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## A simple carbon cycle representation for economic and policy analyses

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**Abstract** Integrated Assessment Models (IAMs) that couple the climate system and the economy require a representation of ocean CO<sub>2</sub> uptake to translate human-produced emissions to atmospheric concentrations and in turn to climate change. The simple linear carbon cycle representations in most IAMs are not however physical at long timescales, since ocean carbonate chemistry makes CO<sub>2</sub> uptake highly nonlinear. No linearized representation can capture the ocean's dual-mode behavior, with initial rapid uptake and then slow equilibration over ~10,000 years. In a business-as-usual scenario followed by cessation of emissions, the carbon cycle in the 2007 version of the most widely used IAM, DICE (Dynamic Integrated model of Climate and the Economy), produces errors of ~ 2°C by the year 2300 and ~ 6°C by the year 3500. We suggest here a simple alternative representation that captures the relevant physics and show that it reproduces carbon uptake in several more complex models to within the inter-model spread. The scheme involves little additional complexity over the DICE model, making it a useful tool for economic and policy analyses.

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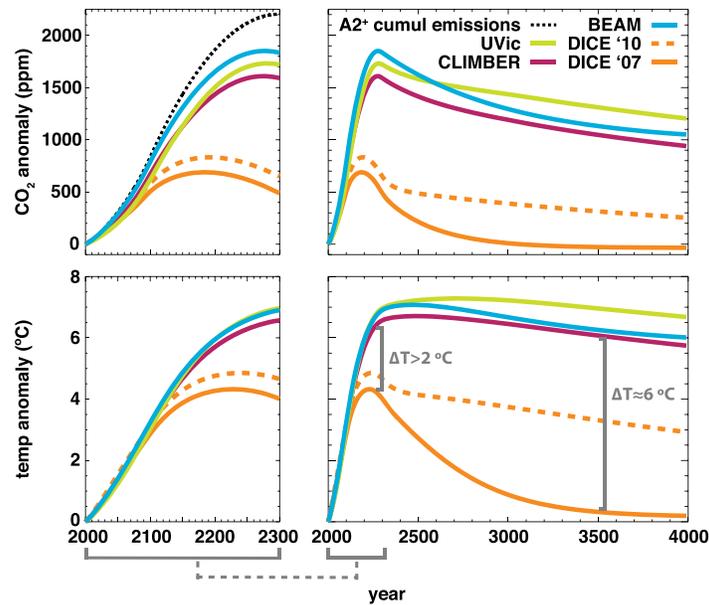
## 1 Introduction

All Integrated Assessment Models (IAMs) that couple the climate system and the economy to evaluate the impacts of climate change require some representation of the global carbon cycle. Anthropogenic climate change is driven primarily by CO<sub>2</sub> emissions produced by human economic activity, but CO<sub>2</sub> does not simply accumulate in the atmosphere, and instead evolves according to the balance between emissions and ocean uptake. IAMs must represent this uptake to translate emissions into atmospheric CO<sub>2</sub> concentrations and in turn climate change. Because state-of-the-art climate models are too computationally expensive for use in economic analyses, IAMs use simplified representations of the physical climate system. (See e.g. Goodess et al. 2003). Care must be taken however to ensure that simplifications do not produce inaccurate behavior that can affect damage estimates.

Many simple IAMs use a linearized representation of ocean carbon uptake (for review, see Hof et al. 2012; van Vuuren et al. 2011), but linearized representations cannot fully reproduce the ocean response to increased atmospheric CO<sub>2</sub>: rapid initial uptake followed by a slow ‘long-tail’ equilibration stage. In state-of-the-art climate models, about half of a CO<sub>2</sub> perturbation is lost in 30 years (Denman et al. 2007) but a significant portion of the remainder persists for thousands of years (Archer et al. 2009). Linearized models that match initial carbon uptake will necessarily produce too-rapid removal of atmospheric CO<sub>2</sub> over the long term and underestimate aggregate climate impacts. In this manuscript we consider the carbon cycle representation in the most widely used simple IAM, DICE (Dynamic Integrated model of Climate and the Economy) (Nordhaus, 1993, 2008, 2010). In a business-as-usual scenario to 2300 followed by cessation of emissions, DICE (2007) produces errors in CO<sub>2</sub> of  $\sim 1000$  ppm and in temperature of  $\sim 6^\circ\text{C}$  on millennial timescales, relative to output of more complex models (Fig. 1; see van Vuuren et al. (2011) for comparison of many IAMs).

The main cause of nonlinear ocean uptake is carbonate chemistry, well-understood since the 1950’s (Revelle and Suess (1957), or see e.g. Sarmiento and Gruber (2006) for review). After any increase in atmospheric CO<sub>2</sub>, uptake will occur until the atmosphere and ocean reach equilibrium. In a simple solution equilibrium, the number of gas-phase molecules leaving the atmosphere would equal the number appearing in solution. In the present-day ocean, about ten times as many CO<sub>2</sub> molecules are removed from the atmosphere as the corresponding increase in aqueous CO<sub>2</sub>, because dissolved inorganic carbon species are partitioned between CO<sub>2</sub>, bicarbonate (HCO<sub>3</sub><sup>-</sup>), and carbonate (CO<sub>3</sub><sup>2-</sup>) in proportions fixed by the ocean’s acidity. Uptake becomes nonlinear because it increases acidity, slowing further uptake by reducing the ocean’s ability to store carbon. Models without this nonlinear chemistry cannot reproduce removal of atmospheric CO<sub>2</sub> over multi-century timescales.

Deficiencies in simple IAM carbon cycle parameterizations have been frequently discussed (Schultz and Kasting 1997; Joos et al. 1999; Warren et al. 2010; Hof et al. 2012), and authors have called for improved representations that better capture known physics (e.g. van Vuuren et al. 2011). One suggested carbon cycle approach that is satisfactory for some purposes is representing CO<sub>2</sub> uptake as a combination of multiple exponential decays (Maier-Reimer and Hasselmann 1987; Joos et al. 2013). However, while multi-exponential representations can reproduce long-term CO<sub>2</sub> uptake for a given emissions scenario, they are not robust across different emissions scenarios. Because CO<sub>2</sub> uptake is a function of acidity, decay timescales depend on the magnitude of CO<sub>2</sub> perturbations (Archer et al. 2009; van Vuuren et al. 2011) and on background concentrations (Joos et al. 2013).



**Fig. 1** CO<sub>2</sub> (*top*) and temperature (*bottom*) anomalies for BEAM and DICE compared to the intermediate complexity models CLIMBER-2 and UVic for the A2<sup>+</sup> scenario (all described in Section 5). BEAM CO<sub>2</sub> matches output of the more complex models well for the duration of the simulation. DICE performs well only for the first several decades but then diverges rapidly. Dotted black line shows cumulative emissions (the CO<sub>2</sub> anomaly if no ocean uptake occurred). DICE removes nearly all emitted CO<sub>2</sub> after several hundred years; in more realistic models, half persists for millennia

Carbon cycle errors can significantly affect IAM policy recommendations (e.g. Joos et al. 1999). In estimations of the social cost of carbon, the choice of carbon cycle representation can produce differences of a factor of two. (See Section 6 and Online Resources Section 3.) Carbon cycle errors become still more significant if climate damages are assumed larger (e.g. Moyer et al. 2014).

To provide a more robust tool for use in IAMs, we describe here a simplified global atmosphere-ocean carbon cycle representation that we term the ‘Bolin and Eriksson Adjusted Model’ (hereafter ‘BEAM’). The scheme provides a computationally inexpensive means of capturing the known nonlinear chemistry of ocean carbon uptake. In the remainder of the manuscript, we describe the BEAM model and compare it to carbon cycle representations in DICE and in Earth system models of intermediate complexity (EMICs). Equations, parameter values, and initial conditions are listed in full in Appendices A.1, A.2 and A.3.

## 2 DICE carbon model

The DICE carbon cycle is a simple box diffusion model of the atmosphere, upper ocean, and lower ocean that assumes constant fractional transfer of CO<sub>2</sub> from each of the three reservoirs. After any CO<sub>2</sub> perturbation, concentrations reach equilibrium when the reservoir concentrations have adjusted so that their transfers are equal. Transfers are described in a system of three linear first-order ordinary differential equations (1–3), which in DICE

are specified discretely assuming 10-year timesteps. All versions of DICE use this same functional form, but parameter values have changed across versions.

$$M_{AT}(t) = CE(t) + \phi_{11} \cdot M_{AT}(t-1) + \phi_{21} \cdot M_{UP}(t-1) \quad (1)$$

$$M_{UP}(t) = \phi_{12} \cdot M_{AT}(t-1) + \phi_{22} \cdot M_{UP}(t-1) + \phi_{32} \cdot M_{LO}(t-1) \quad (2)$$

$$M_{LO}(t) = \phi_{23} \cdot M_{UP}(t-1) + \phi_{33} \cdot M_{LO}(t-1) \quad (3)$$

where the  $\phi_{ij}$ s are transfer coefficients,  $CE(t)$  is cumulative CO<sub>2</sub> emissions over timestep  $t$ , and  $M_{AT}$ ,  $M_{UP}$ , and  $M_{LO}$  are the mass of inorganic carbon (in gigatons) in the atmosphere, upper, and lower ocean, respectively. DICE does not explicitly describe the speciation of inorganic carbon, but the ocean reservoirs  $M_{UP}$  and  $M_{LO}$  can be thought of as comprising dissolved CO<sub>2</sub>, HCO<sub>3</sub><sup>-</sup>, and CO<sub>3</sub><sup>2-</sup>.

Because the model has only four distinct fluxes between reservoirs, only four of the seven transfer coefficients are independent. (For example,  $(1 - \phi_{11}) \cdot M_{AT}$  is the loss of carbon from the atmosphere to the upper ocean in a given timestep, and  $\phi_{12} \cdot M_{AT}$  is that same carbon arriving in the ocean.) To conserve mass, coefficients must be related by:

$$\begin{aligned} \text{atmosphere to upper ocean:} & \quad \phi_{12} & = & (1 - \phi_{11}) \\ \text{upper ocean to atmosphere and lower ocean:} & \quad \phi_{21} + \phi_{23} & = & (1 - \phi_{22}) \\ \text{lower ocean to upper ocean:} & \quad \phi_{32} & = & (1 - \phi_{33}) \end{aligned}$$

### 3 BEAM carbon model

The 3-reservoir carbon cycle model was first outlined by Bolin and Eriksson (1959), who used a 2-layer ocean specification from Craig (1957) and derived transfer coefficients for CO<sub>2</sub> anomalies by considering carbonate chemistry. Bolin and Eriksson (1959) used fixed parameter values, but their model can be readily extended to allow coefficients to change with CO<sub>2</sub> uptake. The three differential equations of BEAM carbon transfer are essentially identical to those of DICE:

$$\frac{dM_{AT}}{dt} = E(t) - k_a \cdot (M_{AT} - A \cdot B \cdot M_{UP}) \quad (4)$$

$$\frac{dM_{UP}}{dt} = k_a \cdot (M_{AT} - A \cdot B \cdot M_{UP}) - k_d \cdot \left( M_{UP} - \frac{M_{LO}}{\delta} \right) \quad (5)$$

$$\frac{dM_{LO}}{dt} = k_d \cdot \left( M_{UP} - \frac{M_{LO}}{\delta} \right) \quad (6)$$

where  $E(t)$  is the emissions rate. The four independent parameters describing the fluxes between reservoirs are now the two  $k_i$ s, which are inverse exchange timescales between atmosphere-ocean ( $k_a$ ) and upper-lower ocean ( $k_d$ ), and two dimensionless parameters:  $\delta$ , the ratio of lower to upper ocean volume ( $\sim 50$ ), and  $A \cdot B$ , the equilibrium ratio of atmospheric to upper ocean inorganic carbon. The term  $M_{AT} - A \cdot B \cdot M_{UP}$  is the disequilibrium between atmospheric and ocean inorganic carbon, eroded with time constant  $1/k_a$ ; and  $M_{UP} - M_{LO}/\delta$  is the disequilibrium between upper and lower ocean inorganic carbon, eroded with time constant  $1/k_d$ .

The only variation in coefficients over time occurs in  $A \cdot B$ , which we separate to distinguish factors with different dependences on environmental conditions.  $A$  is the ratio of atmosphere to ocean CO<sub>2</sub> concentration at equilibrium, which is weakly dependent on temperature: a warmer ocean holds less dissolved CO<sub>2</sub>.  $B$  is the ratio of dissolved CO<sub>2</sub> to total ocean inorganic carbon at equilibrium (see Section 4), a strong function of acidity:

**Table 1** Equivalence of coefficients in DICE and BEAM and corresponding timescales (inverses of transfer coefficients)

Equiv. coefficients		Timescales (years)		
DICE	BEAM	$\tau_{\text{DICE '07 ('10)}}$	$\tau_{\text{BEAM,1994}}$	$\tau_{\text{BEAM,2200}}$
$\phi_{12}$	$k_a$	50 (80)	5	5
$\phi_{21}$	$k_a \cdot A \cdot B$	100 (200)	4.5	1.1
$\phi_{23}$	$k_d$	200 (2,000)	20	20
$\phi_{32}$	$k_d/\delta$	3,000 (13,000)	1000	1000
<i>BEAM params from model year</i>		<i>1800</i>	<i>1994</i>	<i>2200</i>
<i>A</i>		220	150	180
<i>1/B</i>		220	130	31
<i>A · B</i>		1.0	1.1	5.7

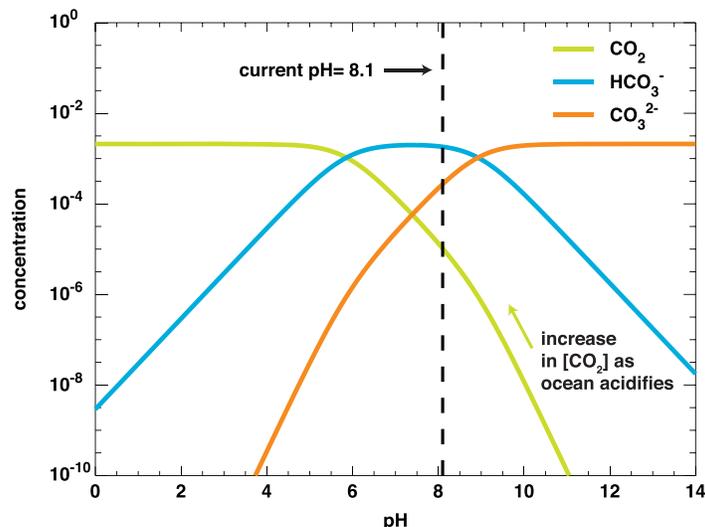
Because its parameters cannot evolve in time, DICE in both 2007 and 2010 versions approximates real-world CO<sub>2</sub> uptake primarily by lengthening exchange timescales. All parameters are shown to 1 or 2 significant figures; see Appendix A.2 to derive BEAM parameters more precisely. DICE timescales are the inverse model parameters x10 since the model uses 10 year timesteps. Timescales are consistent to 1 significant figure with those in a continuous representation

more acidic seawater stores less inorganic carbon. Variation in  $B$  in particular alters uptake rates dramatically. In the business-as-usual scenario of Fig. 1, atmospheric CO<sub>2</sub> rises 5.5 times over present-day concentrations by year 2200, but acidification and warming simultaneously raise  $A \cdot B$  by a factor of 5.2. That is, the ocean's ability to hold inorganic carbon relative to atmospheric CO<sub>2</sub> drops nearly as fast as atmospheric CO<sub>2</sub> rises. This near-cancellation lowers average carbon uptake  $\sim 10$  times below that expected from initial coefficient values.

DICE, with fixed parameters, could not match present-day atmospheric CO<sub>2</sub> uptake rates were it to use realistic values for reservoir sizes and exchange timescales. To approximate uptake slowed by changing acidity, DICE raises exchange timescales substantially above plausible physical values (Table 1). Real-world coefficients continue to evolve in an acidifying ocean, however, so DICE is unable to reproduce uptake rates over all time. In practice, DICE parameter values are set by fitting against relatively short simulations.<sup>1</sup>

Note that BEAM represents ocean CO<sub>2</sub> uptake only, and does not seek to capture the uncertain effects of the terrestrial biosphere. The terrestrial biosphere can impact the carbon cycle in competing ways: CO<sub>2</sub> fertilization generally increases terrestrial CO<sub>2</sub> uptake (e.g. Arora et al. 2013; Joos et al. 2001), but anthropogenic land use change is a source of CO<sub>2</sub> emissions (e.g. Brovkin et al. 2013). The net effect is at present uncertain even in sign (e.g. Sitch et al. 2005; Jones et al. 2013). If the user wants to include the terrestrial carbon cycle, one possibility is to use the parameterization suggested by Joos et al. (2013).

<sup>1</sup>DICE carbon cycle parameters are tuned to match those of another simple model (MAGICC) using an emissions trajectory from 1750-2100 that is a combination of historical emissions and the Intergovernmental Panel on Climate Change (IPCC) A1FI scenario (Nordhaus 2007, 2008). The MAGICC carbon cycle includes ocean uptake described with a multi-exponential function and a four-box model representing the terrestrial carbon cycle. MAGICC parameters are themselves calibrated to output from the C<sup>4</sup>MIP carbon-cycle intercomparison project (Wigley 2008; Meinshausen et al. 2011).



**Fig. 2** Partitioning of dissolved inorganic carbon species in seawater (DIC) as a function of pH. For illustrative purposes, DIC is held constant at  $2100 \mu\text{mol/kg}$  (Zeebe and Wolf-Gladrow 2001). At present, bicarbonate dominates DIC and the carbon storage factor is  $\sim 170$ . In a more acidic ocean (*lower pH*),  $\text{CO}_2$  becomes more significant and the carbon storage factor drops. At  $\text{pH} < 5$  (1000x increase in  $[\text{H}^+]$  over present-day conditions),  $\text{CO}_2$  dominates and the carbon storage factor approaches 1. Present-day seawater contains strong bases that raise pH; pure water interacting with current atmospheric  $\text{CO}_2$  would have  $\text{pH} \sim 5.6$

#### 4 Carbonate chemistry in BEAM

*Ocean carbonate chemistry* Aqueous carbonate chemistry is well understood; we review it briefly only to explain its treatment in BEAM. See any aqueous chemistry textbook (e.g. Sarmiento and Gruber 2006) for more thorough review. Uptake of  $\text{CO}_2$  proceeds when concentrations in the atmosphere and upper ocean are out of equilibrium ( $M_{AT} \neq A \cdot B \cdot M_{UP}$ ). As  $\text{CO}_2$  dissolves, we assume instantaneous repartitioning of inorganic carbon species:



Partitioning is set by the dissociation coefficients  $k_1$  and  $k_2$  and the concentration of hydrogen ions  $[\text{H}^+]$ , i.e. the acidity (pH) of seawater.<sup>2</sup> The ‘carbon storage factor’  $1/B$ , the equilibrium ocean total inorganic carbon relative to dissolved  $\text{CO}_2$ , is then:

$$\frac{1}{B} = \frac{[\text{CO}_2] + [\text{HCO}_3^-] + [\text{CO}_3^{2-}]}{[\text{CO}_2]} = 1 + \frac{k_1}{[\text{H}^+]} + \frac{k_1 k_2}{[\text{H}^+]^2} \quad (8)$$

Higher acidity (higher  $[\text{H}^+]$ , lower pH) reduces the ocean’s ability to store carbon (Fig. 2). As Eq. 7 indicates, dissolved  $\text{CO}_2$  itself acts as a weak acid. Any ocean uptake of  $\text{CO}_2$  therefore intrinsically reduces the efficiency of future uptake.

Solving for  $[\text{H}^+]$  is complicated by the fact that ocean acidity is not simply governed by atmospheric  $\text{CO}_2$ . Seawater contains strong bases (and acids) whose combined effect raises pH above that for a pure water/ $\text{CO}_2$  system. The carbonate system then acts as a buffer against further changes in acidity. To estimate those changes, we assume constant

<sup>2</sup>The pH scale is  $-\log_{10}([\text{H}^+])$ , i.e. the ocean’s current pH of  $\sim 8.1$  means that  $[\text{H}^+] = 10^{-8.1} \text{ mol/kg}$ .

**Table 2** Representative values for the temperature-dependent BEAM parameters  $1/A$ ,  $k_1$ , and  $k_2$  at selected temperatures

Constant	10 °C	12 °C	15 °C	Percentage change 10-15 °C
$1/A$	$7.02 \cdot 10^{-3}$	$6.58 \cdot 10^{-3}$	$6.00 \cdot 10^{-3}$	–15 %
$k_1$	$8.00 \cdot 10^{-7}$	$8.30 \cdot 10^{-7}$	$8.75 \cdot 10^{-7}$	9 %
$k_2$	$4.63 \cdot 10^{-10}$	$4.94 \cdot 10^{-10}$	$5.47 \cdot 10^{-10}$	18 %

Note that the dissociation and solubility effects act in opposite directions on ocean carbon storage

acid-neutralizing capability, or “alkalinity”, approximated as the amount of  $H^+$  that would have to be added to convert all bicarbonate and carbonate to  $CO_2$ :

$$Alk = [HCO_3^-] + 2[CO_3^{2-}] = \left( \frac{k_1}{[H^+]} + \frac{2 \cdot k_1 \cdot k_2}{[H^+]^2} \right) \cdot M_{UP} \cdot B \quad (9)$$

where  $M_{UP} \cdot B$  is the concentration of upper ocean  $CO_2$ . We determine  $Alk$  by assuming equilibrium in the pre-industrial ocean with  $pH=8.29$  (see Appendix A.3). BEAM solves for  $[H^+]$  at each timestep using Eqs. 8–9 and assuming constant alkalinity.<sup>3</sup> Constant alkalinity is a reasonable assumption for several thousand years (Archer et al. 2009; Zeebe and Wolf-Gladrow 2001). On long timescales, dissolution of calcium carbonate would help return  $pH$  to its original value, increasing drawdown of atmospheric  $CO_2$ . BEAM will therefore underpredict  $CO_2$  uptake on  $\sim 10,000$ -year timescales.

*Temperature dependence of coefficients* The discussion above concerned only changes in ocean carbon storage capacity due to changing acidity. To a lesser degree, temperature affects both the carbon storage factor  $1/B$  and  $CO_2$  solubility  $1/A$ . The magnitudes of temperature-induced changes are absolutely smaller than acidity effects, and temperature-dependent changes in  $A$  and  $B$  partially counteract each other: the solubility of  $CO_2$  decreases in a warmer ocean, but the dissociation constants  $k_1$  and  $k_2$  grow with temperature, raising the carbon storage factor (Table 2). (See Appendix A.2 for equations.)

## 5 Model validation and comparison to DICE

Several published experiments comparing carbon cycle representations in Earth system models of intermediate complexity (EMICs) allow us to validate BEAM. The recent EMIC intercomparison (Eby et al. 2013) designed for the IPCC Fifth Assessment Report (IPCC 2013) allows comparison over the historical period.<sup>4</sup> BEAM output is consistent with the range of  $CO_2$  anomalies from seven EMICs over this period. (See Online Resources Section 1.) The most rigorous test of a carbon cycle is however a comparison of long-term behavior after significant anthropogenic  $CO_2$  emissions.

<sup>3</sup>The assumption of a static equilibrium is not strictly true, since the ocean’s “biological pump” can produce short-term variations in alkalinity (e.g. Gangstø et al. 2011), but their effect on  $CO_2$  uptake is small.

<sup>4</sup>The historical period is defined as 850-2005 C.E., but emissions are significant only in the last 200 years.

For such a long-term validation, we use the study of Montenegro et al. (2007), who compared the University of Victoria Earth system climate model (UVic) (Eby et al. 2009) and the CLIMate and BiosphERE 2 model (CLIMBER-2) (Petoukhov et al. 2000). (See Online Resources Section 1 for discussion of limitations of other intercomparisons.) The UVic model consists of a full three-dimensional, 19-layer ocean model and an energy-moisture balanced model of the atmosphere (Weaver et al. 2001). CLIMBER-2 consists of a simpler three-reservoir ocean that includes biogeochemistry (Brovkin et al. 2002) and sedimentation (Archer et al. 1998) coupled to a two-dimensional atmospheric model. In comparisons shown here, models were forced with an A2<sup>+</sup> CO<sub>2</sub> emissions scenario, which reproduces the business-as-usual SRES A2 scenario for 100 years (Nakicenovic et al. 2000) followed by linearly declining emissions that reach zero after another 200 years, for a cumulative emissions total of 5134 GtC.<sup>5</sup> Both models were run for 10,000 years (starting at pre-industrial using historical emissions), although because BEAM does not include long-term adjustment of alkalinity, we compare here only the first 2000 years.

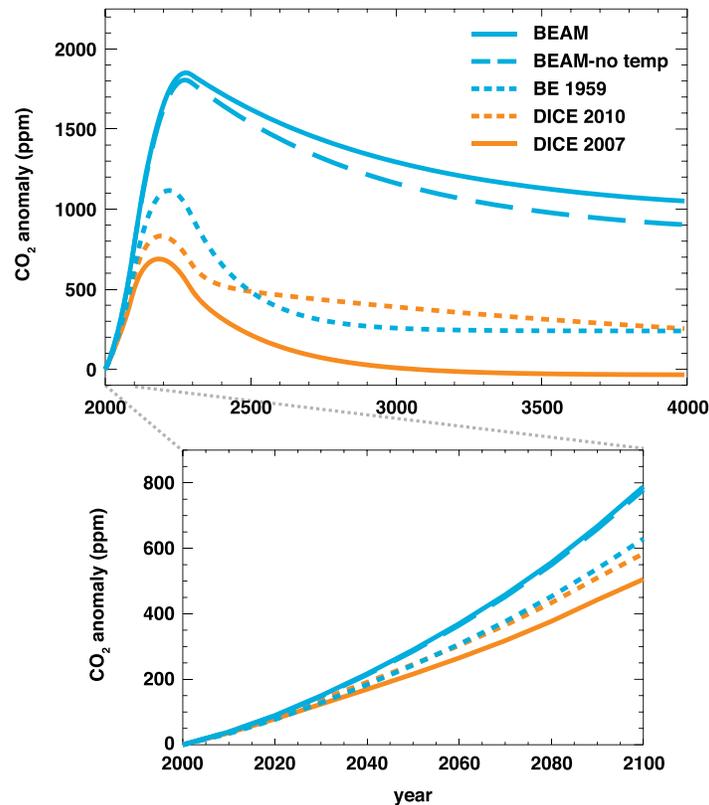
CO<sub>2</sub> evolution in BEAM is consistent with that in UVic and CLIMBER-2 throughout the simulation period (Fig. 1). CO<sub>2</sub> evolution in DICE is consistent for only the first few decades. By 100 years, DICE atmospheric CO<sub>2</sub> is markedly lower than that of the other models (Fig. 1, left). After 300 years, when emissions cease, rapid drawdown of atmospheric carbon in DICE 2007 returns climate to near pre-industrial levels, while BEAM and the intermediate complexity models retain persistent high CO<sub>2</sub> and temperatures elevated by ~6–7 °C for millennia (Fig. 1, right). (DICE 2010 retains a moderate anomaly.) By the end of the simulation period, BEAM does begin to diverge in behavior from the more realistic models: BEAM CO<sub>2</sub> concentrations begin to asymptote while slow CO<sub>2</sub> uptake continues in the intermediate complexity models. For very long simulations, the assumption of constant alkalinity in BEAM would have to be relaxed to allow restoration of pH and continued uptake. (One possible parametrization is that suggested by Kheshgi and Archer 2004.)

The dominant driver of difference between DICE and the more realistic carbon cycle models, including BEAM, is the changing carbon partitioning as the ocean acidifies. Figure 3 shows atmospheric CO<sub>2</sub> evolution for 2007 and 2010 DICE compared to BEAM with and without temperature-dependent coefficients. Omitting the temperature dependence of CO<sub>2</sub> solubility and dissociation constants produces negligible changes for the first few hundred years and only small changes even at millennial timescales.

For perspective, Fig. 3 also shows the original Bolin and Eriksson (1959) linearized model, which is similar to 2010 DICE. While 2007 DICE quickly draws down any CO<sub>2</sub> perturbation to near pre-industrial levels, both 2010 DICE and the Bolin and Eriksson representation retain some anthropogenic CO<sub>2</sub> in the atmosphere for the duration of the simulation. Both achieve this in part by effectively reducing the total ocean volume. The equilibrium ratio of total ocean to atmospheric carbon,  $(\delta + 1)/A \cdot B$ , is ~46 in present-day BEAM and ~32 in 2007 DICE, but only ~20 in 2010 DICE and ~5 in Bolin and Eriksson (1959).<sup>6</sup>

<sup>5</sup>Montenegro et al. (2007) used an older version of historical emissions; BEAM calibration is based on more recent emissions estimates. See Online Resources Section 2 for discussion.

<sup>6</sup>Although 2010 DICE and Bolin and Eriksson (1959) are mathematically equivalent, they are not exactly equivalent in intent: Bolin and Eriksson described evolution of CO<sub>2</sub> anomalies rather than total reservoirs. The reduction in ocean carbon content in 2010 DICE exceeds that produced by any plausible choice of pH.



**Fig. 3** Atmospheric CO<sub>2</sub> anomaly for different versions of BEAM and DICE carbon cycle models for the A2<sup>+</sup> scenario. *BEAM* (solid blue) is the full model described here; *BEAM-no temp* (dashed blue) omits temperature dependence; and *BE 1959* (dotted blue) is our coding of the original linear Bolin and Eriksson model. Temperature dependence in BEAM coefficients has a relatively minor effect and some users may wish to neglect it. Long-term, 2010 DICE retains more CO<sub>2</sub> in the atmosphere than does 2007 DICE because it has a longer ocean equilibration timescale and effectively a smaller ocean. Atmospheric CO<sub>2</sub> in the Bolin and Eriksson model is fully equilibrated by the end of the simulation; in 2010 DICE it is still evolving toward a lower equilibrium level with a time constant of ~13,000 years

## 6 BEAM in economic and policy analyses

Replacing the DICE carbon cycle with BEAM can significantly alter economic projections of the cost of climate change, especially for analyses involving longer timescales. We estimate the importance of BEAM to one measure of the economic impact of climate change, the social cost of carbon (SCC), used in cost-benefit analysis of federal regulations that affect greenhouse gases. The SCC is the present value of the harms (over some integration time) of an additional ton of emitted CO<sub>2</sub>. It is calculated by running a business-as-usual case, repeating the run with an additional ton of CO<sub>2</sub> emissions in one year, calculating the difference in consumption over the modeled period, and discounting those harms back to present value. We estimate SCC in DICE with and without substitution of BEAM<sup>7</sup>, using the DICE emissions assumptions over the modeled 600-year period rather than the A2+ emissions scenario used in previous examples. As before, BEAM leaves more anthropogenic

<sup>7</sup>For simplicity, we use the version of BEAM with no temperature dependence. (See Fig. 3.)

CO<sub>2</sub> in the atmosphere, producing temperature anomalies higher by  $\sim 2\text{--}3^\circ\text{C}$  in the year 2300 (Online Resource Fig. 4, compare to Fig. 1). Those higher temperatures produce larger climate harms, raising the year-2005 SCC by 60 % in 2007 DICE and 40 % in 2010 DICE (Online Resource Table 1).

The accuracy of the long-term carbon cycle is still more important in analyses that weight future harms more heavily. In standard DICE, continued economic growth under climate change leads to an implied discount rate that is relatively high (mean 2.5 % and 2.3 %/year over 600 years for 2007/2010 DICE). If discount rates are lower, either because of the choice of discounting parameters or because future climate harms are larger, substituting BEAM for less-accurate carbon cycle models alters results more strongly. We give several examples in Online Resource Table 1. With alternative discount parameters that produce mean discount rates of 0.5 and 0.6 %/year in 2007/2010 DICE<sup>8</sup>, the use of BEAM raises SCC estimates not by 60 and 40 % but by 160 and 80 %. A similar increase results if damages are assumed to follow a cubic rather than quadratic dependence on temperature increase (following Ackerman et al. 2010). If climate damages are allowed to significantly reduce economic growth, use of BEAM can have order of magnitude effects on SCC estimates (Moyer et al. 2014).

## 7 Conclusions

Many authors have pointed out that the carbon cycle representations in DICE and other simple IAMs do not accurately reproduce the response of more physical models on centennial timescales (e.g. van Vuuren et al. 2011; Hof et al. 2012). We show here that DICE performance worsens still further over time. While recent updates of the DICE carbon cycle reduce discrepancies somewhat, all versions of DICE diverge from predicted real-world behavior within decades. Our analysis here confirms that discrepancies occur because linearized models cannot capture the changing ocean carbon storage potential due to changing ocean acidity. We also show that these discrepancies can be largely eliminated by adding to the DICE framework a single equation describing acidity evolution. While accurate representation of the long-term carbon cycle is most critical in economic analyses with long time horizons and low discount rates, carbon cycle errors can affect policy recommendations even in standard modeling frameworks. The BEAM model offers a simple, computationally tractable carbon cycle representation that retains fidelity over millennial timescales.

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<sup>8</sup>DICE uses Ramsey discounting (discount rate  $r = \eta \cdot g + \rho$ , where  $g$  is the growth rate), with  $\rho$  (the pure rate of time preference) at 1.5 %/year and  $\eta$  (the elasticity of the marginal utility of consumption) at 2 and 1.5 for 2007/2010 DICE, respectively. In our alternate example we assume  $\rho=0$  and  $\eta=1$ , similar to Stern (2008).

## Appendix A: BEAM model equations, parameter values, and initial conditions

### A.1 Equations

BEAM consists of four independent equations: three that track total carbon in each layer (atmosphere, upper, and lower ocean- Eq. 10), and one that tracks acidity (Eq. 13). Carbon transfers in BEAM are described by:

$$\frac{d}{dt} \begin{pmatrix} M_{AT} \\ M_{UP} \\ M_{LO} \end{pmatrix} = \begin{pmatrix} -k_a & k_a \cdot A \cdot B & 0 \\ k_a & -(k_a \cdot A \cdot B) - k_d & \frac{k_d}{\delta} \\ 0 & k_d & -\frac{k_d}{\delta} \end{pmatrix} \begin{pmatrix} M_{AT} \\ M_{UP} \\ M_{LO} \end{pmatrix} + E(t) \quad (10)$$

where the  $M_i$ s represent the mass of carbon (in CO<sub>2</sub> or dissolved inorganic carbon) in the atmosphere ( $_{AT}$ ), upper ocean ( $_{UP}$ ), and lower ocean ( $_{LO}$ ); and  $E(t)$  is rate of anthropogenic CO<sub>2</sub> emissions. (Emissions units must match those of concentrations and are therefore specified in mass of carbon in CO<sub>2</sub>.)

The parameter  $A$  is the ratio of mass of CO<sub>2</sub> in atmospheric to upper ocean dissolved CO<sub>2</sub>, i.e.  $A$  is inversely proportional to CO<sub>2</sub> solubility. Solubility is set by ‘Henry’s law’, which prescribes that in equilibrium, the concentrations of CO<sub>2</sub> in the atmosphere and ocean are related by a coefficient dependent only on temperature. Henry’s Law may be written in various forms; for convenience we define the coefficient  $k_H$  as a dimensionless ratio of the molar concentrations of CO<sub>2</sub> in atmosphere and ocean. The parameter  $A$  is then

$$A = k_H \cdot \frac{AM}{OM/(\delta + 1)} \quad (11)$$

where  $AM$  are  $OM$  are the number of moles in the atmosphere and ocean, respectively, and  $OM/(\delta + 1)$  signifies the upper ocean only.  $B$  is the ratio of dissolved CO<sub>2</sub> to total oceanic carbon, a function of acidity:

$$B = \frac{1}{1 + \frac{k_1}{[H^+]} + \frac{k_1 k_2}{[H^+]^2}} \quad (12)$$

where  $k_1$  and  $k_2$  are dissociation constants. Alkalinity,  $Alk$ , is used to determine  $[H^+]$  by solving the quadratic:

$$\frac{M_{UP}}{Alk} = \frac{1 + \frac{k_1}{[H^+]} + \frac{k_1 k_2}{[H^+]^2}}{\frac{k_1}{[H^+]} + \frac{2k_1 k_2}{[H^+]^2}} \quad (13)$$

Three parameters are temperature-dependent ( $k_1$ ,  $k_2$ , and  $k_H$ , Eqs. 14–16). Because the temperature dependence has a relatively minor aggregate effect, however, some users may wish to omit it.

### A.2 Parameter values

Recommended BEAM parameter values are given in Table 3. Most are well established. Time constants  $k_a$ ,  $k_d$ , and the ratio of upper to lower ocean ( $\delta$ ) are not well constrained; we use reasonable values from Bolin and Eriksson (1959). We determine alkalinity by assuming equilibrium in the pre-industrial ocean at pH=8.29 (see Appendix A.3). It is also possible to specify  $Alk$  and adjust pre-industrial pH,  $k_a$ ,  $k_d$ , and  $\delta$  for best fit to more complex models.

To include temperature-dependent effects, replace  $k_1$ ,  $k_2$ , and  $k_H$  with their temperature-dependent forms of Eqs. 14–16. Following Archer et al. (2004), we assume the global ocean temperature anomaly is equal to the anomaly in mean surface temperature, so this anomaly

**Table 3** Recommended BEAM parameters

Parameter	Representative value	Source
$k_a$ (years <sup>-1</sup> )	.2	Bolin and Eriksson (1959)
$k_d$ (years <sup>-1</sup> )	.05	Bolin and Eriksson (1959)
$\delta$	50	Bolin and Eriksson (1959)
$k_H$	$1.23 \cdot 10^3$	Weiss (1974)
$k_1$ (mol/kg)	$8.00 \cdot 10^{-7}$	Mehrbach et al. (1973)
$k_2$ (mol/kg)	$4.53 \cdot 10^{-10}$	Mehrbach et al. (1973)
AM (mol)	$1.77 \cdot 10^{20}$	Warneck (1999)
OM (mol)	$7.8 \cdot 10^{22}$	Garrison (2009) – (for conversion see <sup>a</sup> )
Alk	767.0 Gt C	(for conversion to $\mu\text{mol}/\text{kg}$ see <sup>b</sup> )

$$^a OM \approx 1.37 \cdot 10^9 \text{ km}^3 \text{ ocean} \cdot \frac{10^{15} \text{ cm}^3}{\text{km}^3} \cdot \frac{1.027 \text{ g seawater}}{\text{cm}^3 \text{ seawater}} \cdot \frac{1 \text{ mol water}}{18 \text{ g}} \approx 7.8 \cdot 10^{22} \text{ moles}$$

$$^b Alk = 767.0 \text{ Gt C} \cdot \frac{1 \text{ mol}}{12 \text{ g}} \cdot \frac{10^{15} \text{ g}}{\text{Gt}} \cdot \frac{1}{OM/(\delta+1)} \cdot \frac{1 \text{ mol water}}{18 \text{ g}} \cdot \frac{1000 \text{ g}}{1 \text{ kg}} \approx 2317 \mu\text{mol}/\text{kg}$$

Temperature-dependent parameters  $k_H$ ,  $k_1$ , and  $k_2$  are calculated here for  $T=10^\circ\text{C}$  (283.15 K), roughly the volume mean ocean temperature. For convenience we state alkalinity in units of Gt C; our value is equivalent to  $\sim 2317 \mu\text{mol}/\text{kg}$ , consistent with standard literature values, e.g.  $\sim 2364 \mu\text{mol}/\text{kg}$  (Sarmiento and Gruber 2006)

is added to the baseline ocean temperature (i.e.  $T=283.15 \text{ K} + \Delta T$ ). This assumption is likely an upper limit for ocean temperature change.

**CO<sub>2</sub> solubility** (Henry's law) (Weiss 1974)

$$k_H = \frac{1}{k_0} \cdot \frac{\text{liter seawater}}{1.027 \text{ kg}} \cdot \left( \frac{55.57 \text{ mol}}{\text{liter}} \right) \quad \text{with} \quad (14)$$

$$k_0 = \exp \left[ \frac{9345.17}{T} - 60.2409 + 23.3585 \cdot \ln \left( \frac{T}{100} \right) + S \cdot \left( .023517 - .00023656 \cdot T + .0047036 \cdot \left( \frac{T}{100} \right)^2 \right) \right]$$

**First and second dissociation constants** (Mehrbach et al. 1973)

$$k_1 = 10^{-pK_1} \quad \text{and} \quad k_2 = 10^{-pK_2}$$

$$\text{with} \quad pK_1 = -13.721 + (0.031334 \cdot T) + \frac{3235.76}{T} + 1.3 \cdot 10^{-5} \cdot S \cdot T - \left( 0.1031 \cdot S^{0.5} \right) \quad (15)$$

$$\text{and} \quad pK_2 = 5371.96 + (1.671221 \cdot T) + (0.22913 \cdot S) + (18.3802 \cdot \log(S)) - \frac{128375.28}{T} \quad (16)$$

$$- (2194.30 \cdot \log(T)) - (8.0944 \cdot 10^{-4} \cdot S \cdot T) - \left( 5617.11 \cdot \frac{\log(S)}{T} \right) + 2.136 \cdot \frac{S}{T}$$

**Table 4** BEAM, DICE initial conditions

	<i>atm. CO<sub>2</sub> (ppm)</i>	<i>M<sub>AT</sub> (GtC)</i>	<i>M<sub>UP</sub> (GtC)</i>	<i>M<sub>LO</sub> (GtC)</i>	<i>pH</i>
BEAM pre-industrial	280	596	713	35,625	8.29
BEAM present-day	380	809	725	35,641	8.18
DICE '07 present-day	380	809	1,255	18,365	(NA)
DICE '10 present-day	370	787	1,600	10,010	(NA)

'Present-day' is year 2005 for DICE 2007 and 2000 for DICE 2010. Note that total ocean carbon content in BEAM is consistent with literature values, e.g.  $\sim 38,000$  GtC (Ciais et al. 2013)

where  $k_0$  has units  $\frac{\text{mol C}}{\text{kg seawater} \cdot \text{atm}}$ ,  $k_H$  is dimensionless,  $k_1$  and  $k_2$  have units  $\text{mol/kg seawater}$ ,  $T$  is temperature (K), and  $S$  is salinity ( $\sim 35$  g/kg seawater).

### A.3 Initial conditions and model implementation

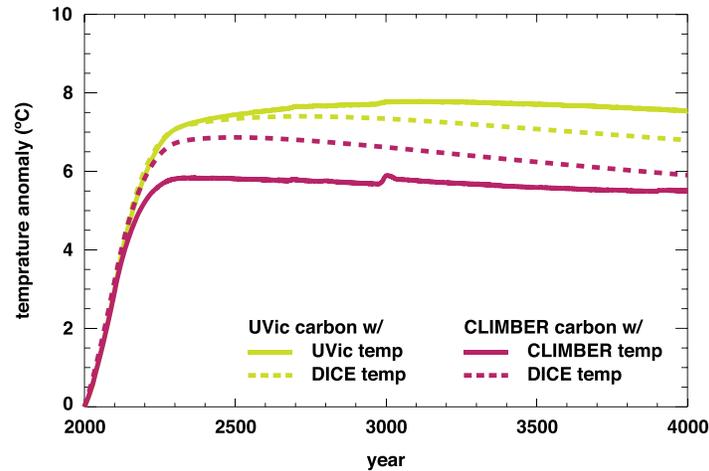
BEAM initial conditions are listed in Table 4. We set pre-industrial ocean carbon content by assuming equilibrium with atmospheric CO<sub>2</sub> at 280 ppm (IPCC 2007) and pH of 8.29. Pre-industrial pH is chosen so that after running forward with historical emissions (Boden et al. 2010), BEAM present-day pH matches that of UVic and CLIMBER-2 ( $\sim 8.16$ , Montenegro et al. (2007)). Matching pH is needed for a valid comparison because pH values markedly affect CO<sub>2</sub> uptake. Both pre-industrial and present-day pH levels are uncertain by  $\pm 0.05$  (Feely et al. 2009). We define 'present-day' as that point where atmospheric CO<sub>2</sub> matches 380 ppm, the 2007 DICE initial condition, which occurs near year 1994 for BEAM. The exact timing is not significant for subsequent CO<sub>2</sub> anomaly evolution.

Because the atmosphere/upper ocean exchange timescale is short in BEAM, the present-day upper ocean is nearly in equilibrium with the atmosphere ( $M_{AT}/M_{UP} \sim A \cdot B \sim 1.1$ ), though the lower ocean is slightly out of equilibrium ( $M_{LO}/M_{UP} \sim 49$  while  $\delta = 50$ ). Both DICE versions begin with excess atmospheric CO<sub>2</sub> relative to the upper ocean. (In 2007 DICE,  $M_{AT}/M_{UP} \sim 0.6$  but  $A \cdot B \sim 0.5$ .) The larger upper ocean carbon reservoir in DICE (2007 and 2010) than in BEAM reflects a larger equilibrium upper ocean/atmosphere carbon ratio and can be thought of as a deeper mixed ocean layer. DICE total ocean carbon is  $\sim 1/2$  the real-world value; this difference can be thought of as a smaller total ocean volume.

The more realistic BEAM representation does have one drawback, that the sensitivity of coefficients mandates fine timesteps to avoid instability during numerical integration. The figures shown here were generated using 0.01 year timesteps. Timesteps as coarse as  $1/10^{\text{th}}$  year can produce oscillation in pH and uptake/release of oceanic CO<sub>2</sub> when emissions change rapidly (e.g. Oeschger et al. 1975). Resulting error in atmospheric CO<sub>2</sub> anomaly for the emissions scenario used here would reach several percent. Note that this instability means that BEAM cannot be used to simulate the response to an abrupt addition of CO<sub>2</sub>. For code of the full BEAM representation, see [www.rdcep.org/carbon-cycle-model](http://www.rdcep.org/carbon-cycle-model).

### Appendix B: Temperature model

For completeness, we describe the 2007 DICE temperature model, which appears to adequately capture temperature evolution (Fig. 4). Just as the ocean takes up CO<sub>2</sub> in response



**Fig. 4** The two-box DICE temperature model appears to adequately capture temperature evolution in more complex models. We drive the 2007 DICE temperature model with atmospheric CO<sub>2</sub> anomalies from UVic (green) and CLIMBER-2 (maroon) from the A2<sup>+</sup> emissions scenario and compare atmospheric temperatures from DICE (dashed) to those models' own temperature representations (solid). DICE temperature evolutions differs somewhat in behavior but lies within the range of uncertainty

to atmospheric CO<sub>2</sub> perturbations, it also takes up heat in response to surface warming, with a long equilibration time because of the large thermal inertia of the ocean. Heat uptake in DICE is represented by a linear model similar to that used for carbon uptake. Because heat uptake is in reality largely linear, this representation adequately reproduces climate behavior. As with the carbon cycle, many of the coefficients in the DICE 2007 temperature model given below are calibrated to the MAGICC model (Wigley et al. 2007) or taken from the IPCC (2001) and IPCC (2007).

The DICE temperature model uses only two layers, the atmosphere and lower ocean; the upper ocean is assumed to follow atmospheric temperature (Eqs. 17–18, but note that we have re-organized parameters to be more intuitive). Radiative forcing  $F$  due to increased atmospheric CO<sub>2</sub> warms the atmosphere (and upper ocean), producing a disequilibrium with the lower ocean that is eroded with timescales  $1/\mu$ :

$$T_{AT}(t) = T_{AT}(t-1) + \mu_{AT} \cdot [\Lambda \cdot (T_{eq}(t) - T_{AT}(t-1)) - \gamma \cdot (T_{AT}(t-1) - T_{LO}(t-1))] \quad (17)$$

$$T_{LO}(t) = T_{LO}(t-1) + \mu_{LO} \cdot \gamma \cdot (T_{AT}(t-1) - T_{LO}(t-1)) \quad (18)$$

where the  $T_i$ s are atmospheric and lower ocean temperature changes (in °C) since pre-industrial times;  $\gamma$  relates atmosphere-ocean heat transfer to temperature anomaly ( $\gamma = 0.3 \text{ W/m}^2/\text{°C}$ );  $\Lambda$  is the general climate sensitivity ( $1.3 \text{ W/m}^2/\text{°C}$ , derived by dividing DICE assumptions of the forcing per doubling of CO<sub>2</sub> ( $\alpha = 3.8 \text{ W/m}^2$ ) by the assumed equilibrium warming after doubling of CO<sub>2</sub> ( $\beta = 3.0 \text{ °C / doubling}$ )); and  $T_{eq}(t)$  is the equilibrium temperature that would be produced by the imposed forcing:  $T_{eq}(t) = F(t)/\Lambda$ .

Forcing  $F(t)$  is assumed to be linear with the logarithm of the fractional change in  $\text{CO}_2$  since pre-industrial times, a standard assumption in climate science<sup>9</sup>:

$$F(t) = \alpha \cdot \log_2(M_{AT}(t)/M_{AT}(PI)) \quad (19)$$

where  $M_{AT}(PI)$  is the mass of pre-industrial atmospheric carbon (596.4 Gt C, equivalent to  $\sim 280$  ppm  $\text{CO}_2$ ).

Note that equilibration timescales for the atmosphere and lower ocean need not be equal since temperature is not a conserved quantity:  $\mu_{AT} = 0.22/10$  years so  $\tau_{AT} \sim 45$  years, while  $\mu_{LO} = (1/6)/10$  years so  $\tau_{LO} \sim 60$  years. The 2010 DICE temperature model uses the same equations with small adjustments to three coefficients:  $\gamma = 0.31 \text{ W/m}^2/\text{°C}$ ,  $\mu_{AT} = 0.208$  ( $\tau_{AT} \sim 48$  years), and  $\beta = 3.2 \text{ °C / doubling}$ . Because DICE 10-year timesteps are long relative to these timescales, DICE coefficient values differ from those of a continuum representation. If the model is rewritten to use 1-year timesteps, Marten and Newbold (2013) recommend using  $\gamma = 0.5072 \text{ W/m}^2/\text{°C}$ ,  $\mu_{AT} = 0.0586$ , and  $\mu_{AT} = 0.018336$  to replicate 2007 DICE.

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<sup>9</sup>In 2007 DICE, radiative forcing is specified as a function of  $\text{CO}_2$  at timesteps ( $t$ ) and ( $t + 1$ ), but that assumption can be relaxed without significant difference. 2007 DICE also adds a constant of  $10^{-6}$  to  $M_{AT}$  to preclude  $F(t) = 0$ , presumably for numerical reasons. We neglect forcing from other greenhouse gases to compare with models driven only by changing  $\text{CO}_2$ , but additional forcings can be added to  $F(t)$ .

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