a return wires, because the system is no longer balanced and certainly there will be net current flow.

Why the inverting? Because 120V was chosen for household wiring to be safe – the lower the voltage, the less dangerous it is if you accidentally span that voltage and shock yourself. But some household appliances actually need higher voltage – things like clothes dryers and electric stoves that pull a lot of power. (Since P = I*V, too low a voltage would require them to draw too high a current). If your building carries two 120V lines whose signals are inverted relative to each other, you can get a normal 120V sine wave by connecting either line to ground (0 volts), or you can get a 240V sine wave by connecting the two hot lines to each other. This strategy allows your house to have the best of both worlds: most of the wiring in your house can be at a safe low 120V, but the occasional high-power device can still be wired to get a higher 240 V.



A. **Go outside and find the transformer that serves your house /dorm / building, photograph the high(er) voltage distribution lines coming in to the transformer, the transformer itself, and the lines coming into your building. Attach the images to your problem set solutions.** (I assume everyone has access to a camera).

B. **Label the picture/s of the distribution lines with all interesting /relevant features.** Refer to the Hayes chapter for more information... Be sure to note whether your local distribution lines have a fourth "neutral" or "return" wire, or whether they carry the three phases without a neutral, as do transmission lines. (Both are possibilities – the high-voltage transmission lines always skip the return, but sometimes it's used for local distribution).

C. **Label the picture/s of wires entering the transformer, and again comment on all interesting features**. How many of the primary phase conductors (the 3 "hot wires" of the distribution system) feed the transformer? If only one conductor feeds it, the transformer can only produce single-phase power.

D. **How many buildings are served by the transformer that serves your house/dorm/apartment?**

E. **Label the picture/s of lines leaving the transformer and entering your building and again comment on features.** From the "low side" of the transformer (the low-voltage power leaving the transformer), how many wires leave the transformer? What are those wires carrying? (See explanation above). Look for where those lines go – how many phases are sent to each building served by the transformer? Remember that since each primary phase conductor would normally become three lines (one phase and its inverse plus a neutral return wire), having 3 wires coming in doesn't mean you're getting 3-phase power. For three-phase you'd need four wires, 3 for the three phases plus a neutral.

F. **Comment on the thickness of the wires leading to your building relative to the distribution lines.** Are they thicker or thinner than the main distribution lines? Explain why. Would they carry more ore less power?

G. **If you can see a max power rating written on the transformer, write it down.** This will be usually written in "kVA", which is essentially kW (as you know if you remember that 1 Volt at 1 Amp = 1 Watt). **Comment on whether that total power is reasonable for the expected number of the people the transformer is serving.**

*Optional problems*

H. ***(Optional)*** **Find and photograph the substation that serves your neighborhood (this is likely a bike ride or even a drive away).** Take pictures of the substation and the high-voltage transmission lines that serve it and discuss the components you see. (You can refer to the Hayes chapter). The substation brings voltage down from the very high levels of long-distance transmission lines (up to > 700 kV) down to ~10s of kV for distribution within a city.

I. ***(Optional)*** **Prove to yourself that if you span one 120V circuit and its inverse, the resultant voltage is a 240V sine wave.** As above, you can do this mathematically or you can do it by hand measurement. For hand measurement: either use the figures from this question or draw out on graph paper two sine waves that are inverted from each other. At some reasonable number of points, find the difference between the two signals and graph it.

J. ***(Optional)*** Within your building, **identify *(if you can)* what appliances are requiring 240V, and *(if you can)* photograph their outlets** to confirm that they are 240V. (It's ideal if you can unplug them and photograph the plugs and outlets separately, but even if you can't unplug them, you should be able to observe that the connections look different from, and bigger than, a standard 120V plug).

## Problem 3: Household electricity

Resistance heating in your household wires in usually not a problem, because the wiring is designed appropriately for typical electricity use. If however you tried to pull a huge current through your household wiring, you could produce enough heat to be a fire danger. For this reason all U.S. houses have built-in protection systems to prevent too much current from melting the wires.

The wiring in your house is arranged in several individual circuits, each of which has thick enough wire to safely carry 15 or sometimes 20 Amps of current, and each circuit is connected to a fuse or circuit breaker that will cut off all current if you try to draw a higher current (if you plug in too many appliances.)

Why? The utilities that provide electricity to your house ensure that the voltage is constant. A space heater or hairdryer typically consumes ~ 1500 W of power when plugged in and draws a corresponding current $I = P/V$. The more appliances you plug in, the more current flows through your wires.

**A. How many space heaters or hairdryers you can plug in to a single household circuit before you blow a breaker?**

**B *(Optional):* Test your answer of A.** (But only if you have found your breaker box and would be comfortable re-setting a circuit breaker.)

**C. If possible: find the breaker box in your apartment, dorm, or house; open it; photograph it; and attach the photo. Identify the main breaker and those for individual circuits.**

If you are in a dorm you can probably not get access to the breaker box unsupervised, but you can possibly talk to the facilities/maintenance personnel and persuade them to let you look at it. People are often very willing to help if it's for a class and if you show interest in their responsibilities and their job

**How many circuits do you have? What is the maximum current draw if you add up the individual circuit breakers? What is the maximum current draw your building can have before blowing the *main* breaker? Is the main breaker equal to the sum of all the individual circuits?** (If the main is less than the sum of individual circuits, you could trip the main even while not overloading any individual circuit.)

**D. *(Optional, continuation of above):* What is the maximum current drawn per inhabitant of your building? Convert that to a maximum power per inhabitant and state whether the value is reasonable, given what you know about U.S. electricity usage.**