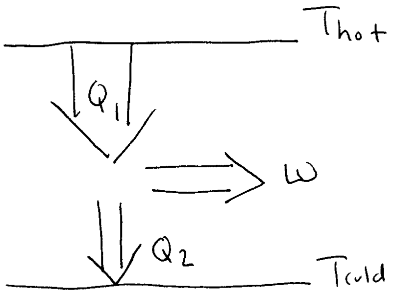
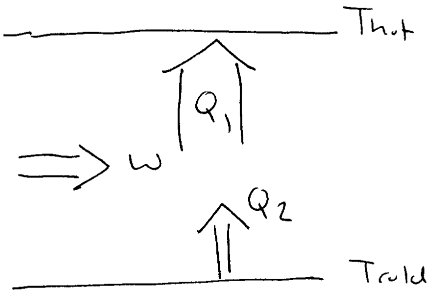
GEOS 24705 / ENST 24705
Problem set #8
Due: Tues. Apr. 28

Problem 1: Heat engines and heat pumps

A heat engine is a device that moves heat down a temperature gradient (from hot to cold) and extracts some of that heat flow as mechanical work. A heat pump is basically a heat engine run in reverse: it takes mechanical work and uses it to push heat up a temperature gradient (from cold to hot). A refrigerator is a heat pump, using mechanical work to move heat from the interior of the refrigerator to the warmer exterior.



heat engine



heat pump

$$\varepsilon = \frac{W}{Q_1} = \frac{(T_{hot} - T_{cold})}{T_{hot}}$$

The best heat engine would be a purely reversible one – that is, it would suffer no frictional or other losses. In this case the values of all the heat flows are identical no matter which direction the engine is run in.

Carnot's thought experiment showed that the Carnot limit was the maximum efficiency possible for ALL reversible heat engines.

A. **What would happen if you could hook up a hypothetical better-than-Carnot-efficiency heat engine with a Carnot engine reversed as a heat pump?** (So that the engine drives the heat pump). Draw the diagram and explain what would happen**. Is this possible?**

B. **What would happen if you hooked up a worse-than-Carnot heat engine with a Carnot heat pump?** Draw the diagram and explain what happens. **Is this possible?**

Problem 2: Evaluating heat pumps

If you want to describe how good a heat pump is, efficiency isn't the right intuitive metric. The efficiency tells you the mechanical work you get out given a certain amount of heat transfer. With a heat pump, you want to do the LEAST mechanical work and transfer the MOST heat.

A **Make up a metric (call it "COP", for "coefficient of performance") that describes how good a heat pump is (i.e. how much heat you get transferred per work done) and write down its definition.**

B Assume that your heat pump is ideal (that is, that it is a Carnot engine run in reverse, the best heat pump you can have). **Now rewrite your definition in terms of T_{hot} and T_{cold} alone.**

C **Does it take more work to pump heat across a large thermal gradient or a small one? Is that consistent with your intuition?**

D **What is the physical meaning if COP > 1? Could you have COP < 1?**

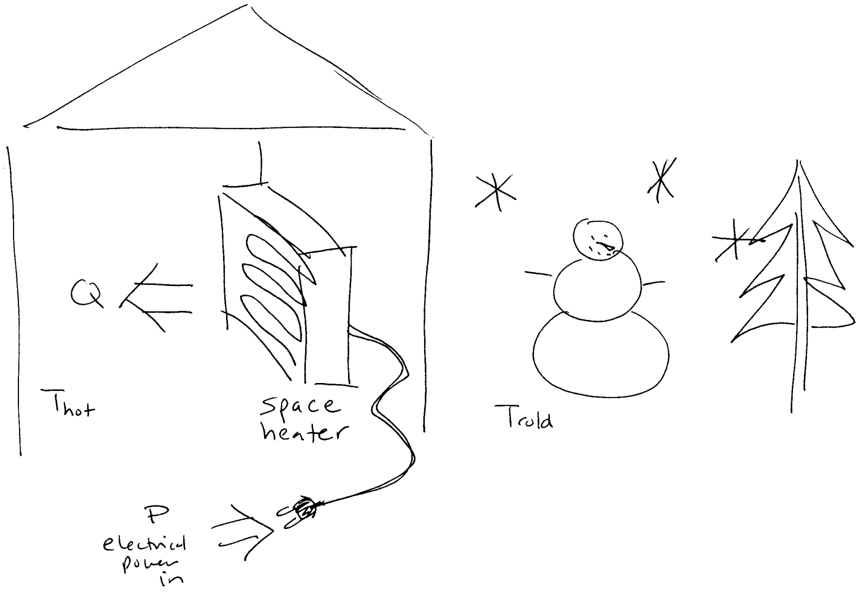
Consider the difference between heating your house with a heat pump vs an electric space heater – see diagram next page. With a space heater, all the chemical or electrical energy is converted to heat. Consider your room in a chilly Chicago winter (make reasonable assumptions). You can assume that in your heat pump system, the input electrical power gets converted to mechanical work (for the heat pump compressor) without any losses.

E **How much more electrical power would you need to heat your room with a space heater than with an ideal heat pump?**

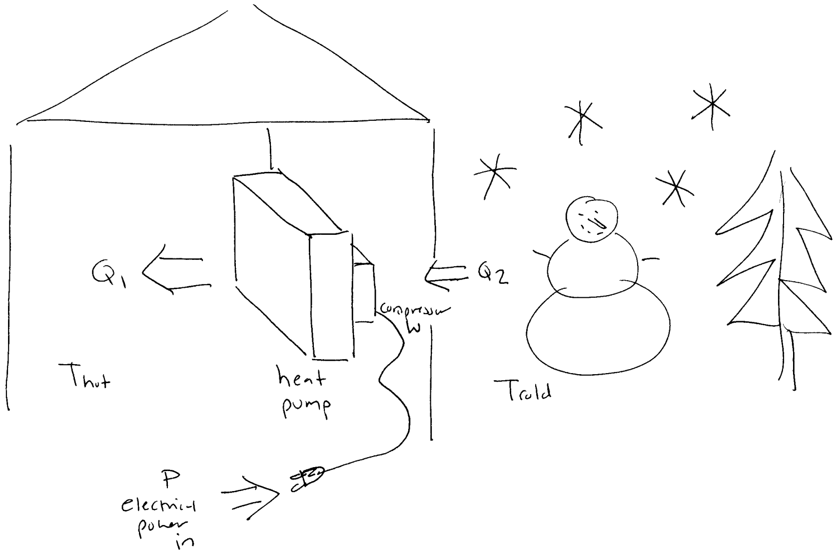
Why don't more people use heat pumps? A real heat pump won't reach ideal efficiency; in practice COP is lower than the Carnot ideal by a factor of 2-4. And people don't usually heat with electric space heaters – they heat with furnaces (big tubs to burn gas in), which are cheap, and burn cheap fuel. The low upfront cost could be luring people to make bad decisions - but it could also actually make the lifetime costs of a furnace lower. Natural gas is cheap right now.

F **Compare the fuel cost of heating with an ideal heat pump to heating with a perfect gas furnace.** First calculate the different price of natural gas and electricity per MJ delivered to your house. Gas retails for about $10 per 1000 cubic feet; 1 ft^3 of natural gas has about a million Joules. Retail electricity rate is ~ 10 cents/kWh. Then apply the energy savings of the heat pump. *(Here you are considering energy delivered at your house, not the primary energy source – as you'll see in Problem 4 and know from class, electricity production involves energy losses to waste heat.)*

Space heater:



And heat pump:



G ***(Optional):*** **At what cost ($/W) for a heat pump system would buying one make financial sense?** Assume a reasonable COP and lifetime for your system, say 25 years. You can ignore the furnace cost, but for more optional credit, also include in your calculation the $/W for a home furnace and the cost of borrowing $ to purchase your heat pump system.

Problem 3: Energy transformation technology

In class we discussed the 17th-18th century energy dilemma, that people could not transform heat to work. In modern life, we now have technologies to transform almost every form of energy to every other form. In this problem you'll try to fill out a grid of energy transformations yourself. In each box, write a technology people have invented (or a natural process) that transforms energy FROM the type on the vertical label and TO the form on the horizontal label. The diagonal boxes represent ways of moving or storing energy. Because everyone could look ahead to the grid you're given in the next problem, we won't grade this problem on how many technologies you could think of, but try for 15-20 minutes, then compare your effort to the grid for problem 4 and write a few sentences on what you missed and why. We'll grade based on that discussion. I filled in two answers to get you started (kinetic → kinetic and heat → kinetic).

To → From ↓	**Electro-magnetic**	**Kinetic**	**Heat**	**Chemical**	**Radiation**	**Nuclear**
Electro-magnetic						
Kinetic		belt drive				
Heat		heat engine				
Chemical						
Radiation						
Nuclear						

Problem 4: Efficiencies of modern energy transformations
Use the grid you're given on the web page to trace out the chain of energy transformations involved in some aspects of your everyday life. For each transformation in the problem below, you'll print out a grid and draw the transformations on it. You'll also book-keep the energy losses that occur during that chain. The table on the following page gives conversion efficiencies for many common transformations. (It gives all the info here you need, except that there are 7% losses in U.S. electricity transmission lines – i.e. on average, electricity transmission and distribution is about 93% efficient). Unless told otherwise, assume for now that all electricity is produced by burning some fossil fuel and spinning a steam turbine. For problems A-G below, you should turn in 1) a printout of the technology grid numbering (or otherwise marking) the progression of transformations, and 2) a calculation of the net efficiency of the entire process. See example on web page.

For transformations that start with some kind of plant matter or fossil fuel, unless told otherwise, ignore the efficiency of converting sunlight to chemical energy in plants – start with the fuel. In one case, I'll tell you to include that ε_photo.

A. Cooking with a gas stove
B. Cooking with an electric stove.
C. Cooking with a microwave (assume efficiency of the microwave itself is ~65%, a typical number. The rest of the power doesn't get into the food but is lost in e.g. turning the fan.)
D. Heating your house with a furnace
E. Heating your house with a heat pump with COP = 3.5. Here you can assume that the rated COP includes the losses of the electric motor that drives the compressor used in the heat pump (i.e. don't include the small-electric- motor transformation; it's already wrapped into the rated COP).
F. Electric lighting, standard incandescent lightbulbs.
G. Electric lighting, the best modern LED bulbs (20% efficient)
H. Electric lighting, best modern LEDs, but now assume you want to power your system on some renewable conversion of solar energy (biofuels, solar photovoltaics, hydro, etc.), and compute your efficiency relative to solar energy. You can pick any renewable that you want, look back through problem sets to find efficiencies. Discuss your choice and the implications of this problem in the context of vertical farms.
I. At what COP would a heat pump use the same *primary* energy as a furnace?
J. ***(Optional):*** The inefficiency of producing electricity is included in the electricity price. Compare the retail price of electricity to the price of the natural gas that (presumably) made that electricity, and discuss.

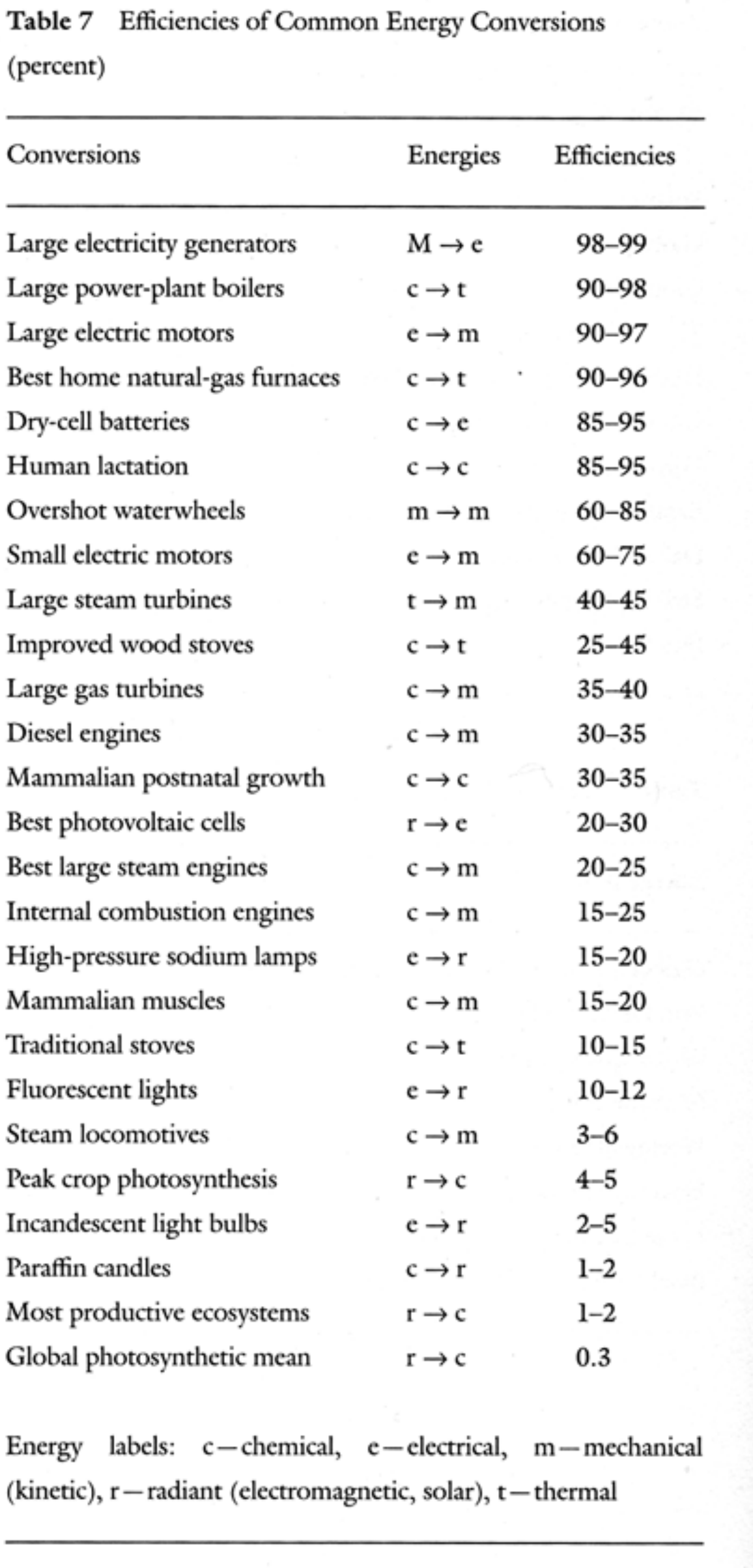


Table 7 Efficiencies of Common Energy Conversions (percent)

Conversions	Energies	Efficiencies
Large electricity generators	M → e	98–99
Large power-plant boilers	c → t	90–98
Large electric motors	e → m	90–97
Best home natural-gas furnaces	c → t	90–96
Dry-cell batteries	c → e	85–95
Human lactation	c → c	85–95
Overshot waterwheels	m → m	60–85
Small electric motors	e → m	60–75
Large steam turbines	t → m	40–45
Improved wood stoves	c → t	25–45
Large gas turbines	c → m	35–40
Diesel engines	c → m	30–35
Mammalian postnatal growth	c → c	30–35
Best photovoltaic cells	r → e	20–30
Best large steam engines	c → m	20–25
Internal combustion engines	c → m	15–25
High-pressure sodium lamps	e → r	15–20
Mammalian muscles	c → m	15–20
Traditional stoves	c → t	10–15
Fluorescent lights	e → r	10–12
Steam locomotives	c → m	3–6
Peak crop photosynthesis	r → c	4–5
Incandescent light bulbs	e → r	2–5
Paraffin candles	c → r	1–2
Most productive ecosystems	r → c	1–2
Global photosynthetic mean	r → c	0.3

Energy labels: c—chemical, e—electrical, m—mechanical (kinetic), r—radiant (electromagnetic, solar), t—thermal

From V. Smil